USE OF ESTIMATION TECHNIQUES FOR FLOOD FORECASTING

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ABSTRACT

An Iterated Extended Kalman Filter (IEKF) is implemented for parameter estimation and flood forecasting. The filter is applied to a conceptual model. The conceptual model selected for study is a generalized one-dimensional kinematic wave model of a watershed which incorporates storages in both surface water and groundwater. Linearization of the nonlinear conceptual model is based upon the Influence Coefficient Method of Becker and Yeh (1972) which performs linearization externally without changing the structure of the conceptual model. Four parameters in the conceptual model are recursively estimated.

A set of numerical experiments are conducted using synthetic, as well as real, watershed data. Sensitivity and correlation analyses of the estimated parameters are carried out. The research results indicate that conceptual models with fewer physically based parameters, such as the kinematic wave model, are feasible for implementations with IEKF for on-line parameter estimation and flood forecasting.

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INTRODUCTION

With the advent of mini-computers, the preparation of flood forecasts is shifting from predominantly manual methods to computerized procedures with automatically selected data from real-time, telemetry reporting systems. These data are used as inputs to a rainfall-runoff model, ranging from complex soil moisture accounting models to time-series types of models, for making real-time flood forecasts. The parameters imbedded in the model are time-varying during a flood situation. Estimation techniques, such as the Kalman filter developed in optimal control, have been used for estimating the states (floods) as well as the time-varying parameters in a rainfall-runoff process.

Most of the published results deal with linear systems where the dynamics are governed by linear and discrete state equations. A closed-form representation of the state equation is essential in the application of the Kalman Filter. Time-series types of models have been used to represent the rainfall-runoff process, and the Kalman Filter has been applied to time-series models for parameter estimation and flood forecasting, such as the work of Todini and Bouillot (1975), Szollosi-Nagy (1976), Szollosi-Nagy et al. (1977), Wood and Szollosi-Nagy (1978), Todini and Wallis (1978), and others.

Application of the Kalman Filter to nonlinear, conceptual watershed models for parameter estimation and flood forecasting has been sparse. Kitanidis and Bras (1980a) and Kitanidis and Bras (1980b) used a linearization technique to effectively linearize the internal structure of the Sacramento model (Burnash, et al., 1973), and the Extended Kalman Filter was used for making real-time flood forecasts. Georgakakos and Bras (1982) also utilized a statistical linearization technique to internally linearize a nonlinear kinematic wave model (Mein et al., 1974). A similar algorithm was then used to make real-time flood forecasts. Moore and Weiss (1980) proposed a simple nonlinear rainfall-runoff model and formulated the model in a different state-space form, that is, the parameters constitute the state vector and the dynamics of the system behavior are represented by the measurement equation. The Extended Kalman Filter was then applied to the linearized measurement equation to obtain the parameter estimates recursively. The flood forecasts are computed in turn based on those parameters.

Some comparisons have been made of the performance of flood forecasting models. Bolzern, et al., (1980) compared the flood forecast performances of an ARMAX model with Kalman Filter of differing complexities. O'Connel and Clarke (1982) made comparisons among four flood forecasting models, ranging from conceptual hydrological models that use unsophisticated parameter estimation procedures to simple time-series types of models that use relatively sophisticated state filtering and parameter estimation procedures.

The objectives of this research are: (1) to develop an algorithm to perform on-line parameter estimation and on-line flood forecasting by applying the Iterated Extended Kalman Filter to conceptual models (e.g., Field and Williams, 1983), (2) to use the Influence Coefficient Method of Becker and Yeh (1972) to linearize externally the nonlinear structure of conceptual models without changing the structure of the models, and (3) to demonstrate the usefulness of the proposed algorithm.

PROBLEM FORMULATION

It has been shown by Szollosi-Nagy (1976) that a general state-space representation of the following form can be used to describe the behavior of a water resource system:

$$X_{k+1} = \xi(X_k, U_k, W_k) \tag{1}$$

$$Z_k = \eta(X_k, V_k) \tag{2}$$

where X_k = the n_X-vector of the states of the system at discrete time k, k=0, 1, 2, ...,

U_k = the n_U-vector of the deterministic inputs,

W_k = the n_W-vector of the system noise,

Z_k = the n_Z-vector of the measurements on the system at discrete time k, k=1, 2, 3, ...,

V_k = the n_V-vector of the measurement noise,

ξ, η = certain functionals characterizing the system.

