

Dynamic one-dimensional modeling of secondary settling tanks and system robustness evaluation

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ABSTRACT

One-dimensional secondary settling tank models are widely used in current engineering practice for design and optimization, and usually can be expressed as a nonlinear hyperbolic or nonlinear strongly degenerate parabolic partial differential equation (PDE). Reliable numerical methods are needed to produce approximate solutions that converge to the exact analytical solutions. In this study, we introduced a reliable numerical technique, the Yee–Roe–Davis (YRD) method as the governing PDE solver, and compared its reliability with the prevalent Stenstrom–Vitasovic–Takács (SVT) method by assessing their simulation results at various operating conditions. The YRD method also produced a similar solution to the previously developed Method G and Enquist–Osher method. The YRD and SVT methods were also used for a time-to-failure evaluation, and the results show that the choice of numerical method can greatly impact the solution. Reliable numerical methods, such as the YRD method, are strongly recommended.

Key words | activated sludge, clarification, control, wastewater treatment

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NOMENCLATURE

A	cross-sectional area of SST [m^2]
C	sludge concentration [g/m^3]
$C_{\min}$	nonsettleable solids concentration [g/m^3]
F	(convection) flux function [$\text{g}/(\text{m}^2\text{h})$]
h	SST inlet depth [m]
H	SST depth [m]
N	number of layers
Q	flow rate [m^3/h]
r	Vesilind settling parameter [m^3/kg]
r_h	Takács settling parameter [m^3/kg]
r_p	Takács settling parameter [m^3/kg]
R	the ratio of solution difference
v	velocity [m/h]
v_0	Vesilind settling parameter [m/h]
$v_{0, \max}$	Takács settling parameter [m/h]
v_s	hindered settling velocity [m/h]
t	time [h]
z	height above SST bottom [m]

Φ	the flux limiter
θ	the averaging factor
δ	the YRD method parameter

Subscripts

B	bottom
e	effluent
f	feed
i	index of model layer
u	underflow
T	top

Superscripts

n	index of time
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Greek letters

Δt	the time step [h]
Δz	the time step [h]

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INTRODUCTION

Activated sludge is the most prevalent secondary treatment process and commonly uses secondary settling tanks (SSTs) to achieve efficient solid–liquid separation. The major functions of SSTs can be described as two

similar but distinct actions: clarification and thickening. Clarification is the removal of suspended particles from effluent, and occurs in the clarification zone (above the inlet), and thickening is the process of increasing the underflow sludge concentration in the thickening zone (below the inlet). Free settling is always observed in the clarification process, while hindered and compression settling dominate the thickening process to produce a more concentrated underflow. Therefore, the settling behavior in the clarification and thickening zones is totally different.

Traditional design and control procedures for SSTs tend to be more empirical and conservative regardless of changes in wastewater characteristics such as flow rate and contaminant concentration. For SST design and operation optimization purposes, mathematical models have been used in engineering practice; for example the one-dimensional (1D) models are used to evaluate the sludge blanket level (Li & Stenstrom 2014), the two-dimensional and three-dimensional models are used for the SST geometry design, such as the inlet structure (Zhou & Mccorquodale 1992; Mazzolani *et al.* 1998).

Although different SST models are available, 1D SST models are most often used for their relative simplicity and low computation cost. Based on solids flux theory (Kynch 1952), 1D SST models describe sludge transport within the SST by the scalar conservation partial differential equation (PDE) with a discontinuous flux, and are able to predict both the effluent and recycling solids concentration as well as the sludge blanket level. However, presently available 1D sedimentation models are highly dependent upon empirical functions to express clarification, thickening and compaction processes and these functions can be an error source that profoundly affects simulation results. A second challenge is lack of reliable numerical methods to provide a high accuracy solution at low computational cost. Further research is still needed to improve the performance of 1D models.

The goal of this paper is to briefly review the development of 1D SST models and currently available numerical techniques used as the model governing PDE solver, then to provide a new, reliable numerical technique (based on the Yee–Roe–Davis method) for accurate numerical solution calculation. The second goal is providing an analysis of SST behavior at different operating conditions (underloading and overloading) based on numerical simulation results. The final goal is to show how the choice of numerical methods impact the model outputs, which has implications on the design and operation strategies.

METHODOLOGY

Model structure development

In order to simplify the problem and satisfy a 1D modeling condition, several assumptions are necessary to be introduced as follows: (1) the SST is circular and central-feed with constant area; (2) reaction rates are zero, and the sludge properties are uniform and constant in the SST; (3) no density currents exist (the hydraulic flow is vertical, and horizontally uniform); (4) loading rate is uniform and there are no wall effects; (5) the mechanical sludge scraper does not affect the sludge settling behavior.

In most previous SST modeling studies, the SST is divided into three functional zones, namely the clarification zone (above the inlet), thickening zone (below the inlet) and inlet zone to characterize the various settling behaviors: clarification, thickening and the mixture of input solids. Because of assumption (3), the hydraulic flow in the clarification zone is an upward effluent flow (Q_e), which conveys the solids toward the SST effluent weir, while the downward underflow (Q_u) in thickening zone transports solids to the SST bottom to produce a concentrated recycle flow. Hence, the 1D SST model should include both the bulk hydraulic transport and gravity settling.

