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Dynamic one-dimensional modeling of secondary settling tanks and design impacts of sizing decisions

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ABSTRACT

As one of the most significant components in the activated sludge process (ASP), secondary settling tanks (SSTs) can be investigated with mathematical models to optimize design and operation. This paper takes a new look at the one-dimensional (1-D) SST model by analyzing and considering the impacts of numerical problems, especially the process robustness. An improved SST model with Yee–Roe–Davis technique as the PDE solver is proposed and compared with the widely used Takács model to show its improvement in numerical solution quality. The improved and Takács models are coupled with a bioreactor model to reevaluate ASP design basis and several popular control strategies for economic plausibility, contaminant removal efficiency and system robustness. The time-to-failure due to rising sludge blanket during overloading, as a key robustness indicator, is analyzed to demonstrate the differences caused by numerical issues in SST models. The calculated results indicate that the Takács model significantly underestimates time to failure, thus leading to a conservative design.

1. Introduction

Biological secondary treatment processes are widely used in wastewater treatment plants to remove organic matter and reduce nutrients such as nitrogen and phosphorus. In all cases, efficient operation requires the sludge to be removed from the wastewater by sedimentation, filtration or other solids–liquid separation processes.

For sedimentation to be successful, the biomass must be composed of large particles or flocs, which have sufficient settling velocity to be removed in a settling tank of manageable size. To achieve this goal, it is necessary to grow the biomass to select floc-forming organisms as well as understanding solids–liquid separation processes (Parker et al., 2004).

Several types of treatment processes can achieve the solids–liquid separation, but secondary settling tanks (SSTs) are most commonly used. SSTs, also known as sedimentation basins or solids–liquid separators, use gravity to separate the biomass from the fluid, and have two similar but distinct functions: clarification and thickening. Clarification is the removal of finely dispersed solids from the liquid to produce a low turbidity effluent; Thickening is the process of increasing the sludge concentration in order for it to be recycled or disposed in less volume. In SSTs, the clarification process occurs in the upper zone while thickening occurs near the bottom. The result is an effluent from the top, low in suspended

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Nomenclature

A	cross-sectional area of SST [m^2]
C	sludge concentration [g/m^3]
C_{min}	non-settleable solids concentration [g/m^3]
C_T	total ASP cost [dollar]
G	flux [$g/(m^2h)$]
G_s	gravity settling flux [$g/(m^2h)$]
h	SST inlet depth [m]
H	SST depth [m]
H_s	Sludge blanket level [m]
n	Veslind settling parameter [m^3/g]
Q	flow rate [m^3/h]
r_h	Takács settling parameter [m^3/g]
r_p	Takács settling parameter [m^3/g]
R_c	relative cost coefficient
S	biodegradable substrate concentration [g/m^3]
v	settling velocity [m/h]
v_0	Veslind settling parameter [m/h]
V	bioreactor volume [m^3]
$v_{0,\ max}$	Takács settling parameter [m/h]
v_s	hindered settling velocity [m/h]
t	time [h]
z	height above SST bottom [m]

Greek letters

Φ	flux [$g/(m^2h)$]
Φ_l	limiting flux [$g/(m^2h)$]
μ/Y	F/M ratio ((kg BOD_5/kg MLSS) d^{-1})

Subscripts

e	effluent
f	feed
i	index of model layer
in	incoming
u	underflow
w	waste

Superscripts

n	index of time

solids, and a second stream of settled, concentrated solids from the bottom, suitable for recycling or disposal.

As one of the most important units in wastewater treatment process, the SST is often a "bottle neck," limiting the capacity of the wastewater treatment process (Ekama et al., 1997; Ekama and Marais, 2002). The SST sizing must be combined with the bioreactor sizing to guarantee the minimum necessary performance to meet the design basis, as well as maintaining required efficiency for contaminant removal. If the SST does not remove solids from the effluent, or fails to produce a recycle stream, process failure occurs with effluent permit violations and loss of biomass from the reactor. Therefore, two commonly used parameters: overflow rate and solids flux, have been developed for SST design and evaluation.

Nevertheless, given the fact that the wastewater characteristics vary, such as flow rate and contaminant concentrations, traditional design procedures for SSTs tend to be more empirical and conservative by introducing averaged parameters with safety factors (Coe and Clevenger, 1916). Therefore SST performance can suffer unanticipated fluctuations, which may cause process control problems and increase the risks of failure. Stringent standards for effluent quality and the need for optimization of WWTP performance have made such variations in effluent quality undesirable, and have encouraged the use of dynamic controls for wastewater treatment process. For the purpose of developing such an automatic control system to provide consistent effluent water quality, great effort has been made to create accurate mathematic descriptions of wastewater treatment process (mathematical models), and the one-dimension (1-D) SST model for predicting the time dependent responses to transient process inputs of SSTs is a good example.

1-D SST models, based on solids-flux theory (Kynch, 1952), describe sludge transport by a scalar conservation partial differential equation (PDE). Although many 1-D SST models are available and some of them, especially Takács model (Takács et al., 1991), have been widely utilized in engineering practice, the predication of the sludge settling characteristics and concentration profiles in and out of a SST is still far from satisfactory.

The presently available 1-D models are highly dependent upon empirical equations to express clarification, thickening and compaction process and these equations or functions can be an error source that can profoundly affect simulation results. A second challenge is the difficulty of making full-scale measurements in working SSTs that has caused a lack of data sets for model calibration and verification. As a consequence, further research is still needed to improve the performance of the 1-D model.

The first goal of this paper is to review the previous, major developments in SST design and analysis to show how they have been used to develop 1-D models. The second goal is to review the 1-D models especially with regard to the numerical methods used to solve the resulting PDE, and to provide an improved method for solving the PDE. The final goal is to show how the 1-D model can be used in the design process to better understand the interaction between bioreactor and SST, particularly with regard to dynamic inputs, such as the time-to-failure after a shock load or appearance of filamentous bulking organisms.