In modeling the rainfall-runoff processes for a watershed in state-space formulation, such as the work of Toyoda, et al. (1960), Todini and Bouillot (1975), Szollosi-Nagy, et al. (1977), Wood and Szollosi-Nagy (1978), and Kitanidis and Bras (1980a), Eqs. (1) and (2) can be rewritten as:

$$X_{k+1} = F(X_k, U_k) + \Gamma(X_k)W_k \tag{3}$$

$$Z_k = H(X_k) + V_k \tag{4}$$

where

W_k = zero mean, independent white Gaussian process, i.e., $W_k \sim N(0,Q_k)$ and $E[W_k,W_i] = 0$ for $k \neq i$,

Q_k = $E[W_k,W_k]$, covariance matrix of W_k,

V_k = additive measurement error which is assumed to be zero mean, independent white Gaussian processes, i.e., $V_k \sim N(0,R_k)$ and $E[V_k,V_i] = 0$ for $k \neq i$,

R_k = $E[V_k,V_k]$, covariance matrix of V_k.

$F(X_k,U_k)$, $\Gamma(X_k)$ and $h(X_k)$ are vector functions which are assumed to be continuous and differentiable. In this paper, $F(X_k,U_k)$ corresponds to a watershed simulation model. Furthermore, $\{V_k\}$ and $\{W_k\}$ are assumed to be independent processes, i.e., $E[V_k\ W_i] = 0$ for all k and i.

The linear form of Eqs. (3) and (4), known as a Gauss-Markov model, can be written as

$$X_{k+1} = \Phi X_k + G\ U_k + \Gamma W_k \tag{5}$$

$$Z_k = HX_k + V_k \tag{6}$$

where Φ is a $(n_X \times n_X)$ matrix,

G is a $(n_X \times n_U)$ matrix,

Γ is a $(n_X \times n_W)$ matrix, and

H is a $(n_Z \times n_X)$ matrix.

This model has been extensively proposed for flood forecasting (e.g., Todini and Buillot, 1975, Szollosi-Nagy, et al., 1977, Wood and Szollosi-Nagy, 1978, et al.)

Given the above-mentioned problem, the objective is to obtain the best estimate of state X_k as we observe the output Z_k , Z_{k-1} , ..., Z_1 . Among all possible criteria (Poljak and Tsypkin, 1980), let us choose $\hat{X}_k$, the estimate of the state X_k , as a function of the observations up to and including Z_k, such that

$$\hat{X}_k = \text{Min } E[(X_k - X_k^*)^T (X_k - X_k^*)/Y_k] \tag{7}$$

where Y_k = $[Z_1, Z_2, \ldots, Z_k]^T$,

X_k^* = any estimate of state X_k ,

E = the statistical expectation,

/ = conditioning notation, meaning that Y_k is given.

It can be shown (Nahi, 1969) that such an estimate is

$$\hat{X}_k = E[X_k/Y_k] \tag{8}$$

The additional requirements for the best estimate are:

1. It must be observed sequentially. At each observation we want to update the estimate without having to reprocess all the data from the beginning of the observation, and

2. It must be computationally feasible; preferably, it should be implementable by a mini-computer or microprocessor.

MODEL DESCRIPTION AND STATE-SPACE FORMULATION OF CONCEPTUAL MODELS

Model Description

The conceptual model selected for study in this paper is a generalized one-dimensional kinematic wave model of a catchment (Field and Williams, 1983) which incorporates storages in both surface and groundwater. This model has been presented by Field (1982) and Field and Williams (1983). The present form of the model assumes that rainfall intensity varies with time, but not spatially in the catchment. The excess rainfall is routed through a nonlinear storage to provide a 'surface supply' and the infiltrated rainfall is routed through a linear storage (with a considerably higher storage coefficient) to provide a 'groundwater supply.' The two supplies together are routed down the main channel using a kinematic wave equation (Field, 1982). The governing equations which describe the above-mentioned processes are:

1. The continuity equation for flow through surface storage:

$$(p-\phi) - s_s = \frac{dh}{dt} \tag{9}$$

where p = rainfall intensity,

ϕ = infiltration index,

s_s = surface supply (discharge per unit area),

h = volume per unit area,

t = time,

2. The continuity equation for flow through groundwater storage:

$$\phi - s_g = \frac{dh}{dt} \tag{10}$$

where ϕ = infiltration index which is set to be equal to p when $\phi > p$ and $s_s = 0$,

s_g = groundwater supply (discharge per unit area),

3. The relationship between storage volume and discharge:

$$h = b\ s_*^{\gamma} \tag{11}$$

where b and γ are constants, and s_* is either surface supply or groundwater supply,

4. The relationship between b and storage coefficient K_c:

$$b = K_c\ p_e^{1-\gamma} \tag{12}$$

where p_e is a characteristic rainfall intensity, which can be taken as the average intensity for the storm duration, and

K_c is either surface water storage coefficient, K_s or groundwater storage coefficient, K_g,

5. The nonlinear form of the kinematic wave equation:

$$\frac{\partial q}{\partial t} = -\ (m+1)\ a^{\frac{1}{m+1}} \left(\frac{q}{B}\right)^{\frac{m}{m+1}} \left\{\frac{\partial q}{\partial x} - Bs\right\} \tag{13}$$

subject to the boundary condition q(0,t) = 0 and prescribed initial values of q(x,0),