In addition to the gravity settling and hydraulic transport, other factors can also impact the continuous settling process, for example the density current in the inlet region (Plosz *et al.* 2007), the hydraulic dispersion around the inlet (Hamilton *et al.* 1992; Watts *et al.* 1996; De Clercq *et al.* 2003; Plosz *et al.* 2007; Bürger *et al.* 2011, 2012), and sludge compression caused by its own weight at the SST bottom (Buscall & White 1987; Landman *et al.* 1988; Landman & White 1992; Cacossa & Vaccari 1994; Kinnear 2002; de Kretser *et al.* 2003; Usher & Scales 2005; Gladman *et al.* 2006, 2010; Usher *et al.* 2006; De Clercq *et al.* 2008; Bürger *et al.* 2011). Any attempt to model hydraulic dispersion and compression must introduce a diffusion term (a second-order derivative term) to the model formula that smoothes concentration profiles (Bürger *et al.* 2011, 2012, 2013). However, the solution may still have discontinuities in the region where local concentration is less than the critical concentration (gel point), which means no compression effect occurs. The governing PDE remains nonlinear hyperbolic in these regions, and cannot be easily discretized due to solution discontinuities. For either the convection dominant model, such as the well-known 10-layer model (Takács *et al.* 1991) only including the convection process, or the

convection-diffusion model which also simulates hydrodynamic dispersion and compression, it is necessary to introduce reliable numerical techniques for accurate numerical solution calculation and discontinuity capture, which is the primary goal of this study. Since solving either the convection dominant model or the convection-dispersion model requires capturing the solution discontinuities and avoiding oscillation at the discontinuity, these two alternative models possess similar characteristics in their numerical solutions. We chose the convection dominant model as our model, because of its greater utility in current engineering practice.

The convection dominant model can be written as the following nonlinear hyperbolic PDEs based on the mass conservation law

$$\frac{\partial C}{\partial t} + \frac{\partial(v_s C - v_e C)}{\partial z} = 0 \quad \text{above the inlet zone} \quad (1)$$

$$\frac{\partial C}{\partial t} + \frac{\partial(v_s C + v_u C - v_e C)}{\partial z} = v_f C_f \quad \text{the inlet zone} \quad (2)$$

$$\frac{\partial C}{\partial t} + \frac{\partial(v_s C + v_u C)}{\partial z} = 0 \quad \text{below the inlet zone} \quad (3)$$

As can be seen, the SST model is one equation with two unknowns (C and v_s). Therefore, an additional constitutive relation is required, and the Kynch's assumption (Kynch 1952) is most often used, which states that the hindered settling velocity is solely determined by the local solids concentration. The two commonly used constitutive formulas are the Vesilind (Vesilind 1968) function, Equation (4), and the double-exponential function (Takács *et al.* 1991), Equation (5)

$$v_s = v_0 \exp^{-rC} \quad (4)$$

$$v_s = \max\left(0, \min\left(v_{0,\max}, v_0 \left(\exp^{-r_h(C-C_{\min})} - \exp^{-r_p(C-C_{\min})}\right)\right)\right) \quad (5)$$

Although both formulas are suitable for hindered settlings, the Vesilind function may overestimate the settling velocity at low solids concentration (Li & Ganczarczyk 1987). The improvement of the two-exponential function relates to the non-settleable fraction in the feed sludge and the discrete settling behavior at the low solids concentration region. Therefore, the double-exponential function is applied in this study for gravity settling velocity calculation,

thus making the solids concentration (C) the only unknown in the model.

The mass conservation law should also hold on the upper and bottom boundaries, which requires the flux of particle leaving the SST to equal the flux entering the effluent and recycling pipes (Diehl 2000; Bürger *et al.* 2012). The mass conservation law of boundaries can be expressed as

$$v_s C_T - \frac{Q_e}{A} C_T = -\frac{Q_e}{A} C_e \quad \text{the top boundary} \quad (6)$$

$$v_s C_B + \frac{Q_u}{A} C_B = \frac{Q_u}{A} C_u \quad \text{the bottom boundary} \quad (7)$$

The sludge settling velocity parameters are site specific and depend upon the condition of the biomass (i.e., filaments, etc.). For this paper, data of Grieves & Stenstrom (1976) are used. The measurement error has been checked to be Gaussian and uncorrelated, and Levenberg–Marquardt algorithm (More 1978) is used for model parameter identification. The results are shown as normal sludge in Table 1.

Numerical technique introduction

Equations (1)–(3) are hyperbolic and cannot be straightforwardly discretized because of the shock problem (discontinuous solutions), which requires determination of unique solutions along the shock, and rejection of unstable discontinuities. To obtain both numerically and physically acceptable solutions, reliable numerical techniques especially designed for scalar conservation PDE are needed to satisfy the three fundamental principles: Courant–Friedrichs–Lewy (CFL) condition, consistent numerical flux and the entropy condition to ensure the calculation stability and accuracy (Bürger *et al.* 2011).

