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A sensitivity and model reduction analysis of one-dimensional secondary settling tank models under wet-weather flow and sludge bulking conditions

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HIGHLIGHTS

- Parameter subsets suitable for the Bürger–Diehl model calibration are identified.
- Parameter interactions are evaluated based on GSA results.
- The imposed simulation conditions impact the sensitivity of model outputs to parameters.
- Reliable reduction of the Bürger–Diehl model can be achieved based on GSA results.
- Uncertainty analysis can be used to evaluate the Bürger–Diehl model reductions.

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ABSTRACT

In this study, we provide the sensitivity and reduction analysis of the current, most advanced one-dimensional (1-D) secondary settling tank (SST) model, the Bürger–Diehl model, under non-ideal flow and settling conditions. Parameter subsets suitable for the Bürger–Diehl model calibration are identified. For example, under the wet-weather condition, all model parameters, except f_n and $D_{c,0}$, are influential to SST performance. When filamentous bulking occurs, the outputs of the Bürger–Diehl model are most sensitive to the hindered settling parameters, v_0 and r_h. In terms of parameter interactions, strong interactions are found among parameters in predicting C_e. However, for C_u, SI and Flux_{op} under the bulking condition, the Bürger–Diehl model is almost additive with negligible parameter interactions. Moreover, the sensitivity of the Bürger–Diehl model outputs to parameters is highly impacted by the imposed simulation conditions, thus resulting in different parameter subsets for model calibration. For example, under the wet-weather condition, the compression settling parameters can be as important as the hindered settling parameters, while bulking of the sludge greatly increases the influence of the hindered settling parameters (v_0 and r_h), and decreases the influence of the compression settling parameters. In terms of model reduction, reliable reduction of the Bürger–Diehl model can be achieved based on GSA results. For example, under the bulking condition, the Bürger–Diehl model can be reduced to the hindered–dispersion model regardless to model outputs. The reliability of the Bürger–Diehl model reduction can be evaluated based on uncertainty, and unreliable reduction can negatively impact the decision making in SST design and control.

1. Introduction

The activated sludge process is the most widely used technique to remove organic matter and reduce nutrients such as nitrogen and phosphorus in wastewater treatment plants (WWTPs). Generally, efficient solids–liquid separation techniques are needed to provide low turbidity effluent by removing the biomass from the liquid, and the secondary settling tanks (SSTs), where biomass is settled by gravity, are the most commonly used [1]. Mathematical modeling approaches, where the activated sludge models, comprised of a set of ordinary differential equations (ODEs), are coupled with the SST models, comprised of a set of partial differential equations (PDEs), are being increasingly used in wastewater treatment process studies for three purposes (1) learning, which means the model simulation results are able to improve

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the understanding of wastewater treatment process; (2) design, the model can be used to evaluate various design alternatives via simulation, and (3) process optimization and control, simulating different sceneries to optimize the process efficiency and avoid possible failure problems [2–4].

The family of activated sludge models [5–7] provide a comprehensive description of the significant biological processes of the activated sludge system, and are widely accepted in the research and industrial communities as a useful tool for scientific study and practical applications. However, compared with the well-developed scientific knowledge on characterizing the metabolic processes and contaminant removal in the bioreactor, various settling behavior occurring in the SST still remain poorly understood, thus making the SST model a potential error source in process simulation [8]. The one-dimensional (1-D) 10-layer model, also known as the Takács model [9], is the most commonly used SST model and has been implemented in most commercial simulators as a reference model. Although the Takács model has achieved a degree of success in predicting the SST performance, its shortcomings are not negligible, such as the insufficient description of various settling behaviors and inaccuracy of numerical solutions, which have been demonstrated in previous studies [8,10–13].

In last two decades, to compensate for the limitations of the Takács model, several advanced SST models have been developed as alternatives, which can be classified into three groups based on their advantages:

1. First-order hindered-only models with reliable numerical techniques: for these models, the model formula remains the same as the Takács model, considering only the hindered settling behavior, but using more reliable numerical techniques. Reliable techniques such as the Godunov numerical flux, the Yee–Roe–Davis (YRD) numerical flux, and finer discretization levels (more than 30-layers), are used to construct both numerically and physically acceptable solutions [10,13,14].
2. Second-order hindered–compression models additionally accounting for compression settling: the improved understanding of activated sludge rheology has facilitated the development of phenomenological theory of sedimentation-consolidation. The phenomenological theory is then expressed in the compression model, which allows a more rigorous description of the compression settling behavior [15,16]. Compared with the hindered-only model, the hindered–compression model is expected to provide more realistic predictions of the sludge blanket level and the underflow concentration.
3. Second-order hindered–dispersion models additionally accounting for hydraulic dispersion: for these models, an explicit hydraulic dispersion term is added to the model formula to account for the potential impact of hydraulics on the biomass settling behavior [17,18]. The hydraulic dispersion model possesses the advantage of simulating the hydraulics of SSTs over a wider range of dynamic flow conditions [17,19]. From the numerical point of view, adding the explicit flow-dependent dispersion term also decreases the difficulty in solving the hindered–dispersion model.

Recently, a new 1-D SST model, the Bürger–Diehl model (the hindered–compression–dispersion model), has been presented [20], which accounts for phenomena that may impact the SST behavior, such as hindered settling, compression settling and hydraulic dispersion. The Bürger–Diehl model is also based on the reliable numerical solution of its governing model formula by appropriate methods [21]. Therefore, the Bürger–Diehl model is able to provide more realistic predictions of the SST performance.

Despite the advantages of the Bürger–Diehl model, its practical application is limited, which can be attributed to two main reasons:

(1) The difficulty of calibration: great efforts have been made to facilitate model calibration, for example by evaluating the hindered-only and hindered–dispersion models, Ramin et al. [18,22] identified the potential parameter subsets suitable for the calibration of WWTP models under various simulation conditions. However, calibrating the 1-D SST models accounting for the compression settling still remains a challenge due to the insufficient understanding of the influence of compression settling on the SST performance.
(2) The increased implementation complexity and computation burden: technically, the currently used hindered-only, hindered–compression and hindered–dispersion models can be considered as the sub-models of the Bürger–Diehl model, and their successful applications in SST simulation implies that the Bürger–Diehl model in some cases can be reduced to these sub-models without sacrificing the quality of prediction. However, how to reliably reduce the Bürger–Diehl model, particularly under non-ideal flow and settling conditions, still remains unclear.

In this study, we provided a comprehensive sensitivity and model reduction analysis of the Bürger–Diehl model under non-ideal flow and settling conditions. The Benchmark Simulation Model No. 1 (BSM1) [23] is used as the simulation platform, because of its well documented model inputs. The influence of the uncertainty of model parameters to the variance of model outputs, such as the sludge blanket level, is quantified by using global sensitivity analysis (GSA), and the reliability of the Bürger–Diehl model reduction is evaluated based on uncertainty analysis.

The main objectives of this paper are (i) identify the suitable parameter subsets for the Bürger–Diehl model calibration under non-ideal flow and settling conditions; (ii) evaluate the influence of imposed flow and settling conditions on the sensitivity of the Bürger–Diehl model outputs to the parameters; (iii) demonstrate how reliable reduction of the Bürger–Diehl model can be achieved based on GSA results; (iv) assess the reliability of the Bürger–Diehl model reduction for different modeling purposes based on uncertainty analysis results.