where q = discharge,

B = width of the elemental strip (Fig. 1),

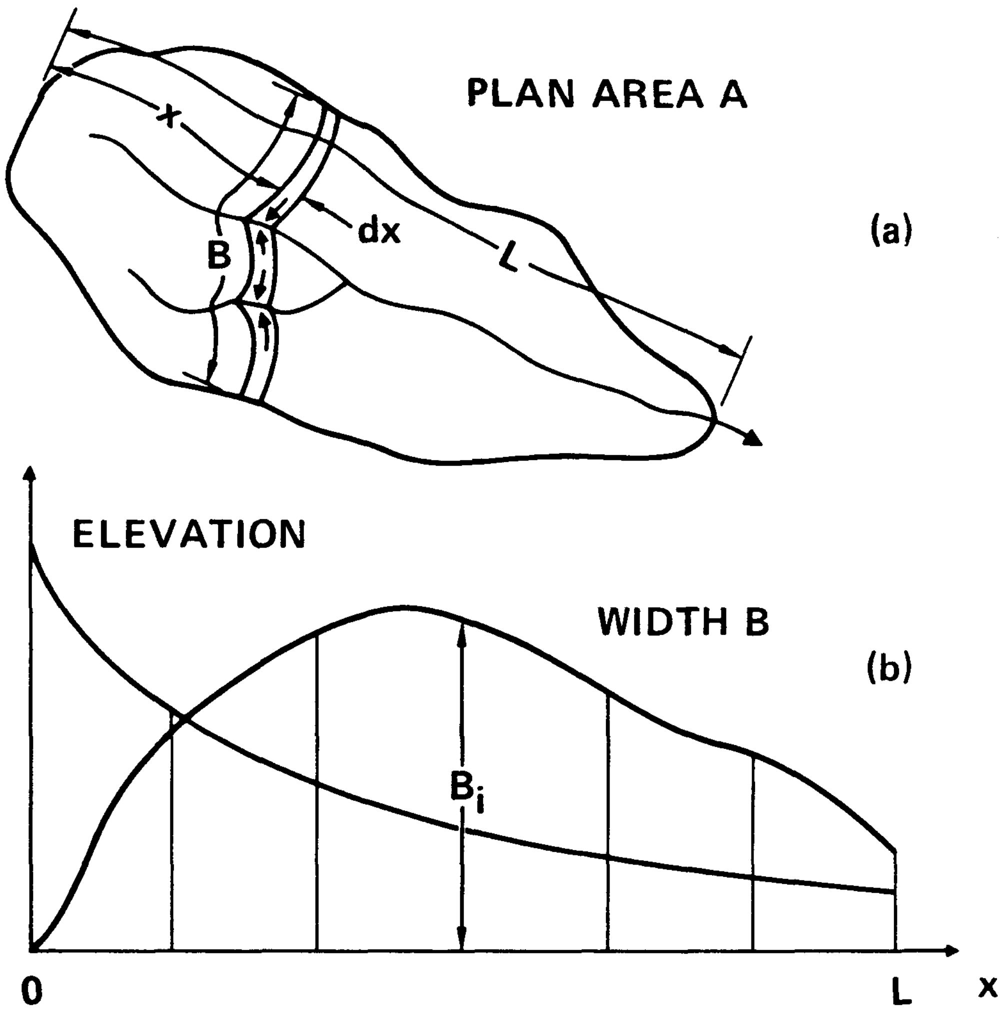

Fig. 1.   Definition Sketch

s = the volumetric rate of total supply from surface storage and groundwater per unit area to channel flow,

x and t = spatial and temporal variables,

a and m = the dimensional coefficient and the exponent respectively in the channel velocity-depth relationship,

$$v = a\, y^{m} \tag{14}$$

For convenience, the preceding equations are non-dimensionalized before they are solved. The resulting equations are of identical form. The dimensionless quantities are defined by Field and Williams (1983). Referring to Fig. 2, letting $x_j = j\, \Delta s$, where Δx is a difference step, $t_k = k\, \Delta t$, writing $q(j\Delta x,\ k\Delta t) = q_j^k$, $a_j = a(j\Delta x)$, $s^k = s\ (k\Delta t)$, etc., and using the Lax-Wendroff solution technique (Kibler and Wolhiser, 1979), which employs the second-order Taylor's expansion of the dependent variable q and the subsequent substitution of spatial derivatives for temporal derivatives using Eq. (13), the following recurrence formula results (Field and Williams, 1983)

$$q_j^{k+1} = q_j^k + \delta + \epsilon \tag{15}$$

$$\text{where } \delta = -\, x_1\, x_2\, x_3 \tag{16}$$

$$x_1 = \frac{\Delta t}{\Delta x}\,(m+1)\, a_j^{\frac{1}{m+1}}\, B_j^{-\frac{m}{m+1}} \tag{17}$$