Table 1 | Parameter sets of gravity settling velocity (normal and deterioration)

Parameter set of settleability	Normal sludge	Deterioration
$v_{0,\max}$ [m/h]	9.63	9.63
v_0 [m/h]	20	20
r_p [m ³ /kg]	0.01	0.01
r_h [m ³ /kg]	0.00063	0.003
$C_{\min}$ [g/m ³]	10	12

Kynch (1952) first introduced the characteristics (iso-concentration line) analysis in a vessel with constant cross-section area to capture the path of concentration gradients (shocks) in batch settling tests. Petty (1975) extended Kynch's procedure to continuous sedimentation, and provided an explicit shock analysis for the transient state, while Bustos *et al.* (1990) constructed the global weak solutions based on method of characteristics for various initial data and operating conditions. Diehl (2000) complemented the characteristics analysis by resolving the problem with special boundary conditions at top, bottom and inlet, as well as considering the conical effect near the SST bottom. Successful examples of the characteristics analysis are the estimate of the batch-settling flux function from experimental data (Diehl 2007), and the mathematical analysis of the well-known solids-flux theory (Diehl 2008). On the basis of the method of characteristics, Bürger *et al.* (2004) also developed a front tracking method, which is efficient for shock capture. As a conclusion, the method of characteristics or the characteristics analysis is currently the only available approach to obtain exact solutions of the nonlinear hyperbolic governing PDEs; however, it requires considerably more effort for its implementation in engineering practice, and further investigations are needed.

Compared with analytical approaches, numerical techniques have advantages in dynamic process simulations. One of the earliest numerical flux descriptions used in 1D SST modeling is the Stenstrom–Vitasovic–Takács (SVT) flux (Stenstrom 1976; Vitasovic 1986; Takács *et al.* 1991) shown as follows

$$F_{i+1/2}^{\text{SVT}} = \min(v_{s,i}C_i, v_{s,i+1}C_i) \quad (8)$$

Several studies used the SVT flux, and the most well-known one is the 10-layer model (Takács *et al.* 1991) with the SVT flux as the key ingredient. Bürger *et al.* (2011, 2012, 2013) showed that the SVT flux can invalidate the entropy condition, and generates unphysical solutions in the low concentration region. The Godunov numerical flux, shown as Equation (9), is another widely used numerical technique in 1D SST modeling, which is derived from the unique exact solutions (Jeppsson & Diehl 1996), and also used by Plossz *et al.* (2007)

$$F_{i+1/2}^G = \begin{cases} \min_{C_i \leq C \leq C_{i+1}} \left(v_s C - \frac{Q_e}{A} C \right) & \text{if } C_i \leq C_{i+1} \\ \max_{C_{i+1} \leq C \leq C_i} \left(v_s C - \frac{Q_e}{A} C \right) & \text{if } C_i > C_{i+1} \end{cases} \quad (9)$$

An explicit numerical method (Method EO) with the Enquist–Osher numerical flux (Engquist & Osher 1981) was presented by Bürger *et al.* (2005), and De Clercq *et al.* (2008) employed it for batch settling simulation. Another numerical technique presented by Bürger *et al.* (2010) is Method G, based on the Godunov numerical flux. Although both Method G and Method EO are reliable for SST modeling, which means they are able to provide approximate solutions that converge to the unique physically relevant solutions, and in many cases they yield similar, even identical, solutions, the selection as a PDE solver is subjected to three competing principles: the complexity of implementation, the solution accuracy, and the computation cost. The comparison study (Bürger *et al.* 2012) showed that the Method EO is too complicated for application as the PDE solver in practical engineering problems, and for a given discretization level, Method G is capable of producing solutions faster than the Method EO. However, the Method EO reduces numerical error more efficiently than the Method G, which means the larger CPU time needed by Method EO results in higher quality numerical solutions.

Numerical discretization and integration

Because of the possible solution discontinuities (shocks) during the calculation, the nonlinear hyperbolic governing PDE cannot be straightforwardly discretized, and specific numerical techniques designed for scalar conservation PDE solving are often applied to avoid the shock, for example the flux averaging technique. Rather than choosing one method such as a first-order upwind method, the flux averaging starts with two or more established methods, then chooses one method or averages them. The averaging flux can be shown as follows

$$\hat{F}_{i+1/2}^n = \theta_{i+1/2}^n \hat{F}_{i+1/2}^{(1)} + (1 - \theta_{i+1/2}^n) \hat{F}_{i+1/2}^{(2)} \quad (10)$$

where $\hat{F}_{i+1/2}^n$ is the averaging numerical flux, $\hat{F}_{i+1/2}^{(1)}$ is the conservative numerical flux of numerical Method 1, $\hat{F}_{i+1/2}^{(2)}$ is the conservative numerical flux of numerical Method 2, and $\theta_{i+1/2}^n$ is the averaging factor, sometimes called the shock switch. An equivalent way of writing Equation (10) is Equation (11) shown as follows