2. Materials and methods

2.1. Model structure and simulation condition description

As shown by Fig. 1, BSM1 is used as the simulation platform, where ASM1 is combined with the SST model to describe the biological and settling processes of the activated sludge system. For further details about ASM1, the reader is referred to literature [5]. With regards to the SST model, the Bürger–Diehl model is used to replace the Takács model.

The formula of the Bürger–Diehl model can be expressed as Eq. (1) on the basis of the mass and momentum conservation:

$$\frac{\partial C}{\partial t} + \frac{\partial}{\partial x} F(C, x, t) = \frac{\partial}{\partial x}\left((d_{\mathrm{disp}}(v_{ov}) + d_{\mathrm{comp}}(C)) \frac{\partial C}{\partial x} \right) + \frac{Q_f(t) C_f(t)}{A} \delta(t) \quad (1)$$

where C is the solids concentration, t is time, x is the depth from the feed layer, v_{ov} is the overflow velocity, Q_f is the feed flow rate, A is the SST surface area, C_f is the feed solids concentration, δ is the Dirac delta distribution and the transport flux F can be written as Eq. (2) [24]:

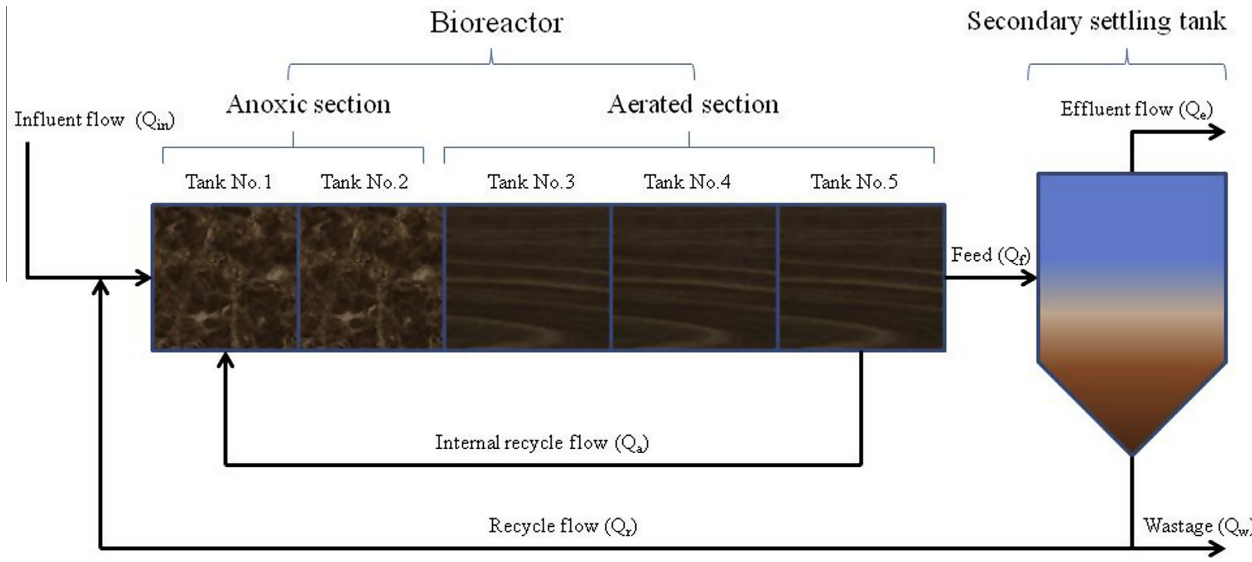

Fig. 1. Layout of the Benchmark Simulation Model No. 1 (BSM1).

$$F(C,x,t) = \begin{cases} -\frac{Q_e}{A}C_e & \text{effluent region} \\ v_{hs}(C)C - \frac{Q_e}{A}C & \text{clarification zone} \\ v_{hs}(C)C + \frac{Q_u}{A}C & \text{thickening zone} \\ \frac{Q_u}{A}C_u & \text{underflow region} \end{cases} \quad (2)$$

where Q_e is the effluent flow rate, Q_u is the underflow rate, C_e is the effluent solids concentration and C_u is the underflow concentration. The hindered settling velocity, v_{hs}, is calculated by the double-exponential equation [9]:

$$v_{hs} = v_0\left(e^{-r_h(C-f_{ns}C_f)} - e^{-r_p(C-f_{ns}C_f)}\right) \quad (3)$$

The compression function is approximated by Eq. (4) recommended by previous publications [20,25]:

$$d_{\text{comp}}\ (C) = \begin{cases} 0 & 0 \leqslant C < C_g \\ \frac{\rho_s \cdot \alpha \cdot v_{hs}(C)}{g(\rho_s - \rho_f)(\beta + C - C_g)} & C \geqslant C_g \end{cases} \quad (4)$$

where ρ_s is the solids density, ρ_f is the liquid density. The dispersion function developed by Plósz et al. [17] is used to characterize the hydraulic dispersion, shown as Eq. (5):

$$D_{\text{disp}}\ (v_{ov}) = \begin{cases} D_{c,0} & v_{ov} < v_{ov,c} \\ D_{c,0} + \gamma(v_{ov} - v_{ov,c})^2 & v_{ov} \geqslant v_{ov,c} \end{cases} \quad (5)$$

Table 1
Uncertainty of the Bürger–Diehl model parameters under good settling and bulking conditions.

	Unit	Good settling		Bulking	
		Min	Max	Min	Max
v_0	m/d	355.5	592.5	177.75	296.25
r_h	m^3/kg	0.432	0.721	0.75	1.25
r_p	m^3/kg	2.7	10	2.7	10
f_{ns}	–	0.00123	0.00259	0.00123	0.00259
C_g	kg/m^3	6.06	10.12	6.06	10.12
α	Pa	0	20	0	20
β	kg	1	10	1	10
$D_{c,0}$	m^2/d	1.98	5.92	1.98	5.92
γ	d	1.1E−2	3.3E−2	1.1E−2	3.3E−2
$v_{ov,c}$	m/d	10	22	10	22

Uncertainty is determined based on literature review and expert knowledge.

Quantifying prior uncertainty of all model parameters is one of the most important but difficult task for reliable sensitivity analysis. Table 1 shows the definition and prior uncertainty of parameters. The uncertainty of hindered settling parameters (v_0, r_h, r_p and f_n), compression settling parameters (C_g, α and β) and dispersion parameter, $v_{ov,c}$, are determined based on literature references and expert knowledge. Since the dispersion parameters, $D_{c,0}$ and γ, are not well documented in previous studies, we introduce a relative uncertainty (50%) based on the default values reported by Plósz et al. [17] to reasonably quantify their uncertainties. The probability distributions of all model parameters are assumed uniform because of no prior knowledge [22].