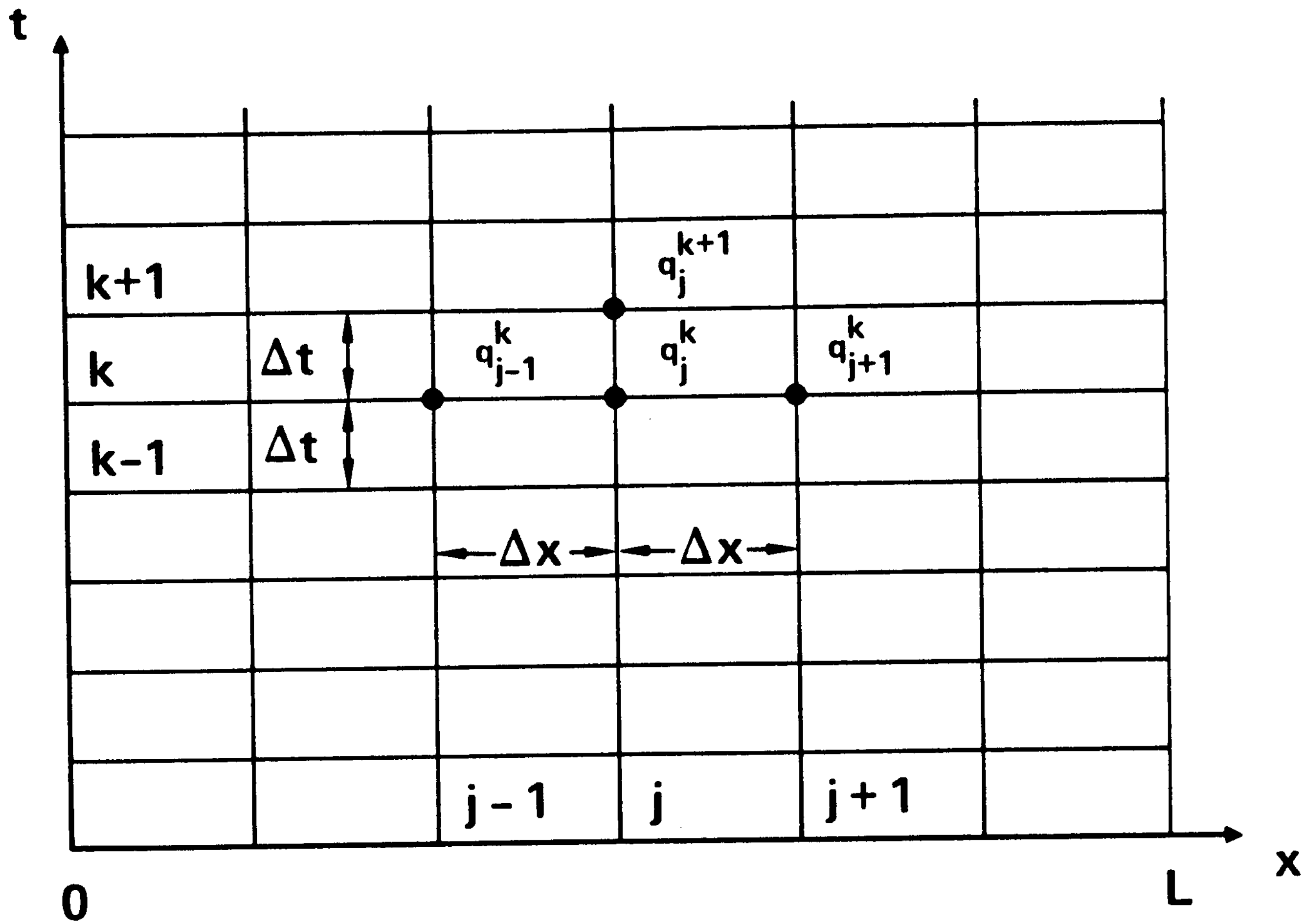

Fig. 2. Discretization of x-t Plane

$$x_2 = (q_j^k)^{\frac{m}{m+1}} \tag{18}$$

$$x_3 = \frac{q_{j+1}^k - q_{j-1}^k}{2} - \Delta x \; B_j \; \frac{(s^k + s^{k+1})}{2} \tag{19}$$

$$\varepsilon = \frac{1}{2}(x_1 \; x_2)^2 \; (x_3)^2 \left\{ \frac{m}{(m+1)\, q_j^k} + \frac{1}{x_3} \left\{ \frac{1}{(m+1)\, a_j} \frac{(a_{j+1} - a_{j-1})}{2} \right.\right. \tag{20}$$

$$+ \frac{m}{(m+1)\, q_j^k} \frac{(q_{j+1}^k - q_{j-1}^k)}{2} - \frac{m}{(m+1)\, B_j} \frac{(B_{j+1} - B_{j-1})}{2}$$

$$\left.\left. + \frac{1}{x_3}\left[(q_{j+1}^k - 2q_j^k + q_{j-1}^k) - \Delta x \frac{(B_{j+1} - B_{j-1})}{2} \frac{(s^k + s^{k+1})}{2} \right] \right\} +$$

$$\left. \frac{1}{(x_1 \; x_2)} \cdot \frac{1}{(x_3)^2} \Delta x \; B_j \; \frac{(-3s^k + 4s^{k+1} - s^{k+2})}{2} \right\}$$

At the catchment outlet (i.e., $j = N$), the term in the recurrence equation (20) is

$$\varepsilon = \frac{1}{2}(x_1 \; x_2)^2 \; (x_4)^2 \left\{ \frac{m}{(m+1)\, q_N^k} + \frac{1}{x_4} \left\{ \frac{1}{(m+1)\, a_N} \frac{(3a_N - 4a_{N-1} + a_{N-2})}{2} \right.\right. \tag{21}$$

$$+ \frac{m}{(m+1)\, q_N^k} \frac{(3q_N^k - 4q_{N-1}^k + q_{N-2}^k)}{2} - \frac{m}{(m+1)\, B_N} \frac{(3B_N - 4B_{N-1} + B_{N-2})}{2}$$

$$+ \frac{1}{x_4} \left[(2q_N^k - 5q_{N-1}^k + 4q_{N-2}^k - q_{N-3}^k) - \Delta x \frac{(3B_N - 4B_{N-1} + B_{N-2})}{2} \frac{(s^k + s^{k+1})}{2} \right] \Bigg\}$$

$$+ \frac{1}{(x_1\, x_2)} \frac{1}{(x_4)^2} \Delta x\, B_N \frac{(-3s^k + 4s^{k+1} - s^{k+2})}{2} \Bigg\}$$

where

$$x_4 = \frac{3q_N^k - 4q_{N-1}^k + q_{N-2}^k}{2} - \Delta x\, B_N \frac{(s^k + s^{k+1})}{2} \tag{22}$$