2. Background

2.1. *Flux theory and state point analysis*

As theoretical foundations of solids–liquid separation, flux theory and state point analysis are widely used in SST studies, such as SST design, capacity analysis, and optimizing daily operations. For the purposes of quantifying biosolids settling characteristics, the starting point of both flux theory and state point analysis is usually the batch settling test. Table 1 lists the major contributors to solids flux theory and shows that Coe and Clevenger (1916) performed one of the earliest batch settling studies. Their major contribution was a comprehensive method to understand and utilize batch settling test

Table 1 – Major contributions to SST behavior analysis and comments.

Author	Major contribution	Comments
Coe and Clevenger (1916)	Conducted one of the earliest batch settling tests and developed a comprehensive method to understand and utilize batch settling results.	Their work connected batch settling tests and continuous settling study, and had a profound impact on the SST studies for the first half of the 20th century.
Kynch (1952)	Assumed that gravity settling velocity is only determined by the local solids concentration, and mathematically expressed the mass conveyance as a partial differential equation (PDE).	The velocity assumption made the settling process mathematically describable, thus greatly improving the understanding of solid-liquid separation.
Talmage and Fitch (1955)	Suggested the slope of the tangent to the interface subsidence curve of a batch settling test was equal to the settling velocity of the layer with the same solid concentration, and developed a procedure to obtain the settling velocity information.	It was one of the earliest ramifications of Kynch theory, and the batch settling velocity can be easily determined. The accuracy of this procedure was highly depended on the determination of the compression point.
Yoshioka et al. (1957), Hassett (1958)	Independently developed two widely accepted graphical methods of analyzing batch flux data (the gravity flux and bulk flux), and the flux analysis was related to a concentration profile in SST.	Both are convenient methods to estimate several significant parameters, such as underflow solids concentration and limiting flux, therefore making the SST performance predictable.
Tory and Shannon (1965), Dick and Ewing (1967)	Recommended using a slow speed mixer to offset the wall effects and to reduce lag time, and showed that the batch settling results were independent to the initial settling height, the particles' shape and size.	Since the free settling velocity obtained in batch settling tests can represent the observed velocity in full-scale continuous SST, the batch settling results can be used as reference of continuous SST deign and control.
Keinath (1985)	Extended the flux theory to the state point analysis, a comprehensive analysis of the relationships among various operating parameters (MLSS, overflow rate, underflow rate et al.)	The state point analysis provided a fundamental description of solids handling characteristics of the SST, and could be used to develop effective SST control strategy and optimize its performance.

results, and confirm SSTs' limited capacity in clarification/thickening, now known as limiting flux theory.

Because of the difficulty in analyzing the details of forces on particles during sedimentation, a complete theoretical analysis of solids settling cannot be completed. Though Coe and Clevenger's method was widely accepted, it was still considered an empirical rather than a "first-principles" analysis. One of the most well-known theoretical analyses was given by Kynch (1952), also shown in Table 1. Kynch approached the problem by introducing a simple but critical assumption that hindered settling velocity is only determined by the local solids concentration, thus making the solids concentration the dominant factor in sedimentation processes. The solids transport then can be calculated using mass flux ($\Phi = \upsilon_s C$, a function only related to C), which is the rudiment of flux theory. Therefore, the batch settling process can be mathematically expressed as a continuous function based on scalar mass conservation law:

$$\frac{\partial C}{\partial t} + \frac{\partial(\upsilon_s C)}{\partial z} = 0 \quad (1)$$

Talmage and Fitch (1955) and Fitch (1962) interpreted Kynch's result by suggesting the slope of the tangent to the interface subsidence curve of a batch settling test was equal to the settling velocity of the layer with the same solid concentration. Their experimental results matched Kynch's theory.

However, one remaining question in flux theory was whether the free settling velocity obtained in laboratory scale batch settling conditions can accurately represent the observed velocity in the full-scale continuous SSTs, since batch settling can be affected by various factors, such as the size of cylinders, wall effects, the non-uniform shape and size particles. Dick (1970) compared batch settling processes with similar initial concentrations in different sized cylinders, and recommended using a slow speed mixer to offset the wall effect and reduce lag time. Tory and Shannon (1965) investigated the non-uniform shape and size problem by using Gaussian distribution spheres, and found that zone settling was independent of particle shape and size.

Yoshioka et al. (1957) and Hassett (1958) independently developed two widely accepted graphical methods of analyzing batch flux data on the basis of flux theory. The former plots the gravity flux only, while the later shows on both gravity and total fluxes. However, both of them use batch test results for continuous settling predication, such as underflow solids concentration and limiting flux, thus making the SST performance predictable.

One of the most well known concepts in flux theory is a limiting flux that is normally used to estimate SST solids handling capacity. Thickening and clarification failure may occur when the solids loading flux exceeds the limiting flux, which can be caused by either hydraulic shock loading or deterioration of settling characteristics. Keinath (1985) extended flux theory, creating what has become known as state point analysis, by integrating clarification requirements into the flux diagram. State point analysis considers feed flow rate, underflow rate and feed solids concentration, and is now routinely used for operation analysis of activated sludge systems and solids inventory control strategies. Table 1 summarizes the major advances of these pioneering studies and summarizes the contribution of each researcher.

2.2. *Modeling of continuous settling process*

The previous discussion traced the development of flux theory for SSTs and all the analyses were steady state analysis. To extend these results to more realistic conditions, a dynamic procedure is needed. Considerable efforts have been made to develop dynamic models to better understand and predict SST performance. Bryant (1972) improved Kynch's batch continuity equation by adding extra terms to simulate the bulk mass thickening near the SST bottom and established the following fundamental model structure:

$$\frac{\partial C}{\partial t}+\left(\upsilon_u+\frac{\partial(\upsilon_s C)}{\partial C}\right)\frac{\partial C}{\partial z}=0 \quad (2)$$

However, Bryant realized that Eq. (2) cannot predict a limiting flux and solids overloading condition without proper PDE solver.

Tracy (1973) developed an inventory model by dividing the SST into clarification, dilution, thickening, and compression zones. Tracy adjusted the volume of each zone to conserve mass and adhere to the limiting flux by using solids concentrations calculated from Hasselt's flux graphing method. The utility of his model is the ability to simulate overloading by adjusting the thickening zone volume to represent sludge blanket level rise as a function of solids overloading, and even predicting time-to-failure with continued overloading. The limitation of this model is its inability to dynamically change concentrations in each zone as a function of hydraulic changes. It is interesting to note that Tracy never actually solved Eq. (1) or (2).