To comprehensively evaluate the 1-D SST models, three scenarios of non-ideal flow and settling conditions are selected to run the BSM1 as following:

1. Wet-weather inflow but good biomass settleability: based on the values of ASM1 inputs and parameters follows the dry-weather data set provided by Alex et al. [23], and a constant inflow rate (18,446 m^3/d), a 150-day simulation is conducted to obtain steady-state, and then a wet-weather condition (14.4-h) is imposed by increasing the influent flow rate to four times of the average dry-weather inflow rate.
2. Dry-weather inflow but filamentous bulking: the same strategy as scenario 1 is used to reach steady state. Then, the filamentous bulking condition is simulated as long as 9.6 h on the basis of the parameter uncertainty of bulking as shown in Table 1; (Since no significant association is found between compression parameters and filamentous abundance [26], same uncertainty ranges of compression parameters are used under bulking condition as those used under good settling condition).
3. Wet-weather inflow and filamentous bulking: the steady-state is obtained by 150-day simulation similar as scenarios 1 and 2. A 4.8-h simulation of the wet-weather inflow and bulking condition is conducted by increasing the influent flow rate twice of the average dry-weather inflow rate, and using the settling parameter set of bulking.

2.2. Global sensitivity analysis

To improve the model understanding, global sensitivity analysis is usually introduced to investigate the dependence of model outputs on the uncertainties of model factors (defined as both model parameters and inputs). This is informative for various purposes, such as quantifying the individual contribution of the uncertain parameters to the model output uncertainty to identify influential parameters.

The global sensitivity technique used in this study is the Extended-Fourier Amplitude Testing (Extended-FAST), originally developed by Cukier et al. [27] and Schaibly and Shuler [28], and later extended by Satelli et al. [29]. Compared with frequently used regression-based and screening methods, the Extended-FAST possesses the advantages of providing accurate sensitivity measures in various model structures (linearity, monotonicity etc.) without the necessary of making any assumptions of model behaviors. As a variance-based approach, the Extended-FAST has its root in the general theorem that the total variance can be decomposed into conditional variances, as shown in Eq. (6):

$$\mathrm{Var}(sy) = \mathrm{Var}(E(sy|\theta_i)) + E(\mathrm{Var}(sy|\theta_i)) \tag{6}$$

where Var and E is the variance and expectancy operator respectively, sy denotes a vector of scalar values for the model output and θ_i is the ith model factor. The Extended-FAST converts the multidimensional integral over all the uncertain parameters in a one-dimensional integral using a transformation function that scans the entire parameter space, and the contribution of the individual parameters to the variance of the model output is calculated by using a Fourier decomposition [30,31]. The Extended-FAST implementation strategy used in this study is based on Satelli et al. [29], and the transformation function is given as Eq. (7):

$$\theta_i = \frac{1}{2} + \frac{1}{\pi}\arcsin(\sin(\omega_i s + \varphi_i)) \tag{7}$$

where s ranges from $-\pi/2$ to $\pi/2$, ω is a set of different frequencies and φ_i is a random phase-shift. The total number of model evaluation required can be determined by Eq. (8):

$$N_s = m(2M\omega_{\max} + 1) \tag{8}$$

where m is the number of model parameters, M is the interference frequencies, and $w_{\max}$ is the maximum frequency. For further information about Extended-FAST implementation strategy, such as the selection of ω, the reader is referred to the literature [29].

Generally, Extended-FAST provides three kinds of sensitivity measures: the first-order effect index (S_i), which represents the contribution of the variance of individual parameters to the output variance without considering the interaction with other parameters, and can be used to identify the most significant factors (factors prioritization); the total effect index (S_{Ti}), which accounts for the total contribution of the parameter to the output variance, and is informative for determining factors that can be fixed without greatly reducing output variance (factor fixing); the interaction (S_{Si}), which can be used to evaluate the interactions among parameters.

2.3. Uncertainty analysis with Monte Carlo procedure

Uncertainty analysis can be understood as the propagation of the uncertainty of parameters to model outputs, which is beneficial for design and control decision-making. The model uncertainty analysis in this study involves the following steps as listed by Sin et al. [32]:

1. Specifying input uncertainty: for each scenario, only the SST model parameters are considered as uncertainty source;
2. Sampling input uncertainty: Latin hypercube sampling is applied;
3. Propagating input uncertainty to obtain prediction uncertainty: Monte Carlo simulation is used;
4. Representation and interpretation of results: the predicted uncertainty results are represented using mean, percentiles and cumulative distribution functions.

The possible correlations among parameters are not considered here, since there is no detailed information available about the correlation matrix.

2.4. Numerical setting and comparison of SST models

The discretization level of SST models is 30-layer. Given that the GSA and reduction analysis are made based on the numerical solutions of the Bürger–Diehl model, reliable numerical techniques are needed: the solids transport flux is approximated by Yee–Roe–Davis numerical flux, and the compression and dispersion terms are approximated following the strategy provided by Bürger et al. [33]. For Extended-FAST, M and $\omega_{\max}$ is 4 and 8 respectively. Only factors with S_i larger than 0.01 or S_{Ti} larger than 0.1 are considered to be important according to Cosenza et al. [34]. To provide an effective coverage of model output uncertainty, the BSM1 model with different SST models is simulated 500 times for each scenario following the benchmark simulation strategy.

Five significant model outputs: sludge blanket height (SBH), C_e, C_u, sludge inventory (SI) and operating flux (Flux$_{op}$), are used to characterize the SST performance, since they are closely related to the SST robustness, the mostly interesting topic under non-ideal flow and settling conditions.

3. Results and discussion

3.1. Global sensitivity analysis of the Bürger–Diehl model under non-ideal flow and settling conditions

In this section, the GSA results of Bürger–Diehl model are provided in order to identify the potential parameter subsets suitable for model calibration. Table 2 shows the sensitivity measures (S_i and S_{Ti}) of the Bürger–Diehl model under the wet-weather condition (scenario 1). The high sensitivity indices ($S_i > 0.01$) of v_0 and r_h indicate their strong influence on the model outputs as well as implying the important role hindered settling plays in determining the SST performance. In contrast, the hindered settling parameter, r_p, is non-influential to SST behavior, with only one notable exception: C_e, where r_p contributes more than 50% percent of the total variance. It means that to accurately predict C_e, r_p needs to be carefully calibrated. Regarding the compression settling parameters, the gel concentration, C_g, is another significant parameter strongly impacting the SST performance, particularly, in the case of C_u, SI and Flux$_{op}$, where C_g contributes more than 30% of the total variance. The other two compression settling parameters, α and β, are moderately influential to C_u, SI and Flux$_{op}$ as well. The high sensitivity of C_u, SI and Flux$_{op}$ to the compression settling parameters can be interpreted by the fact that the sludge with high compressibility can be easily compacted in the thickening zone, thus leading to the increase of C_u and Flux$_{op}$, but a decrease of SI. The hydraulic dispersion parameters, γ and $v_{ov,c}$, are important for SST behavior too, especially in the case of C_e, where γ ranks the second of most influential parameters. This is due to the explicit hydraulic term accounts for some of the variations in hydraulic feathers of SST under the wet-weather condition, as reported by Ramin et al. [22]. f_n and $D_{c,0}$, are identified as non-influential parameters, since their sensitivity measures are much smaller than the corresponding thresholds.

The sensitivity measures of model parameters under the bulking condition are shown in Table 3. It is interesting to find that the sensitivity measures of compression settling parameters, C_g, α

Table 2
Parameter sensitivity indices of the Bürger–Diehl model outputs in scenario 1. (The sensitivity indices larger than thresholds are in bold.)