$$\hat{F}_{i+(1/2)}^n = \hat{F}_{i+(1/2)}^{(1)} + \phi_{i+(1/2)}^n (\hat{F}_{i+(1/2)}^{(2)} - \hat{F}_{i+(1/2)}^{(1)}) \quad (11)$$

where $\phi_{i+1/2}^n$ equals to $(1 - \theta_{i+1/2}^n)$, and is called the flux limiter. This flux averaging method is called the flux-limiter method. After determining the two first-generation methods, the next step is choosing a suitable flux limiter, which strongly depends on distinguishing shocks from the smooth regions. Generally, shocks are indicated by the ratios of solution differences, which can be expressed as Equation (12)

$$R_i^+ = \frac{C_i^n - C_{i-1}^n}{C_{i+1}^n - C_i^n}, R_i^- = \frac{C_{i+1}^n - C_i^n}{C_i^n - C_{i-1}^n} \quad (12)$$

where R is the ratio of solution difference, and has the following properties:

- $R_i^+ \geq 0$ if the concentration is monotone increasing or decreasing
- $R_i^+ \leq 0$ if the solution has a maximum or a minimum
- $|R_i^+|$ is large and $|R_i^-|$ is small if the solution differences decrease dramatically from left to right
- $|R_i^+|$ is small and $|R_i^-|$ is large if the solution differences decrease dramatically from right to left

A large decrease or increase of the ratio of solution differences always indicates shocks. The flux-limit technique directly leads to the popular total variation diminishing (TVD) methods, which enforce the nonlinear stability by using the freedom of flux averaging. The Yee–Roe–Davis (YRD) numerical technique introduced in this study is a typical flux-limited method, which has the TVD property. The two first-generation methods used in the YRD numerical technique are the forward-time central-space (FTCS) method (Equation (13)) and Roe's first-order upwind method (Equation (14))

$$\hat{F}_{i+1/2}^{\text{FTCS}} = \frac{F(C_{i+1}^n) + F(C_i^n)}{2} \quad (13)$$

$$\hat{F}_{i+1/2}^{\text{ROE}} = F(C_{i+1}^n) - |a_{i+1/2}^n| (C_{i+1}^n - C_i^n) \quad (14)$$

where

$$F(C_i^n) = \begin{cases} C_i^n v_i^n - \frac{Q_e}{A} C_i^n & \text{the clarification zone} \\ C_i^n v_i^n + \frac{Q_u}{A} C_i^n & \text{the thickening zone} \end{cases} \quad (15)$$

In the original Roe's first-order upwind method, $a_{i+1/2}^n$ is given by Equation (16)

$$a_{i+1/2}^n = \begin{cases} \frac{F(C_{i+1}^n) - F(C_i^n)}{C_{i+1}^n - C_i^n} & \text{for } C_i \neq C_{i+1} \\ F'(C_i^n) & \text{for } C_i = C_{i+1} \end{cases} \quad (16)$$

In the YRD method, $|a_{i+1/2}^n|$ is replaced by $\psi(a_{i+1/2}^n)$, as Equation (17) shows

$$\psi(a_{i+1/2}^n) = \begin{cases} \frac{a_{i+1/2}^n + \delta^2}{2\delta} & \text{for } |a_{i+1/2}^n| < \delta \\ |a_{i+1/2}^n| & \text{for } |a_{i+1/2}^n| > \delta \end{cases} \quad (17)$$

Here, δ is an arbitrary small value, which is determined as 10^{-20} in this study. The final step is determining the flux limiter $\phi_{i+1/2}^n$, and Yee *et al.* (1990) suggested three possible flux limiters

$$\phi(R_i^+, R_{i+1}^-) = \min\text{mod}(1, R_i^+, R_{i+1}^-) \quad (18)$$

$$\phi(R_i^+, R_{i+1}^-) = \min\text{mod}\left(2, 2R_i^+, 2R_{i+1}^-, \frac{1}{2}(R_i^+ + R_{i+1}^-)\right) \quad (19)$$

$$\phi(R_i^+, R_{i+1}^-) = \min\text{mod}(1, R_i^+) + \min\text{mod}(1, R_{i+1}^-) - 1 \quad (20)$$

where minmod is the minimum modulus. The minmod function returns the argument closest to zero if all of its arguments have the same sign, and it returns zero if any two of its arguments have different signs. In this study, we choose the first one, Equation (18), as the flux limiter, and the explicit YRD method is

$$C_i^{n+1} = C_i^n - \frac{\Delta t}{\Delta z} (\hat{F}_{i+1/2}^n - \hat{F}_{i-1/2}^n) \quad (21)$$

where

$$\hat{F}_{i+1/2}^n = \frac{1}{2} (F_i^n + F_{i+1}^n) + \frac{1}{2} \psi(a_{i+1/2}^n) (\phi_{i+1/2}^n - 1) (C_{i+1}^n - C_i^n) \quad (22)$$

The YRD method determines what to do in terms of the solution gradient rather than considering the solution's stability and accuracy in the same fashion throughout the entire domain. Therefore, the YRD method can work well in both regions simultaneously with small tradeoffs, and possesses

second-order accuracy. Since the SVT numerical flux is most often used in current engineering practices, we use it as a reference method to show the improvement of applying the YRD method.