Stenstrom (1976) modified Bryant's model Eq. (2) in order to make it conform to the limitations of solids flux theory by using an empirical constraint Eq. (3) on gravity settling flux.

$$G_{s,i+1/2}=\min\left(\upsilon_{s,i}C_i,\upsilon_{s,i+1}C_i\right) \quad (3)$$

He divided the SST into layers or finite differences, and limited the solids flux out of each layer into the lower layer. Fluxes were calculated from the batch settling velocity relationship, and an upper layer was limited to the flux that the lower layer could pass. The constraint, while based on a physical concept, does have a theoretical basis, since it satisfies a mathematical principle called 'consistent', which means the numerical flux should be a function related to adjacent layers instead of the local single layer (Burger et al., 2011). Vitasovic (1986) found this constraint may cause oscillations at low concentrations, and recommended to setting a threshold concentration below which the constraint is not active. He chose 3000 g/m^3 and noted the value is site specific. The best well-known work using Stenstrom and Vitasovic's work is the Takács model (Takács et al., 1991), which has become the most widely used 1-D SST model.

Although the studies discussed above have partially solved the one-dimension SST modeling problem, the increasingly strict standard for WWTPs stability and reliability calls for efforts to provide more accurate and practicable continuous models; and approaches to improve model quality can be identified into two categories: improvements to the numerical solution technique, and more accurate description and implementation of the sedimentation mechanism.

The settling model, a typical nonlinear hyperbolic PDE, is not easy to solve because of possible solution discontinuities, which are difficult to detect. Authors (Jeppsson and Diehl, 1996; David et al., 2009; Burger et al., 2011) have discussed better methods for providing more reliable solutions of Eq. (2).

In addition to the gravity and hydraulic flux, other effects can also influence continuous settling process: for example dispersion, compression, density currents and short-circuiting. Several approaches now are available to estimate possible impacts. Dispersion is usually modeled by a second-order dispersion term (Watts et al., 1996; Plosz et al., 2007; Burger et al., 2011), and compression is caused by solid-solid and solid-water interaction in a high concentration zone, and modeled by the second-order compression term (De Clercq et al., 2008). Usually, 1-D models have difficulty in accounting for hydraulic flow impacts, but the density current and short-circuiting are predictable with inlet height and feed flow rate adjustment (Dupont and Dahl, 1995).

3. Model improvement

3.1. *Model structure development*

The SST is idealized as one dimension with constant cross-sectional area, no net organism growth, no density currents and no short-circuiting. Therefore the SST feed solids concentration can be assumed to equal the bioreactor solids concentration.

Other effects, such as dispersion and compression affect SST performance, but modeling them can greatly increase the model's complexity. Therefore, to maintain the model complexity at a reasonable level for practical application, in this study, the model mathematic expression is determined, giving

$$\frac{\partial C}{\partial t}+\frac{\partial(\upsilon_s C-\upsilon_e C)}{\partial z}=0 \quad \text{above the inlet layer} \quad (4)$$

$$\frac{\partial C}{\partial t}+\frac{\partial(\upsilon_s C+\upsilon_u C-\upsilon_e C)}{\partial z}=\upsilon_f C_f \quad \text{the inlet layer} \quad (5)$$

$$\frac{\partial C}{\partial t}+\frac{\partial(\upsilon_s C+\upsilon_u C)}{\partial z}=0 \quad \text{below the inlet layer} \quad (6)$$

A relationship between the C and υ_s is required. Based on Kynch's settling velocity assumption (Kynch 1952), two commonly used functions are Vesilind's equation (Vesilind 1968) and the double-exponential equation (Takács et al., 1991):

$$\upsilon_s=\upsilon_0 e^{-nC} \quad (7)$$

$$\upsilon_s=\max\left(0,\min\left(\upsilon_{0,\max},\upsilon_0\left(e^{-r_h(C-C_{\min})}-e^{-r_p(C-C_{\min})}\right)\right)\right) \quad (8)$$

Both of equations are suitable in hindered settling zone, while Vesilind's equation may overestimate the settling velocity at low solids concentration (Li and Ganczarczyk, 1987; David et al., 2009). Therefore, the double-exponential formulation is used in this study.

The sludge settling velocity parameters are site specific and depend upon the condition of the biomass (i.e., filaments or no filaments, etc). For this paper, the data collected by Grieves and Stenstrom (1976) and Tracy (1973) were used. The measurement error has been checked to be Gaussin and uncorrelated. Therefore, the estimation used the Levenberg–Marquardt algorithm (More, 1978). The results are shown as normal sludge in Table 2.

For most situations where an SST is overloaded, the limit occurs because of the rise of the sludge blanket in the hindered settling zone, which is less than the gel point concentration. Compression settling only occurs near the SST bottom region, which can impact the sludge blanket level rise and sludge recycling. However, the rise in the compression zone usually does not limit operation. In cases where more thickening is involved, such as with gravity thickeners, modeling this compression zone will be more important. Notice that the major improvement of this model is the use of reliable numerical techniques, and the solids settling description still follows the double-exponential formulation, which is not designed for compression settling modeling. Several researchers (Buscall and White, 1987; Cacossa and Vaccari, 1994; Burger, 2000; Kinnear, 2002; De Clercq et al., 2008; Gladman et al., 2010) have studied the compression zone, and provided approaches and guidance for modeling two-phase flow and compression settling.

3.2. Numerical discretization and integration

The first-order nonlinear hyperbolic PDE mathematic structure can produce numerical discontinuities during the calculation. However, computing these discontinuities generates severe challenges, since numerical oscillations may occur near the discontinuous point.

Though great efforts have been done to improve model solution accuracy, problems still remain. For example, when using the flux constraint (Stenstrom, 1976) the result is sensitive to the number of discretization layers used in the model, and can cause numerical oscillation at low concentrations; Both Godunov's first-order method and Engquist and Osher's upwind method routinely treat every part of numerical solutions same, regardless of how these solutions behave, thus causing a sharp tradeoff between accuracy and stability.