Wet-weather and good settling											
		v_0	r_h	r_p	f_n	C_g	α	β	$D_{c,0}$	γ	$v_{ov,0}$
SBH	S_i	**0.071**	**0.461**	0.004	0.002	**0.046**	9.1E−4	6.7E−4	0.003	**0.085**	**0.067**
	S_{Ti}	0.086	**0.471**	0.014	0.016	0.063	0.004	0.005	0.007	0.097	0.086
C_e	S_i	**0.199**	**0.209**	**0.544**	3.5E−4	**0.011**	0.001	1.7E−4	0.001	**0.299**	**0.093**
	S_{Ti}	**0.438**	**0.453**	**0.593**	0.039	0.048	0.027	0.011	0.008	**0.578**	**0.186**
C_u	S_i	**0.046**	**0.261**	0.002	8.1E−4	**0.333**	**0.018**	0.008	1.6E−4	0.009	**0.031**
	S_{Ti}	0.067	**0.299**	0.012	0.013	**0.394**	0.027	0.016	0.002	0.013	0.049
SI	S_i	**0.019**	**0.282**	0.008	9.4E−4	**0.374**	**0.024**	**0.011**	2.9E−4	0.001	0.007
	S_{Ti}	0.052	**0.427**	0.021	0.012	**0.438**	0.036	0.024	0.004	0.005	0.029
$Flux_{op}$	S_i	**0.046**	**0.254**	0.001	7.8E−4	**0.351**	**0.017**	0.007	1.6E−4	0.008	**0.031**
	S_{Ti}	0.067	**0.295**	0.012	0.012	**0.411**	0.027	0.015	0.002	0.012	0.048

Table 3
Parameter sensitivity indices of the Bürger–Diehl model outputs in scenario 2. (The sensitivity indices larger than thresholds are in bold.)

Dry-weather and filamentous bulking											
		v_0	r_h	r_p	f_n	C_g	α	β	$D_{c,0}$	γ	$v_{ov,0}$
SBH	S_i	**0.125**	**0.895**	0.001	5.6E−4	0.004	8.7E−4	0.001	**0.051**	0.001	0.001
	S_{Ti}	**0.143**	**0.911**	0.008	0.006	0.029	0.006	0.011	0.071	0.009	0.007
C_e	S_i	**0.191**	**0.138**	**0.823**	**0.063**	0.003	0.003	0.002	**0.062**	0.006	0.001
	S_{Ti}	**0.355**	**0.336**	**0.866**	**0.142**	0.022	0.054	0.021	**0.103**	0.043	0.029
C_u	S_i	**0.076**	**0.931**	8.2E−5	5.4E−6	4.9E−5	1.6E−5	5.7E−6	**0.016**	6.4E−6	4.1E−6
	S_{Ti}	0.078	**0.933**	0.003	0.001	0.004	0.001	0.003	0.021	0.003	0.001
SI	S_i	**0.079**	**0.933**	1.7E−5	1.2E−5	8.9E−5	1.4E−5	1.4E−5	**0.019**	1.1E−5	7.7E−6
	S_{Ti}	0.081	**0.935**	0.003	0.001	0.004	0.001	0.003	0.022	0.003	0.001
$Flux_{op}$	S_i	**0.083**	**0.924**	0.001	5.5E−6	6.7E−5	2.1E−5	6.7E−6	**0.016**	9.4E−6	7.2E−6
	S_{Ti}	0.085	**0.926**	0.003	0.001	0.004	0.001	0.003	0.021	0.003	0.001

and β, are smaller than the thresholds regardless of the model outputs, which means that the compression settling is not influential to SST performance under the bulking condition. The hindered settling parameters, v_0 and r_h, are the most influential parameters, which contributes more than 95% of total variance to most model outputs, thus demonstrating the importance of reliably calibrating v_0 and r_h under the bulking condition. According to Table 3, C_e is most sensitive to the change of r_p, which contributes more than 80% of total variance of C_e. Therefore, reliable calibration of r_p is highly needed for accurate prediction of C_e when sludge bulking occurs. Similar as r_p, f_n is only influential to C_e with 6% contribution of total variance. The hydraulic dispersion parameter found to be important is $D_{c,0}$, even though its contribution of variance is relatively small compared those made by hindered settling parameters. Consequently, hindered settling process is most influential to the SST performance under the bulking condition, while the effect of compression settling is almost negligible.

According to Table 4, four parameters, v_0, r_h, r_p and $D_{c,0}$, are important for model calibration under the wet-weather and filamentous bulking condition, since their sensitivity measures are larger than the thresholds. For SBH and C_e, they are highly sensitive to the change of both hindered and hydraulic dispersion parameters, which means they are results of hindered settling and hydraulic dispersion effects. However, in contrast to scenarios 1 and 2 where C_e is most sensitive to r_p, C_e in scenario 3 is mostly determined by r_h, which accounts for more than 50% of the total variance. The hindered settling process primarily impacts C_u, SI and $Flux_{op}$, because the hindered settling parameters, v_0 and r_h, are the only influential parameters. Consequently, the reliable calibration of v_0 and r_h can greatly reduce the uncertainty of model outputs, hence producing an adequate description of the SST behavior under the wet-weather and bulking condition.

3.2. Parameter interactions of the Bürger–Diehl model under non-ideal flow and settling conditions

Although the analysis of S_i greatly facilitates the SST model calibration by identifying influential parameters suitable for prioritization, it is important to emphasize that the analysis of S_{Ti} is still required; for parameters having small S_i, they cannot be simply fixed as non-influential parameters because of the possible interactions reflected by their high S_{Ti} values. In this study, the interac-

Table 4
Parameter sensitivity indices of the Bürger–Diehl model outputs in scenario 3. (The sensitivity indices larger than thresholds are in bold.)

Wet-weather and filamentous bulking											
		v_0	r_h	r_p	f_n	C_g	α	β	$D_{c,0}$	γ	$v_{ov,0}$
SBH	S_i	**0.132**	**0.929**	9.8E−4	2.1E−4	7.6E−4	4.7E−4	8.3E−4	**0.013**	0.001	0.006
	S_{Ti}	**0.152**	**0.947**	0.008	0.004	0.018	0.003	0.006	0.022	0.007	0.012
C_e	S_i	**0.222**	**0.509**	**0.025**	0.004	6.8E−4	0.001	0.004	**0.039**	0.008	0.003
	S_{Ti}	**0.591**	**0.801**	0.091	0.019	0.029	0.005	0.059	**0.181**	0.089	0.018
C_u	S_i	**0.088**	**0.952**	1.3E−4	2.6E−5	1.1E−4	3.4E−5	2.2E−5	0.006	4.9E−4	0.003
	S_{Ti}	0.091	**0.954**	0.003	0.002	0.005	0.001	0.003	0.01	0.004	0.006
SI	S_i	**0.056**	**0.937**	4.7E−4	7.2E−4	3.1E−4	1.1E−4	3.7E−4	0.007	4.1E−4	0.006
	S_{Ti}	**0.106**	**0.987**	0.004	0.004	0.006	0.003	0.002	0.011	0.003	0.009
$Flux_{op}$	S_i	**0.115**	**0.919**	0.001	2.4E−5	2.2E−4	4.2E−5	3.6E−5	0.005	3.1E−4	0.002
	S_{Ti}	**0.118**	**0.922**	0.004	0.002	0.005	0.001	0.003	0.009	0.003	0.005

tions among parameters are characterized by the ratio of the sum of S_i to the sum of S_{Ti}, as shown in Fig. 2 (left). As can be seen, the sum of S_i is always smaller than the sum of S_{Ti}, which indicates the existence of interactions. This result is more pronounced for C_e, where the ratio of the sum of S_i to the sum of S_{Ti} is smaller than 60% for all scenarios. However, for C_u, SI and $Flux_{op}$ in scenarios 2 and 3, the sum of S_i is close to the sum of S_{Ti}, which means the model is almost additive without parameter interactions affecting model outputs.