SST behavior investigation (underloading and overloading conditions)

Wastewater flow rate and contaminant concentration vary, which means control strategies for SST must make appropriate adjustments. Hence, it is significant to understand SST's behavior in different operating conditions. SSTs are usually operated at underloading conditions, which requires the operating flux to be less than the limiting flux. Overloading can occur from hydraulic shock loading or sludge bulking.

In this study, we use both the SVT flux model and the YRD flux model to investigate the SST's response to different operating conditions (parameter set shown in Table 2). According to a discretization sensitivity study that numerical solution converges when the number of layers exceeds 50 (Li & Stenstrom 2014), the discretization level is determined as 50 layers.

System robustness study

SSTs may experience failure due to two primary causes: hydraulic shock loading and deterioration of sludge settleability. Time-to-failure is defined as the time interval between the beginning of an upset and failure, and can be used as an important indicator for system robustness evaluation (Diehl 2005, 2006). The longer time-to-failure indicates a more robust process. System robustness is closely related to SST size, since SST size can greatly impact several important operating factors, such as operating flux and limiting flux. To quantitatively investigate the relationship between system robustness and SST size, we simulated solids overloading for both hydraulic shock loading and sludge

settleability deterioration, for SST surface area from 100 to 400 m². All variations are imposed as step functions with the initial condition of zero concentration throughout the SST.

- Hydraulic shock loading: At $t = 0$ h, $Q_e = 200$ m³/h to reach steady state. At $t = 2$ h, Q_e is increased from 200 to 800 m³/h. C_f is fixed as 2,000 g/m³.
- Sludge settleability deterioration: Q_e and C_f are fixed as 200 m³/h and 2,000 g/m³. At $t = 0$ h, the settling parameters are set to normal as shown in Table 1. At $t = 2$ h, the settling velocity parameters change to deterioration (Table 1) in order to model a change to poor settleability condition (e.g., bulking).

RESULTS AND DISCUSSION

Numerical solution accuracy

To evaluate solution accuracy, we created a hypothetical but typical overloading condition ($A = 100$ m², $C_f = 4,000$ mg/l), with normal settling parameters as shown in Table 1. As can be seen from the predicted concentration profiles (Figure 1), both models are able to predict the sludge blanket level movement; however, the model solved by the SVT method provides smooth profiles rather than sharp discontinuities shown in the YRD one. The predictions also diverge with differences in the sludge blanket level, solids concentration in each layer and the underflow concentration. The sludge blanket level predicted by the SVT method is higher, while the concentration profile solved by the YRD method has an increased solids concentration in each layer, including the bottom one (the underflow concentration). Using the YRD method also provides a more accurate prediction of the discontinuities at the edge of the blanket. It is also significant to notice that the overestimation of the sludge blanket level may encourage designing larger SSTs.

In order to further demonstrate the reliability of the YRD method, we ran both the YRD method and the SVT method with the same scenario as in Figure 7 of Bürger *et al.* (2012), and the simulation results are shown as Figure 2. The concentration profiles constructed by the YRD method and the Method G are similar, which demonstrates that the YRD method is reliable to produce entropy-satisfying solutions, and can be an equivalent alternative to the G and EO methods. However, the SVT method provides solutions different from the YRD, G and EO methods, and it is also sensitive to the discretization level.

Table 2 | Parameter set to generate different operating conditions

	Underloading condition	Overloading condition 1	Overloading condition 2
A [m ²]	100	100	100
H [m]	4	4	4
h [m]	2	2	2
Q_e [m ³ /h]	200	200	200
Q_u [m ³ /h]	60	60	60
C_f [g/m ³]	2,500	4,000	9,000

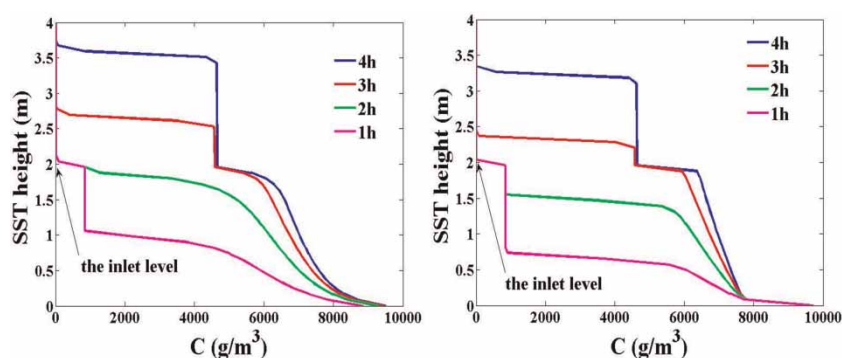


Figure 1 | Typical overloading concentration profiles (1, 2, 3 and 4 h) (left: the SVT method; right: the YRD flux method).