In this study, the SST model is improved by introducing a second-order accurate total variation diminishing (TVD) numerical technique- the Yee–Roe–Davis scheme (Yee et al., 1990), which is specifically designed for the scalar conservation law solving. To capture the solution discontinuity and avoid oscillation across the shock, the solution difference calculation and flux limiter technique are used. For smooth region, the solution difference can be very small, while it can be relative large through the shock. And instead of the straightforward discretization, the flux limiter is used to satisfy the nonlinear stability condition. Eq. (9) is used as the flux limiter in this study.

Table 2 – Parameter sets of ASP design and gravity settling velocity (normal and deterioration).

Parameter set of ASP design		Parameter set of settleability		
Q_f	260		Normal sludge	Deterioration
Q_e	200	$v_{0,max}$	20	20
Q_u	60	v_0	9.63	9.63
S_{in}	300	r_p	0.01	0.01
F/M	0.3	r_h	0.00063	0.003
H	4	C_{min}	10	12
h	2			

$$\phi(r_i^+, r_{i+1}^-) = \text{minmod}(1, r_i^+, r_{i+1}^-) \quad (9)$$

where

$$r_i^+ = \frac{C_i^n - C_{i-1}^n}{C_{i+1}^n - C_i^n}, \quad r_i^- = \frac{C_{i+1}^n - C_i^n}{C_i^n - C_{i-1}^n} \quad (10)$$

The explicit Yee–Roe–Davis flux limited scheme is

$$C_i^{n+1} = C_i^n - \frac{\Delta t}{\Delta z}\left(\overline{G}_{i+1/2}^n - \overline{G}_{i-1/2}^n\right) \quad (11)$$

where

$$\overline{G}_{i+1/2}^n = \frac{1}{2}\left(G_i^n + G_{i+1}^n\right) + \frac{1}{2}\psi\left(a_{i+1/2}^n\right)\left(\phi_{i+1/2}^n - 1\right)\left(C_{i+1}^n - C_i^n\right) \quad (12)$$

$$\psi(a) = \left\{\frac{a^2 + \delta^2}{2\delta}, |a| < \delta; \quad |a|, |a| > \delta\right\}, \quad a = \frac{dG}{dC} \quad (13)$$

In this study, δ is determined as 10^{-20}. As a solution sensitive technique, the Yee–Roe–Davis technique is able to determine what to do in terms of the solution gradient rather than considering the solution's stability and accuracy in the same way throughout the entire solution domain. Therefore, the improved model can work well at both discontinuity and smooth regions simultaneously with small tradeoffs.

Given the fact that solids concentration depends on two variables (z, t), discretization is required for both time axis and spatial axis: the SST is divided into several layers with equal height, volume and a uniform concentration within the layer. Specifying the number of layers must be done carefully, because it greatly impacts the calculated concentration profile (Jeppsson and Diehl, 1996). Sensitivity to the number of layers is a numerical artifact; the solution should be independent of the number of layers. Hence, a favorable 1-D model is required to be consistent with respect to number of layers. Fig. 1 shows discretization sensitivity of the improved model and Takács model in a solids overloaded simulation.

3.3. Numerical solution accuracy

For the SST modeling, what interests us most is the SSTs' response to solids overloading, since effluent violations are likely to occur with solids overloading. To evaluate sensitivity, we created a hypothetical but typical overloading condition ($A = 100\ m^2$, $C_f = 4000$ mg/l), with normal settling parameters as shown in Table 2. As can be seen from the predicted concentration profiles (Fig. 1), both models are able to detect the sludge blanket propagation regardless of the number of layers, but the results diverge with differences in the sludge blanket level, solids concentration in each layer and the recycling solids concentration. The Takács model predicts a higher sludge blanket height (~3 m), while the improved model has a larger solids concentration in each layer, including the bottom layer. Using the Yee–Roe–Davis technique as PDE solver, the improved model more accurately predicts the discontinuities at the edges of the blankets. It is also important to notice the overestimation of the sludge blanket height (~3 m versus ~2.5 m) that may lead to designing for a larger SST area, to accommodate the excessive blanket height.

3.4. Layer number sensitivity test

Fig. 1 is the solids concentration profile for four different layer-number conditions. Except for the case of 10 layers, no

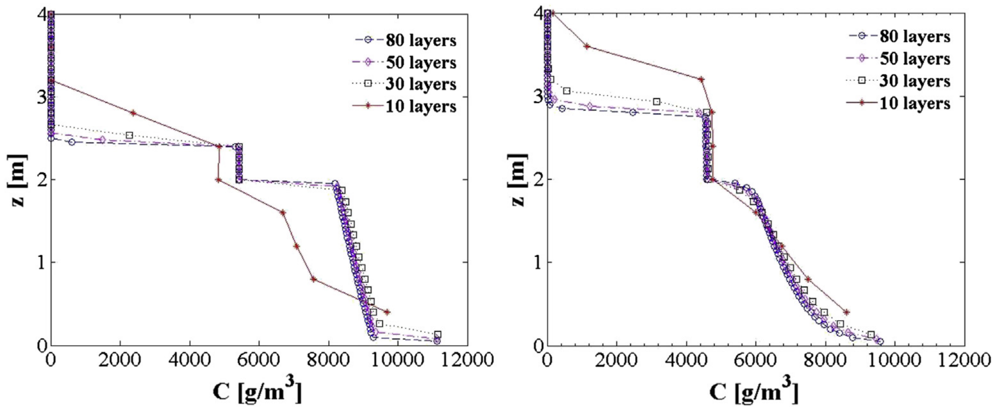

Fig. 1 – Typical overloading concentration profiles of different discretization levels (left: the improved model; right: Takács model).

obvious differences within each model's results can be observed. Clearly, 10-layer is not enough for either model. The 10-layer model, especially for the Takács model, predicts too much sludge storage within the SST. When the model layer number exceeds 30, the predicted concentration profiles within each model, will be similar.