To further investigate the parameter interactions, we take the parameter interaction analysis of C_e under the wet-weather condition as an example, where the ratio of the sum of S_i to the sum of S_{Ti} for important parameters are shown as Fig. 2 (right). According to Fig. 2 (right), strong parameter interactions are observed in most parameters, which implies that the effluent concentration under the wet-weather condition is strongly influenced by hindered settling, compression settling and hydraulic dispersion. What is surprising is that even though r_p is the most influential parameter on C_e, its interactions with other parameters are negligible, as indicated by the high ratio of S_i to S_{Ti}. This may be attributed to the fact that r_p is a parameter used to describe settling behavior in low solids concentration range, while other parameters, especially the compression parameters, mostly affect the medium and high concentration domains. Based on the results of parameter interaction analysis, simple GSA techniques, such as the Standard Regression Coefficients method or Morris screening method, are sufficient for the sensitivity analysis of C_u, SI and $Flux_{op}$ in scenarios 2 and 3, while the advanced GSA methods, such as Extended-FAST, are needed for the sensitivity analysis of C_e.

3.3. Influence of imposed flow and settling conditions on the sensitivity of the Bürger–Diehl model outputs to parameters

In this study, the influence of imposed flow and settling conditions on the sensitivity of the Bürger–Diehl model outputs to the parameters are analyzed on the basis of the similarity of parameters identified to be important as well as the similarity of ranking of sensitivity indices. The Venn diagrams, Fig. 3, are used for the comparison of important parameter identified in different scenarios. The significant hindered settling parameters found in scenarios 1–3, are almost identical regardless to the model outputs, with only one exception: f_n which is only important for the prediction of C_e in scenario 2. The important parameters found in scenarios 2 and 3 are similar, and moreover, all influential parameter in scenario 3 are important in scenario 2. The main difference between scenarios 2 and 3 is that the dispersion parameter, $D_{c,0}$, is influential for the predication of C_u and SI in scenario 2, which implies the SST performance under the bulking condition is sensitive to the hydraulic dispersion. Great differences can be observed between scenario 1 and the other two scenarios in terms of the significance of the compression parameters. The compression settling parameters, especially C_g, are strongly influential on SST performance in scenario 1, while none of the compression parameters are important in scenarios 2 and 3. Another interesting observation is that significant hydraulic dispersion parameters found in scenarios 1 and 2 are different: γ and $v_{ov,c}$ are influential on SST behavior in scenario 1, while $D_{c,0}$ is the only hydraulic dispersion parameter important in scenario 2.

The similarity of ranking of sensitivity indices in scenarios 1 to 3 is quantitatively investigated based on the Spearman's rank correlation index (N_S), and the high similarity of ranking leads to large N_S. According to Table 5, the rankings of sensitivity measures obtained in scenarios 2 and 3 are similar in terms of the model output variables SBH, C_e and $Flux_{op}$, where N_s values are larger than 0.7. However, for most model outputs, the ranking of sensitivity measures obtained in scenario 1 differs greatly from those obtained in scenarios 2 and 3, which is demonstrated by the small N_s. For instance, C_g is the most important parameter for the change of C_u, SI and $Flux_{op}$ in scenario 1, while in scenarios 2 and 3, most of the variance of C_u, SI and $Flux_{op}$ are contributed by r_h. Therefore, it should be noted that for the Bürger–Diehl model, sensitivity of model outputs to parameters strongly depends on the imposed flow and settling conditions, and suitable parameter subsets used for model calibration need to be determined based on flow and settling conditions to improve the calibration efficiency and reliability.

3.4. Reduction of the Bürger–Diehl model based on GSA results

Reduction of the Bürger–Diehl model, to some extent, is useful to facilitate its practical application, such as inclusion in the popular commercial simulators. Therefore, in this section, we focus on how to reliably reduce the Bürger–Diehl model for specific modeling purposes based on the GSA results.

According to Table 2, C_g is the only compression settling parameters influential on SBH and C_e in scenario 1 with sensitivity measures close to the threshold. It implies that if the primary modeling interests are SBH and C_e under the wet-weather condition, the Bürger–Diehl model (hindered–compression–dispersion) can be reduced to the hindered–dispersion model without greatly deteriorating prediction quality. However, in the case of SI, which is sen-

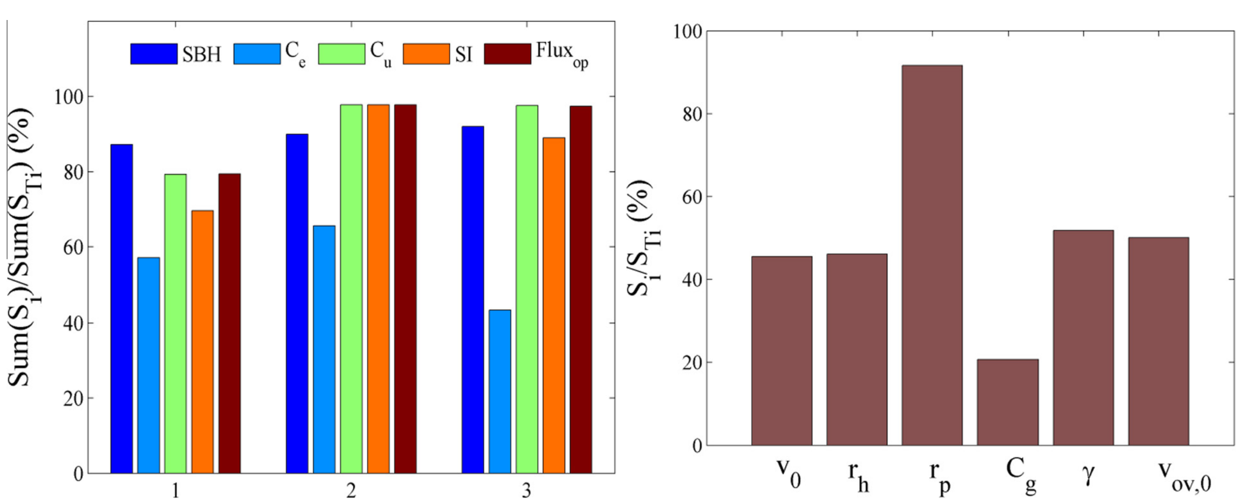

Fig. 2. Ratio of sum (S_i) to sum (S_{Ti}) of the Bürger–Diehl model outputs in scenarios 1 to 3 (left), and ratio of S_i and S_{Ti} of the Bürger–Diehl model parameters in scenario 1.