SST behaviors in various operating conditions

As can be seen in Figure 3, SSTs can convey most feed sludge towards the bottom and produce low turbidity effluent in the underloading case, which matches previous experiment observations (Tracy 1973). Sludge is thickened in the thickening zone for further recycle and disposal. Since the SVT numerical flux limits the gravity settling flux, the downward bulk flux is the only source for sludge transfer during the initial thickening time, which can cause a numerical delay. Therefore, an obvious sludge accumulation occurs in the SVT method results compared with the normal smooth concentration prediction of the model solved by the YRD method.

When C_f is $4,000 \text{ g/m}^3$ (overloading 1), the operating flux is larger than the limiting flux, and overloading occurs. Both models show that the sludge blanket will rise, although the predicted sludge blanket growth rate is different (2.7 m for the SVT method versus 2.2 m for the YRD method). This result supports the earlier statement that the model solved by the SVT method overestimates the sludge blanket height. Another key variable is the underflow concentration (C_u). Figure 3 shows that C_u is independent of sludge blanket height, and is approximately $10,000 \text{ g/m}^3$, matching the flux diagram prediction (Hassett 1958).

The SST behavior can be totally different after C_f increases to $9,000 \text{ g/m}^3$ (overloading 2), although the operating condition is still defined as solids overloading. In this case, instead of settling to the thickening zone, most sludge will be directly conveyed to the SST effluent weir by the effluent flow. Rather than a gradual sludge blanket growth from the SST bottom, we can observe sludge blanket rise in both thickening and clarification, and the latter one is even more rapid than the former one as shown in Figure 3. Finally, the sludge blanket will exceed the effluent weir, and cause an effluent validation, known as clarification failure.

The solutions solved by these two methods are totally different in this case. The predicated concentration difference in the clarification zone is $2,000 \text{ g/m}^3$ ($8,500 \text{ g/m}^3$ vs $6,500 \text{ g/m}^3$). The recycling concentration solved by the SVT method is $8,000 \text{ g/m}^3$, while if the YRD method is used as the PDE solver, it remains the same as overloading 1 ($10,000 \text{ g/m}^3$). For the sludge blanket level, the SVT method provides a higher value in the clarification zone, but lower value in the thickening zone compared to the solutions solved by the YRD method.

System robustness

SSTs with larger surface area are usually considered to be more robust compared with smaller ones in terms of offering more sludge storage capacity and smaller operating flux. However, this cannot always be correct, since the associated limiting flux can also decrease with the increase of size. Hence, in order to quantitatively investigate this problem, time-to-failure is selected as a system robustness indicator. Generally, a lengthy time to reach failure implies a more stable process. Figure 4 illustrates time-to-failure after a 20-h hydraulic shock loading simulation (Figure 4 left) and deterioration of sludge settleability (Figure 4 right).

It is notable that the estimated time-to-failure based on the solutions solved by the SVT method is much smaller than what the YRD method provides. This corresponds well to the conclusion presented earlier in the numerical accuracy section that the model solved by the SVT method overestimates the sludge blanket height due to numerical inaccuracies. As a consequence, the time-to-failure solved by the YRD method is used for system robustness analysis.

According to Figure 4 (left), the hydraulic shock loading failure time of smaller SST alternatives ($A = 100$ to 135 m^2) is less than 0.1 h. It increases to 1.5–4 h, a great improvement, when SSTs are enlarged to medium size ($A = 140$ to