3.5. *Model verification of SST responses to solids flux overloading*

In current engineering practice, the underflow solids concentration and the sludge blanket level predications are the two key 1-D SST model outputs, since they are closely related to ASP system robustness and contaminant removal efficiency. To further investigate the model performance, the data collected by Tracy (1973) in continuous ferric hydroxide suspension settling is introduced for both the Takács model and the improved model verification. The response of the height of the sludge blanket level and the predicted and actual underflow solids concentration are shown in Fig. 2. For the sludge blanket level predication, the improved model simulation (6.82 cm/h) closely approximates the actual sludge blanket rising rate (6.71 cm/h), while the Takács model predication is 7.25 cm/h, which agrees well with the conclusion above that the Takács model overestimates the sludge blanket level. The underflow solids concentration predicted by the Takács model (5850 g/m^3) is also much smaller than the measured concentration, as expected. The improved model provides a steady state value 6480 g/m^3, which is virtually the same as the measured value 6650 g/m^3, and the underflow concentration change tendency predicated by the improved model is more rapid than data. These incongruities can be explained by the impact of compression settling. Because of the compression settling impact, the thickening process will be retarded during the initial solids overloading period, which can accelerate the sludge blanket rise and decelerate the underflow concentration increasing rate. However, with the sludge blanket increase, the weight of the overlying sludge in the compression zone promotes thickening, which can decelerate the sludge blanket rise but can accelerate the increase in underflow concentration.

4. Practical model application

4.1. *Activated sludge process design*

Mathematic models including ordinary differential equations (ODEs) for bioreactor and PDEs for SST are frequently used for activated sludge process (ASP) design and evaluation (Diehl and Faras, 2012). Here, we present a traditional ASP design to demonstrate practical value of the improved SST model. Assume that dissolved oxygen in bioreactor is sufficiently high; the bioreactor is completely mixed; Q_w and S at steady state are much smaller than Q_e and S_0, respectively. Design parameters and sludge settling parameters (normal sludge) are given in Table 2.

The substrate mass balance around the bioreactor yields the following ODE:

$$V\frac{dS}{dt} = Q_eS_{in} - (Q_e - Q_w)S - Q_wS_w - \frac{\mu}{Y}C_fV \quad (14)$$

Sedimentation process in SST is modeled by a series of PDEs, Eqs. (4)–(6). At steady state, S_{in}, Q_e, V, F/M ratio are constant, and Q_w and S are much smaller than Q_e and S_0. Thus, the steady state C_f can be expressed as:

$$C_f = \frac{Q_eS_{in}}{V(\mu/Y)} \quad (15)$$

In solids flux theory, the SST solids input is defined as the operating flux, and the maximum SST solids handling capacity must equal to or be less than the limiting flux. To maintain ASP stability and reliability, process loading at critical or under loading conditions is preferable, which means operating flux must be less than or equal to the limiting flux. Otherwise, thickening or clarification failure may occur. In this study, we set this requirement as a safety constraint safeguarding ASP efficiency and stability, and express it as the following inequality:

$$\frac{(Q_e + Q_u)}{A}\cdot\frac{Q_eS_{in}}{V(\mu/Y)} \le \Phi_l \quad (16)$$

If steady state Q_u is also set as a constant 30% of Q_e, Φ_l can be obtained from the graphic total flux method (Hassett, 1958) for

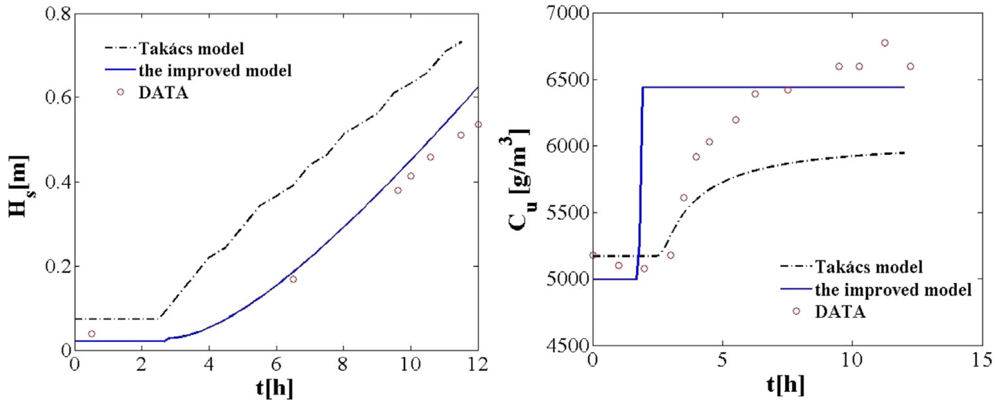

Fig. 2 – SST response to step increase in influent solids flux (Data from Tracy, 1973).

various areas indicated in Fig. 3. A restricted range (100–400 m^2) was evaluated which is within the proper overflow rate (0.5–2 m/h). As a result, the bioreactor volume and SST area are the two only variables in this inequality, and their effect on process performance can be plotted.

4.2. *Selection of different-sized ASP*

Various combinations of *V* and *A* can satisfy the safety constraint, so three extra criterions are established to evaluate the quality of the design:

1. Economic plausibility;
2. Contaminants removal efficiency at steady state;
3. Robustness to solids overloading;

Most ASP cost optimizations treat the designed treatment capacity as the dominant components, while few recognized the capital cost is closely related to reactors' size (Keinath et al., 1977). In this study, the cost estimation strategy follows the latter one, and unit cost of bioreactor and SST are respectively expressed as dollar/m^3 and dollar/m^2 in terms of their volume and area. Providing an explicit cost analysis is beyond the scope of this research, but a hypothetical yet realistic example is achieved by fixing the bioreactor unit cost as 1 dollar/m^3, and then introducing a relative-cost coefficient for SST cost calculation as Eq. (17):

$$C_T = 1 \cdot V + R_c \cdot A \quad (17)$$

The relative cost coefficient will vary and be site specific, and three typical values (1, 1.5, 3) are selected here (David, 1968).