To be consistent with earlier developments, a second-order finite difference scheme is required for the solution of the supply from surface storage of Eq. (9). Replacing the derivative of the right-hand side of Eq. (9), substituting for h from Eq. (11) and substituting for b from Eq. (12) yields

$$\frac{3K_s}{\Delta t} (s_s^{k+1})^\gamma + s_s^{k+1} = \frac{K_s}{\Delta t} \left\{ 4(s_s^k)^\gamma - (s_s^{k-1})^\gamma \right\} + (p^{k+1} + p^k) - 2\phi - s_s^k \tag{23}$$

The recurrence formula, Eq. (23), requires values of s_s^k and s_s^{k-1} to generate the solution for s_s^{k+1}.

The supply from groundwater storage is computed in the same way using Eqs. (10), (11) and (12) with $\gamma = 1$. The recurrence equation is then

$$\frac{3k_g}{\Delta t} (s_g^{k+1}) + s_g^{k+1} = \frac{K_g}{\Delta t} \left\{ 4(s_g^k) - s_g^{k-1} \right\} + 2\phi - s_g^k \tag{24}$$

Thus at any instant in time the model can predict the surface and groundwater supplies and discharges at a predetermined number of increments (N) in the main channel.

The parameters which determine the size of these supplies and discharges are: (1) the time of concentration, t_c, which is used to non-dimensionalize the dimension 'time' of all quantities, (2) the surface water storage coefficient, K_s, (3) the groundwater storage coefficient, K_g, and (4) the infiltration index, ϕ. The only other parameters in the model are the exponents in the storage-discharge equations and the velocity-depth relationship, and these parameters have been assumed fixed from physical considerations (Field and Williams, 1983).

State-Space Formulation

In order to progress from one time step to the next, the model requires information about the current and next rainfall intensity, the current level of the two supplies and also their level at the preceding time step, in addition to N discharges in the main channel. Using the state-space formulation, Eqs. (15) - (24) can be rearranged to form the following set of non-linear equations with additive errors

$$\begin{bmatrix} q_*^{k+1} \\ s_s^{k+1} \\ s_s^{k} \\ s_g^{k+1} \\ s_g^{k} \end{bmatrix} = \begin{bmatrix} f_1(q_*^k,\ s_s^k,\ s_s^{k-1},\ s_g^k,\ s_g^{k-1}) \\ f_2(s_s^k,\ s_s^{k-1},\ p^{k+1},\ p^k) \\ s_s^k \\ f_3(s_g^k,\ s_g^{k-1},\ s_s^k,\ s_s^{k-1}) \\ s_g^k \end{bmatrix} + W_k \tag{25}$$

where q_*^k = $[q_1^k, q_2^k, \ldots, q_N^k]^T$,

f_1 = N-vector function,

f_2, f_3 = scalar functions,

q_j^k = discharge at time k at the jth spatial point,

s_s^k, s_s^{k-1} = surface water supply at time k and $k-1$ respectively,

s_g^k, s_g^{k-1} = groundwater supply at time k and $k-1$ respectively, and

p^{k+1}, p^k = rainfall intensity at time $k+1$ and k respectively.

Corresponding to the system equation (3), the state vector and the input vector can be defined specifically as:

$$X_k = [q_1^k, q_2^k, \ldots, q_N^k, s_s^k, s_s^{k-1}, s_g^k, s_g^{k-1}]^T \tag{26}$$

$$U_k = [p^{k+1}, p^k]^T \tag{27}$$

The noise vector W_k is a (N+4)-vector which is subject to the same assumptions as that in Eq. (3) and $\Gamma(X_k)$ is assumed to be an identity matrix.