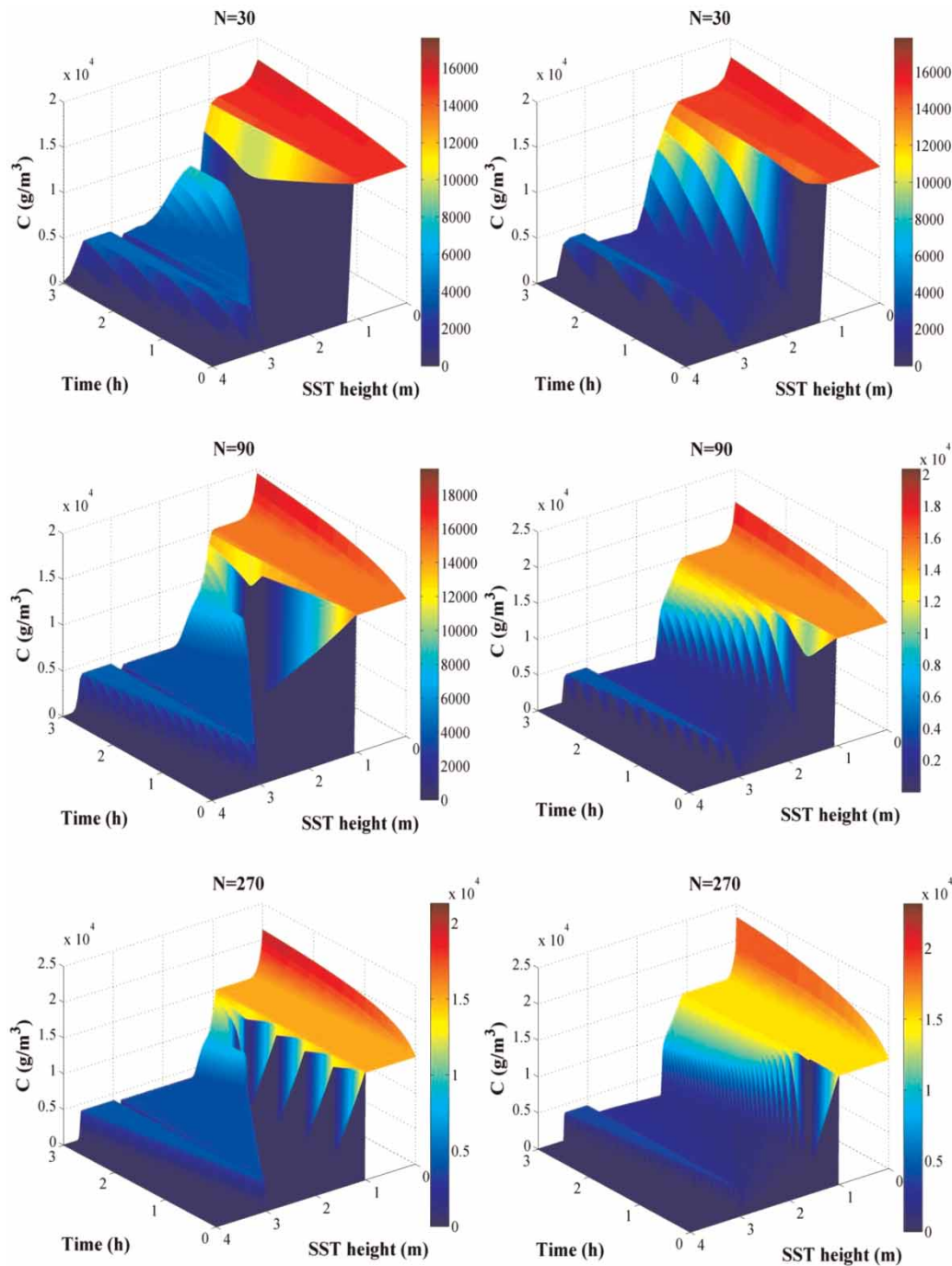


Figure 2 | Concentration profiles of the SVT method (left) and the YRD method (right).

250 m²). No failure will occur if the SST is larger than 250 m². For the case of a small SST, most biomass is directly conveyed to the clarification zone by the overflow instead of settling to the thickening zone, causing a clarification failure in less than 0.1 h. This helps explain why small SSTs have extremely short time-to-failure. A gradual sludge blanket

rise is observed in medium SSTs, and causes a thickening failure when it reaches the feed point. An area of 140 m² is the demarcation point between clarification failure and thickening failure. Compared with a clarification failure, the thickening failure is a relatively slow process as the sludge blanket must rise from the bottom to top, which