At steady state, the biomass distribution between the bioreactor and SST will reach equilibrium. Total biomass in the reactor is an important factor in contaminants removal evaluation, and retaining most biomass in bioreactor is preferable for maximizing overall treatment efficiency. The steady state C_f is solved by the following ODE:

$$\frac{dC_f}{dt} = \frac{C_u Q_u - C_f Q_f}{V} \quad (18)$$

Here, we only discuss the contaminant removal efficiency in the underloading condition, because it is most prevalent. A typical underloading condition is modeled by fixing $Q_e = 100$ m^3/h, half of the designed capacity with a simulation time interval as 3 h.

Though the safety constraint considerably improves system stability and reliability, SST may still experience failure problems attributed to two chief causes: hydraulic shock loading and deterioration of sludge settleability. The time-to-failure or failure time is defined as the time interval between the beginning of an upset and failure, and can be used as an important indicator for process robustness evaluation. Obviously, a longer time-to-failure indicates a more robust process. All variations are imposed as step functions in solids overloading endurance test:

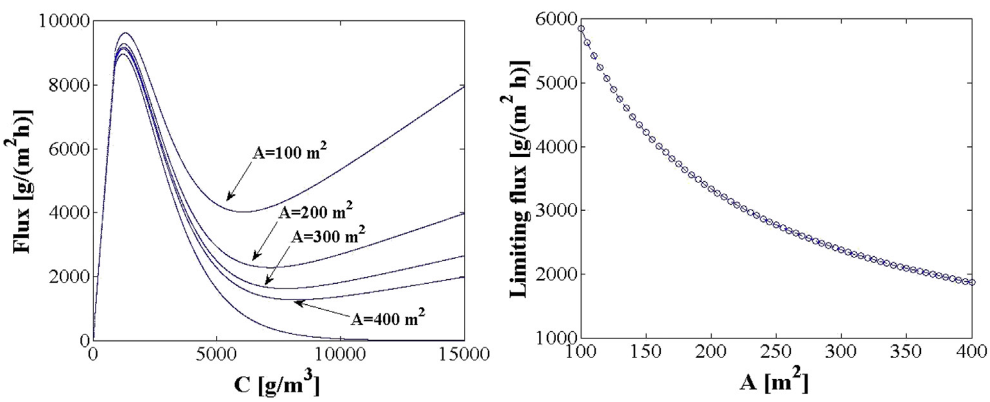

Fig. 3 – Total settling flux profile (left) and the limiting flux profile (right) in different SST size conditions.

- Hydraulic shock loading: At $t = 0$ h, $Q_e = 200$ m^3/h to reach steady state. At $t = 2$ h, Q_e is increased from 200 m^3/h to 800 m^3/h, four times of the designed flow rate.
- Slurry settleability deterioration: Q_e is fixed as 200 m^3/h. At $t = 0$ h, the settling parameters are set to normal as shown in Table 2. At $t = 2$ h, the settling velocity parameters change to deterioration (Table 2) in order to simulate a change to poor settleability condition (e.g., bulking).

5. Process size results and discussion

5.1. ASP unit sizes

Fig. 3 shows the limiting flux decreasing with increasing SST area. This occurs because a larger area provides smaller hydraulic bulk flux toward SST bottom. A smaller limiting flux does not simply mean less reliability, because enlarging the SST's area meanwhile decreases the operating flux to less than or equal to the limiting flux.

As shown in Fig. 4 (left), the required volume decreases with increasing area. Adjustment of either bioreactor or SST's size can requires a compensating modification of the other, thereby restricting the whole ASP size.

5.2. Economic plausibility

For all relative cost conditions, the system capital cost decreases with the increase of SSTs' area initially, and then increases, therefore an economic optimal point exists, as shown in Fig. 4 (right). What differs is the location of the least cost point. The relative cost is a significant weighting factor greatly impacting the relative size of bioreactor and SST, and a large relative cost shifts the economic optimal point toward the smaller SST surface area. If economic plausibility is the only consideration, the least cost point reduces capital investment (10~20% reduction). However, an important consideration is whether the least cost design can provide sufficient contaminant removal and simultaneously avoid solids overloading failures, which we will discuss in the next section.

5.3. Overall treatment efficiency

In an underloading condition, biomasses are shifted from bioreactor to SST, and finally reach a balance. And as shown in Fig. 5 (left), the stored biomass amount in larger SSTs is 2–3 times more than it in smaller ones.

Nevertheless, the gradient of SST biomass storage actually has little effect on the total slurry distribution: the maximum SST biomass is no larger than 5% of the total. This indicates that all alternatives ($A = 100 \sim 400$ m^2) are able to retain most biomass (95~98%) in bioreactors. Additionally, since the total biomass amount is almost the same, there is no significant difference in the total bioreactor biomass storage regardless of the small variations in the SST biomass storage. This implies all alternative ASP designs can meet the high overall treatment efficiency requirement.

5.4. System robustness

A lengthy time to reach biomass distribution equilibrium between bioreactor and SST implies a more stable process, since a longer time to reach equilibrium provides more time to respond in the event of an overload. This section illustrates this point by evaluating the time-to-failure of a 20-h hydraulic shock loading simulation (Fig. 6 left) and deterioration of sludge settleability (Fig. 6 right).

It is notable that the estimated time-to-failure provided by Takács model is much smaller than what the improved model predicts. This corresponds well to the conclusion presented earlier in the numerical accuracy section of this paper that the Takács model overestimates the sludge blanket height due to numerical inaccuracies. This shortage can eventually cause a conservative design. For instance, the Takács model shows for a hypothetical hydraulic shock loading, the SST surface area should be at least 285 m^2 to provide sufficient capacity, while the improved model indicates 195 m^2 is sufficient.