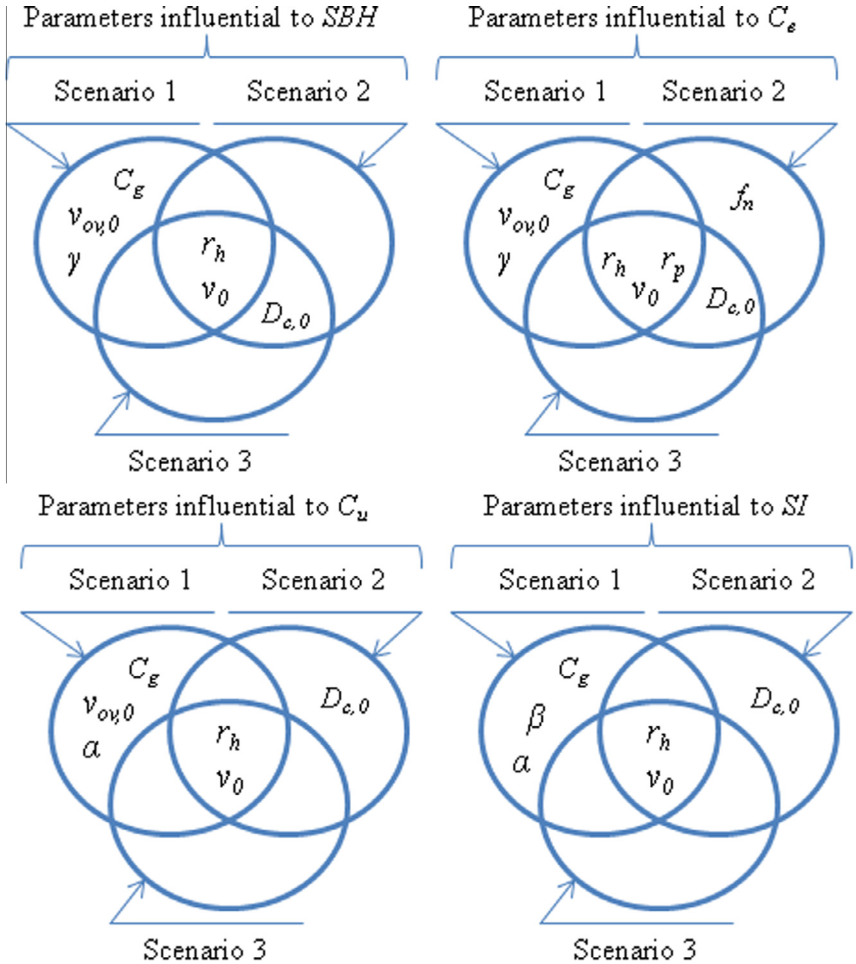

Fig. 3. Venn diagram related to the comparison of important parameters identified in scenarios 1 to 3.

Table 5
Spearman's rank index of the comparison of the similarity of sensitivity measure ranking.

Model outputs	N_s (rank of wet *vs.* rank of bulking)	N_s (rank of wet *vs.* rank of wet + bulking)	N_s (rank of bulking *vs.* rank of wet + bulking)
SBH	0.539	0.709	0.831
C_e	0.515	0.479	0.769
C_u	0.261	0.333	0.661
SI	0.442	0.224	0.467
$Flux_{op}$	0.358	0.333	0.842

sitive to all of the compression settling parameters, it is not reliable to reduce the Bürger–Diehl model to the hindered–dispersion model. To demonstrate this point, we compare the Bürger–Diehl model and the hindered–dispersion model based on the prediction of SBH, C_e and SI. The Monte Carlo simulations were run 300 times for each model. For each run, the hindered–dispersion and Bürger–Diehl models shared the same hindered and dispersion parameters. The similarity of model outputs obtained from the Bürger–Diehl and hindered–dispersion models is characterized by the scatter plot and the Pearson correlation index (N_p), as shown in Fig. 4. As can be seen, in terms of the variable C_e, the predictions of the hindered–dispersion model are almost identical to those obtained from the Bürger–Diehl model, which is also confirmed by the high value of N_p, close to 1. The comparison of the SBH prediction shows that the SBH values predicated by the hindered–dispersion model agree well with those obtained from the Bürger–Diehl model. The discrepancy for these two models in the prediction of SBH is larger than it in the prediction of C_e. This can be expected, since C_g is more influential on SBH than it on C_e. When it comes to SI, the predictions obtained from the Bürger–Diehl model and the hindered–dispersion model differ greatly with a low N_p (0.647), which means the reduction of the Bürger–Diehl model to the hindered–dispersion model is not reliable in terms of the SI prediction.

Under the filamentous bulking condition, all compression settling parameters are non-influential regardless of model outputs, as shown in Table 3, thus making the reduction of the Bürger–Diehl model to the hindered–dispersion model reliable for all model outputs. As Fig. 4 shows, in terms of model outputs SBH, C_e and SI, the performance of the hindered–dispersion model is equivalent to the Bürger–Diehl model, which is confirmed by the high value of N_P. This means under the bulking condition, the Bürger–Diehl model can be reliably reduced to the hindered–dispersion model to reduce the implementation complexity and computation cost.

Further reduction of the Bürger–Diehl model to hindered-only model can, to some degree, deteriorate the quality of model prediction; for instance an obvious discrepancy can be observed between the Bürger–Diehl and hindered-only models in the prediction of C_e, where N_p is as low as 0.871. This is due to the strong sensitivity of C_e to the hydraulic dispersion parameter $D_{c,0}$ under the bulking condition, thus making the hydraulic effect not negligible in SST modeling if C_e is the primary interest.