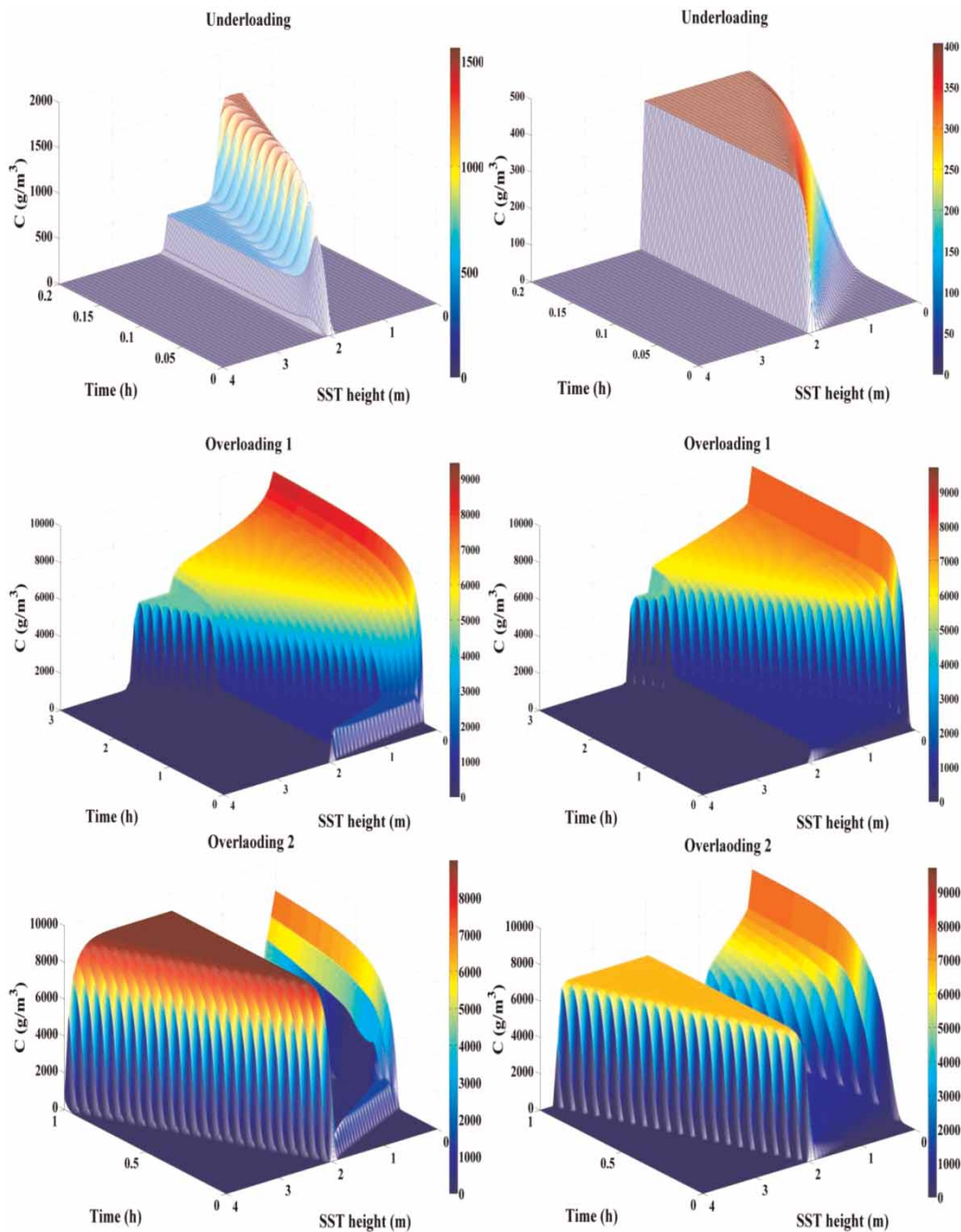


Figure 3 | Concentration profiles of different solids loading conditions (left: the SVT method; right: the YRD method).

usually occurs over several hours. If the SST can afford a large enough limiting flux, the system can always maintain an underloading condition. For this reason, neither clarification nor thickening failure occurs when the SST area is greater than 260 m².

Compared to hydraulic shock loading, where the failure is caused by a sudden increase of operating flux, failure due

to poor biomass settleability (sludge bulking), is attributed to a decrease in the limiting flux. [Figure 4](#) (right) shows a similar failure time change tendency observed in hydraulic shock loading: a rapid to gradual process. In this case, failure can be avoided only by increasing the limiting flux, such as changing the recycle rate or contacting pattern ([Stenstrom & Andrews 1979](#)).

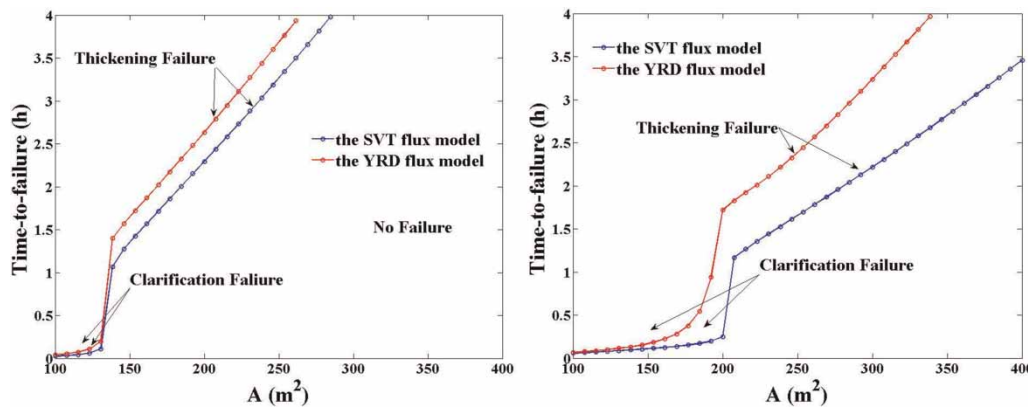


Figure 4 | Failure time in the hydraulic shock loading (left) and the settleability deterioration (right).

CONCLUSIONS

The conclusions of this study can be summarized as follows:

- Instead of applying the empirical SVT method as the non-linear hyperbolic governing PDE solver, the YRD method determines the calculation behavior in terms of the solution gradient, and provides both numerically and physically acceptable solutions that satisfy the CFL condition and entropy condition. Therefore, the YRD method is a reliable numerical technique for solving the nonlinear hyperbolic PDE of the SST model, and can be an acceptable alternative to the G and EO methods.
- Both clarification and thickening failure can occur during overloading, with the magnitude of the overloading determining the type of failure. Clarification failure occurs with greater overloading. The model solved by the SVT method is likely to produce unrealistic solids accumulation during underloading but both models perform well in thickening failure prediction (overloading 1). For clarification failure (overloading 2), the model solved by the YRD method provides more accurate recycle solids concentration and sludge blanket level prediction.
- The choice of numerical methods can greatly impact the model solutions, for instance the time-to-failure evaluation. Compared with the exact time-to-failure solved by the YRD method, the SVT method can underestimate the time-to-failure, and lead to conservative design and operation strategies. Therefore, reliable numerical techniques, such as the YRD method, are strongly recommended for 1D SST model solving.

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