The time-to-failure predicted by the improved model can be used for system robustness analysis. According to Fig. 6 (left), the hydraulic shock loading failure time of smaller SST alternatives ($A = 100$ to 185 m^2) is less than 0.1 h. It increases to 2–5.8 h, a great improvement in overloading endurance, when SSTs are enlarged to medium size ($A = 190$ to 260 m^2). No

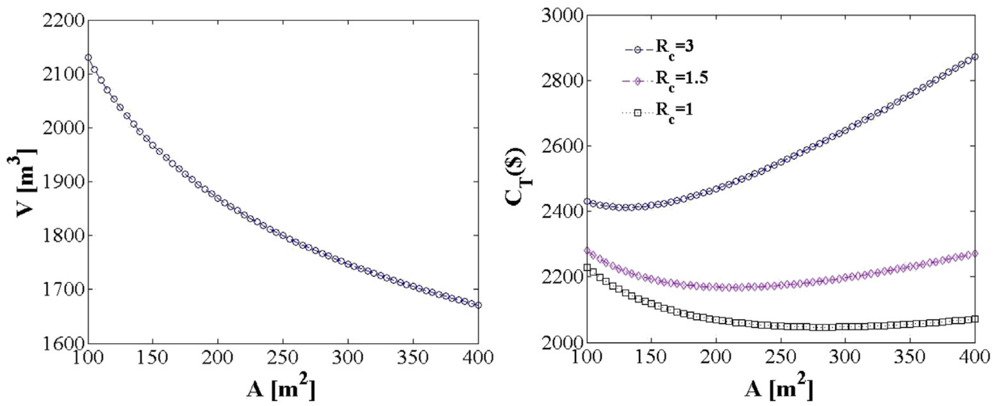

Fig. 4 – Scale relationship between SSTs and bioreactors (left) and total cost of alternative ASP designs in three typical relative cost conditions (right).

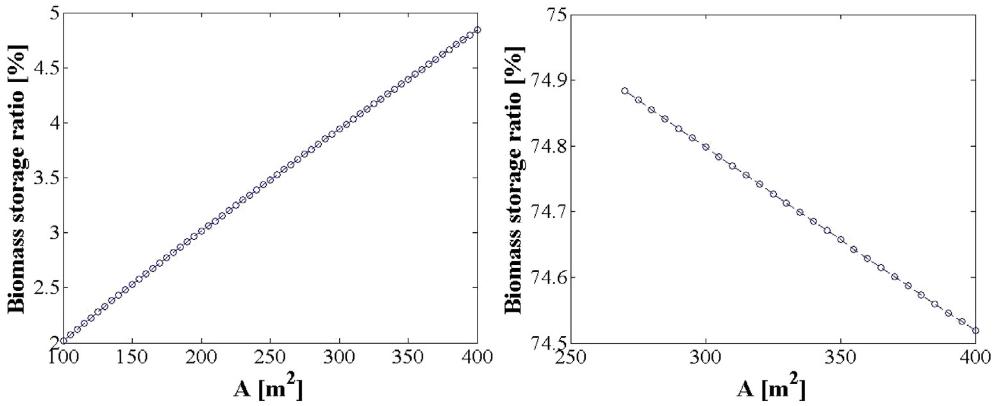

Fig. 5 – SST biomass storage ratio in underloading steady state (left) and large size SST biomass storage ratio when the flux loading statues changes from overloading to critical loading (right).

failure will occur if the SST is larger than 260 m^2. Fig. 7 presents the 20-h dynamic biomass accumulation processes in small, medium and large SSTs (A = 120, 230, 350 m^2). For the case of a small SST, most biomasses are directly conveyed to the clarification zone by the overflow instead of settling to the thickening zone, causing a clarification failure in less than 0.1 h. This helps explain why small SSTs have extremely short time-to-failure. A gradual sludge blanket rise is observed in medium SSTs, and causes a thickening failure when it reaches the inlet. An area of 195 m^2 is the demarcation point between clarification failure and thickening failure, and a state point analysis also shows this distinction (Keinath, 1985). Compared with a clarification failure, the thickening failure is a relatively slow process as the sludge blanket must rise from the bottom to top, which usually occurs over several hours. If the SST can afford sufficient storage capacity of biomass in the thickening zone, the solids loading condition can be reduced from overloading to critical loading. This explains why neither clarification failure nor thickening failure occurs when the SST area is greater than 260 m^2. Fig. 5 (right) shows the biomass distribution for a large, overloaded SST and most (75%) of the biomass is stored in the thickening zone, which is much larger than for the under loaded case (2∼5%).

Compared to hydraulic shock loading, where the failure is caused by a sudden increase of operating flux, failure due to poor biomass settleability (sludge bulking), is attributed to a decrease in the limiting flux. In this case, failure can be avoided only by increasing the limiting flux, such as changing the recycle rate or contacting pattern (Stenstrom and Andrews, 1979).

5.5. *Comprehensive selection*

An overall selection of SST size relative to the bioreactor size must include considerations other than capital investment. Simultaneously achieving an economically justifiable sizing, high contaminant removal and system stability can be difficult. It may be tempting to use the least cost alternative, such as an area of 130 m^2 when $R_c = 3$ (Fig. 4 right), but this selection produces a situation with no ability to tolerate overloading from either a hydraulic shock or deterioration in sludge settling properties. When the SST is less expensive relative to the bioreactor (R_c = 1), a larger SST can be provided (A = 285 m^2) and much greater stability is obtained. Comprehensive selection of ASP design using a 1-D SST model is a tool for the designer to evaluate capital cost, overall treatment efficiency and process stability.

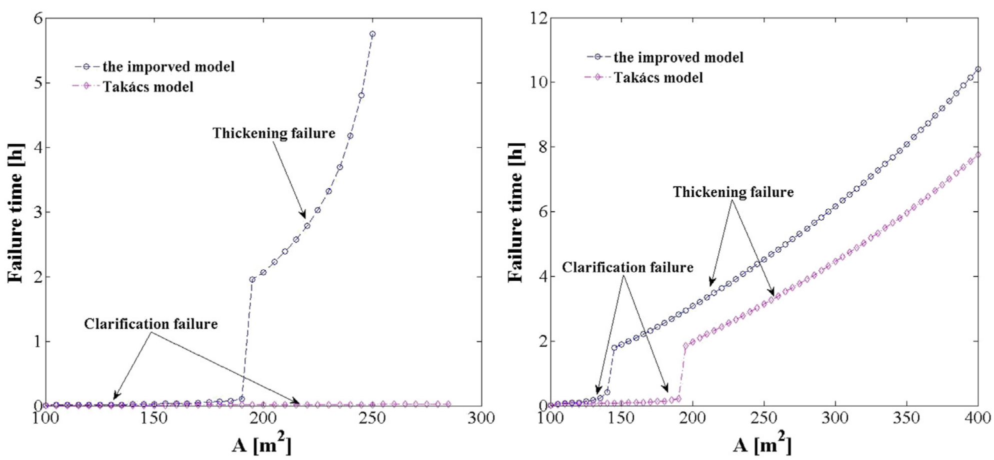

Fig. 6 – SSTs' time-to-failure corresponding to hydraulic shock loading (left) and settleability deterioration (right).