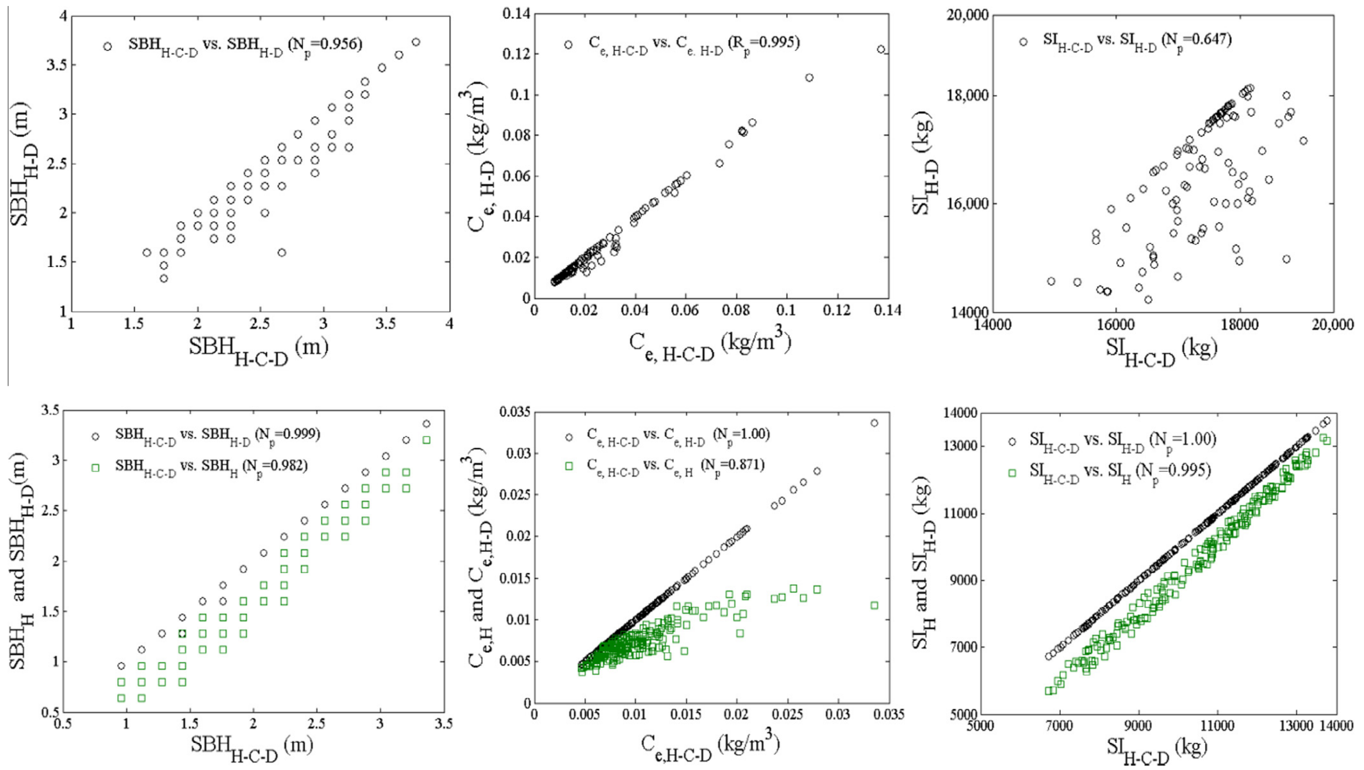

Fig. 4. Scatter plots and the Pearson correlation index comparing the similarity of model outputs of the Bürger–Diehl model and the reduced models (upper: scenario 1, bottom: scenario 2, and subscripts H-C–D, H-D and H denote the Bürger–Diehl model, the hindered–compression model and the hindered-only model respectively).

3.5. Investigating the reliability of the Bürger–Diehl model reduction based on uncertainty analysis

In this section, the reliability of the Bürger–Diehl model reduction under non-ideal flow and settling conditions is further evaluated based on uncertainty analysis of the prediction of SBH, C_e, C_u and SI. Three typical reduced models, the hindered-only model, the hindered–dispersion model and the hindered–compression model, are considered, and the Bürger–Diehl model is used as the reference model to evaluate reliability of model reduction.

Fig. 5 is the boxplot of uncertainty of SST model outputs under the wet-weather condition. It is easy to observe that there is considerable uncertainty concerning all model outputs. With regard to SBH and C_e, reduction of the Bürger–Diehl model to the hindered-only and hindered–compression models cannot produce reliable predictions; for example the 75th percentiles of SBH and C_e predicted by the hindered-only and hindered–compression models are lower than the 25% percentile predicted by the Bürger–Diehl model. In contrast, the uncertainties of SBH and C_e obtained from the hindered–dispersion model are similar to those of the Bürger–Diehl model, which reveals that it is reliable to reduce the Bürger–Diehl model to the hindered–compression model in terms of model outputs SBH and C_e. In the case of C_u and SI, reducing the Bürger–Diehl model to hindered–compression model is acceptable, since the hindered–compression model can provide satisfactory uncertainty results similar as those of the Bürger–Diehl model.

The uncertainties of SST model outputs under the filamentous bulking condition are shown as Fig. 6. As expected, the prediction uncertainties of the hindered–dispersion model are identical to those obtained from the Bürger–Diehl model, which agree with the conclusion that the SST performance under the bulking condition is not sensitive to the compression settling, and the Bürger–Diehl model can be reliably reduced to the hindered–dispersion model without significantly deteriorating the accuracy of model predictions. However, the prediction uncertainty of the hindered-only and hindered–compression models differ from those obtained from the hindered–dispersion and Bürger–Diehl models as Fig. 6 shows; for example SBH, C_e and SI predicted by the hindered-only and hindered–compression models are much smaller than those of hindered–dispersion and Bürger–Diehl models, while in the case of C_u, the hindered-only and hindered–compression models provide higher predictions than the hindered–dispersion and Bürger–Diehl models. Therefore, reducing the Bürger–Diehl model to the hindered-only and hindered–compression models is unreliable under the bulking condition, which may introduce considerable errors to the model uncertainty analysis.

For scenario 3 (wet-weather and bulking), to show the influence of the Bürger–Diehl model reduction on the decision making in SST design and control, the cumulative function distribution plots of SBH and C_e are given as Fig. 7 to represent the uncertainty results. When wet-weather and sludge bulking occur, the sludge blanket can propagate from the SST bottom to effluent weir, which will cause system failure. If the effluent limit of TSS is set as 0.03 kg/m^3, Fig. 7 shows that based on the uncertainty results, violation of effluent TSS limit always has the opportunity to occur under the wet-weather and bulking condition regardless of the SST model structure. However, it is notable that using different SST models leads to inconsistent probabilities of violation: the probabilities predicted by the hindered-only and hindered–compression models are 37%, which are smaller than 45% obtained from the hindered–dispersion and Bürger–Diehl models. If a more strict system robustness requirement is imposed, for example no thickening failure is allowed, the primary interest of uncertainty analysis is to investigate probability that the sludge blanket will rise above the feed inlet (SBH > 2.2 m). According to the Fig. 7, the probabilities of thickening failure predicted by the hindered-only and hindered–compression models are 70% which are much

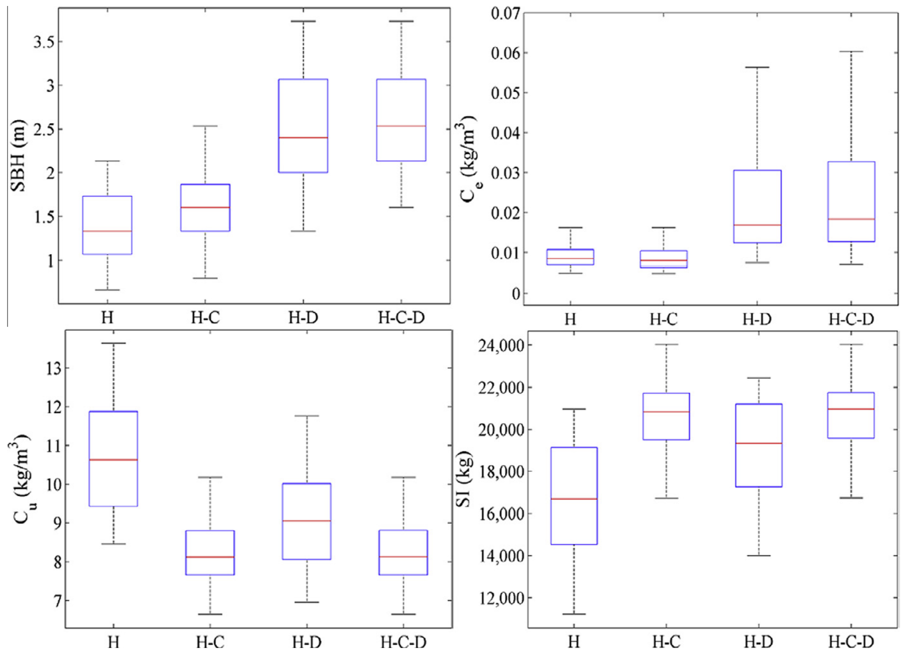

Fig. 5. Results of the Monte Carlo simulations of BSM1 for scenario 1. On each box, the central mark is the mean value, the edges of the box are the 25th and 75th percentiles, and the whiskers extend to the most extreme data points (subscripts H, H-C, H-D and H-C–D denote the hindered-only, hindered–compression, hindered–dispersion and Bürger–Diehl models respectively).