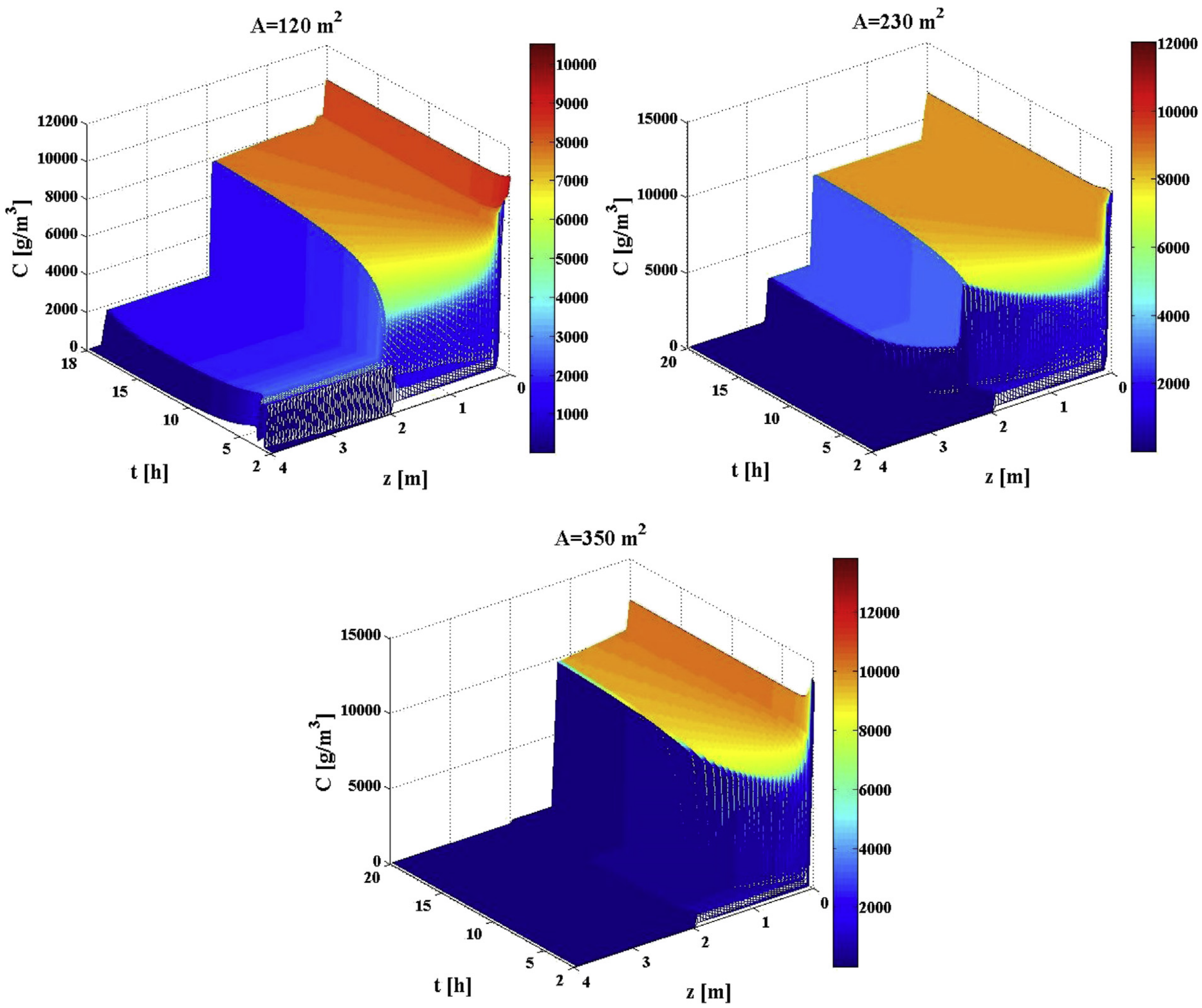

Fig. 7 – Dynamic solids concentration profiles in hydraulic shock loading for different surface areas: 120 m^2, 230 m^2 and 350 m^2.

6. Conclusion

The conclusions of this study can be summarized as following:

- The improved model with solution-sensitive PDE solver can determine the calculation behavior in terms of the solution gradient, therefore providing more numerically and physically acceptable solutions. The discretization level sensitivity test demonstrates that the improved model can provide uniform solids concentration and storage predication, while Takács' model is sensitive to number of layers, and can even overestimate the sludge blanket level and underestimate the underflow concentration.
- For an activated sludge process design, the bioreactor and SST should be designed as a whole, and a safety constraint can be introduced in the design process to greatly improve the system's efficiency and reliability. The designed alternatives based on the safety constraint show that the requirement of bioreactor volume decreases with an increase of SST size, and this can help to prevent over-designing the ASP size and land waste.
- A comprehensive selection of the designed alternatives should consider three aspects: economic plausibility, contaminant removal efficiency, and system robustness. Least-cost points can usually be attained, but their locations will vary depending on the weighting of the relative cost factor. In a solids underloading condition, all designed alternatives are able to provide sufficiently high contaminant removal with only 2%∼5% total of the biosolids storage in the SST.
- Both hydraulic shock loading and settleability deterioration can cause solids overloading problems. ASPs with small size SSTs may suffer clarification failure in less than 0.1 h, and the medial size ones can have a thickening failure problem in 2∼5 h due to the continuously rising sludge blanket level. If the SST is large enough to store 75% of the biomass in the thickening zone, no failure occurs, and ASP system will be robust. In most cases, it is difficult to prioritize these three criterions, which requires trade-offs for system optimization.

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