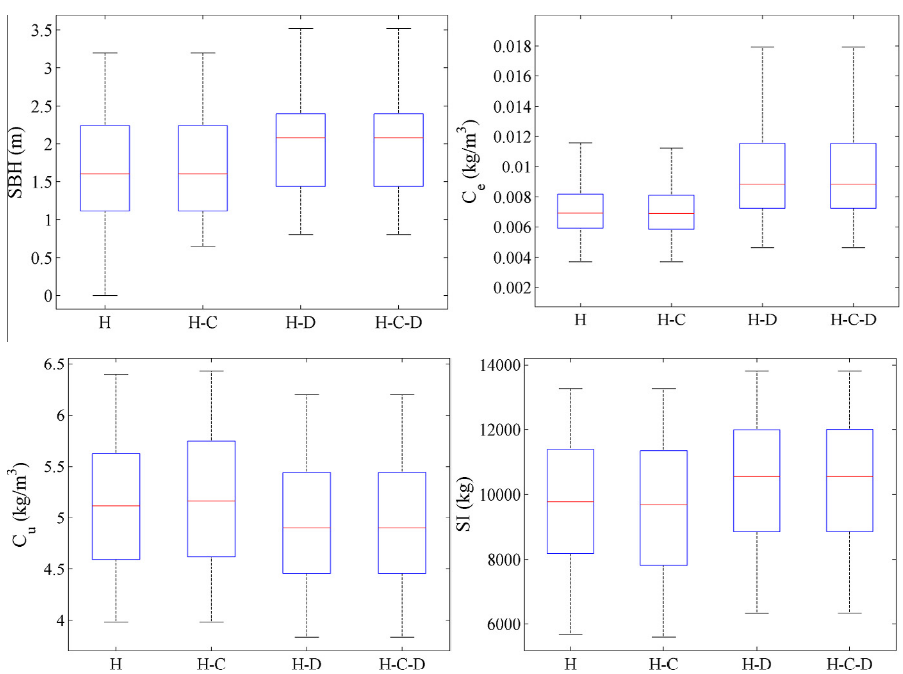

Fig. 6. Results of the Monte Carlo simulations of BSM1 for scenario 2. On each box, the central mark is the mean value, the edges of the box are the 25th and 75th percentiles, and the whiskers extend to the most extreme data points (subscripts H, H-C, H-D and H-C–D denote the hindered-only, hindered–compression, hindered–dispersion and Bürger–Diehl models respectively).

smaller than 83% predicted by the hindered–dispersion and Bürger–Diehl models. Therefore, it is noteworthy that in terms of the violation of effluent TSS limit and thicken failure, the unreliable reduction of the Bürger–Diehl model to the hindered-only and hindered–compression models can lead to the underestimation the risk of system failure, thus negatively impact the decision making of the system design and control under the wet-weather and bulking condition.

4. Conclusions

In the last decade, great efforts have been made to improve the SST simulation. In this study, by using the Benchmark Simulation Model No. 1 as the simulation platform, we provide the sensitivity and reduction analysis of the Bürger–Diehl model under non-ideal flow and settling conditions. The following specific conclusions can be made:

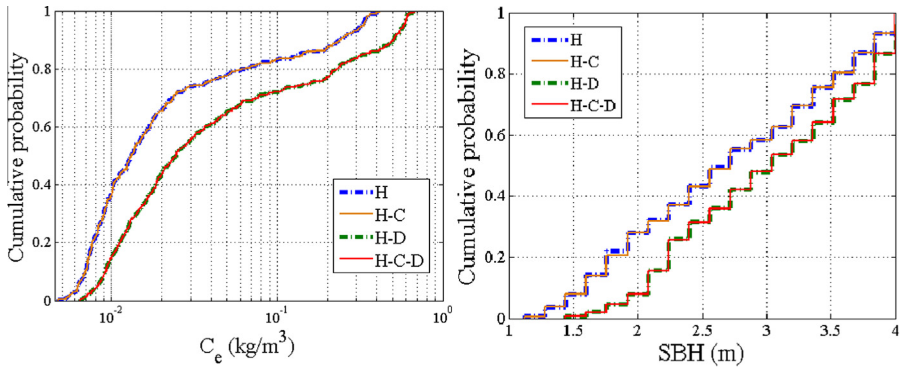

Fig. 7. Representation of the uncertainties of C_e and SBH for scenario 3 by the cumulative distribution function (subscripts H, H-C, H-D and H-C–D denote the hindered-only, hindered–compression, hindered–dispersion and Bürger–Diehl models respectively).

1. Based on the GSA results, the important parameters are identified for the Bürger–Diehl model calibration under non-ideal flow and settling conditions. All model parameters, except f_n and $D_{c,0}$, are influential to SST performance under the wet-weather condition. When filamentous bulking occurs, the outputs of the Bürger–Diehl model are most sensitive to the hindered settling parameters, v_0 and r_h, which need to be accurately calibrated.
2. The analysis of the total sensitivity measure (S_{Ti}) shows that the parameter interactions impact the model output differently. In the case of C_e where strong parameter interactions exist, advanced GSA techniques, such as Extended-FAST, are required for reliable GSA results. However, for C_u, SI and Flux$_{op}$ under the bulking condition, the model is almost additive with negligible parameter interactions. Therefore, simple GSA techniques, such as Standard Regression Coefficients method or Morris screening method, are sufficient to provide reliable GSA results.
3. The sensitivity of the Bürger–Diehl model outputs to parameters is highly impacted by the imposed simulation conditions, resulting in different parameter subsets for model calibration. For example, under the wet-weather condition, the compression settling parameters can be as important as the hindered settling parameters, particularly in the cases of C_u, SI and Flux$_{op}$. Imposing the sludge bulking in scenarios 2 and 3 greatly increases the influence of the hindered settling parameters (v_0 and r_h), while decreasing the influence of the compression settling parameters. Different simulation conditions can also lead to different influential dispersion parameters; for example γ and $v_{ov,c}$ are found to be influential in scenario 1, while $D_{c,0}$ is important in scenarios 2 and 3.
4. Reliable reduction of the Bürger–Diehl model can be achieved based on GSA results. For example, under the wet-weather condition, in terms of the prediction of C_e, the Bürger–Diehl model can be reduced to the hindered–dispersion model without deteriorating model performance, since the compression settling parameters are not as influential to C_e as the hindered settling and dispersion parameters. Under the bulking condition, the Bürger–Diehl model can be reduced to the hindered–dispersion model without impacting model outputs, which occurs because none of model outputs are sensitive to the compression settling parameters.
5. The reliability of the Bürger–Diehl model reduction can be evaluated based model uncertainty analysis. Unreliable reduction of the Bürger–Diehl model can introduce considerable errors to model predictions, thus negatively impact SST design and control.

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