In on-line flood forecasting, the parameters of the system must also be estimated during each time step. For this purpose, they can be added to the state vector. The augmented state vector becomes

$$X_k = [q_1^k, q_2^k, \ldots, q_N^k, s_s^k, s_s^{k-1}, s_g^k, s_g^{k-1}, t_c, K_s, K_g, \phi]^T \tag{28}$$

where t_c = the time of concentration,

K_s = the surface water storage coefficient,

K_g = the groundwater storage coefficient,

ϕ = the infiltration index.

In this paper, the observations are discharges, which in fact are some of the states containing measurement errors. Hence, the measurement equation is of the form of Eq. (6), i.e.,

$$Z_k = HX_k + V_k \tag{6}$$

where H = observation matrix of dimension $n_Z \times n_X$ with entries of values 0 and 1,

n_Z = number of observation stations,

n_X = number of states of augmented systems.

SOLUTION TECHNIQUE

The Iterated Extended Kalman Filter (e.g. Wishner, et al., 1969) is chosen and implemented for state and parameter estimation because of its generality. Linearizing $F(X_k, U_k)$ about a reference state vector X_k^* yields

$$X_{k+1} = F(X_k^*, U_k) + F'(X_k^*, U_k)(X_k - X_k^*) + W_k \tag{29}$$

where F' is the Jacobian matrix which can be computed by the technique of influence coefficients of Becker and Yeh (1972). The reference state vector, X_k^*, from which point the linear trajectory is projected, is updated iteratively at each time increment.

Let X_k^* be $\hat{X}^i_{k/k+1}$, the updated reference state vector. The Iterated Extended Kalman Filter can be represented by a set of recursive equations as:

1. State Prediction

$$\hat{X}^i_{k+1/k} = F(\hat{X}^i_{k/k+1}, U_k) + F'(\hat{X}^i_{k/k+1}, U_k)(\hat{X}_{k/k} - \hat{X}^i_{k/k+1}) \tag{30}$$

2. Covariance prediction

$$P^i_{k+1/k} = F'(\hat{X}^i_{k/k+1}, U_k)P_{k/k}\,[F'(\hat{X}^i_{k/k+1}, U_k)]^T + Q^i_k \tag{31}$$

3. Kalman gain

$$K^i_{k+1} = P^i_{k+1/k}\, H^T \left\{HP^i_{k+1/k}\, H^T + R^i_k\right\}^{-1} \tag{32}$$

4. <u>Covariance update</u>

$$P^{i}_{k+1/k+1} = [I - K^{i}_{k+1}H]\, P^{i}_{k+1/k} \qquad (33)$$

5. <u>State update</u>

$$\hat{X}^{i+1}_{k+1/k+1} = \hat{X}^{i}_{k+1/k} + K^{i}_{k+1}\,[Z_{k+1} - H\hat{X}^{i}_{k+1/k}] \qquad (34)$$

$$\hat{X}^{i+1}_{k/k+1} = \hat{X}^{i}_{k/k} + P^{i}_{k/k}\,[F'(\hat{X}^{i}_{k/k+1}, U_k)]^T \cdot$$

$$[P^{i}_{k+1/k}]^{-1}\,(\hat{X}^{i+1}_{k+1/k+1} - \hat{X}^{i}_{k+1/k}) \qquad (35)$$

where the superscript i represents the ith iteration, "^" represents estimation and k-1/k represents the value at time k-1 based on the information from the initial stage up to time k. Also, we will assume, for each time k,

$$\hat{X}^{0}_{k+1/k} = \hat{X}_{k/k} \qquad (36)$$

$$\hat{X}^{0}_{k+1/k+1} = \hat{X}_{k+1/k} \qquad (37)$$

Given the input, e.g., precipitation and the initial conditions

$$E\,[X_0] = \hat{X}_{0/0} \qquad (38)$$

$$E[(X_0 - \hat{X}_{0/0})(X_0 - \hat{X}_{0/0})] = P_{0/0} \qquad (39)$$

in which $P_{0/0}$ is the covariance matrix of the error of the initial states, the flood forecasting and parameter estimation can be carried out by repeated applications of the eqs. (30)-(35).

Note that if i takes the value 0 the filter reduces to the Extended Kalman Filter.

In the application of the IEKF, the true value of the system noise covariance Q_k and the measurement noise covariance R_k are unknown. However, these values can be updated sequentially, from arbitrary initial estimates, with adaptive estimation algorithms. The measurement noise covariance R_k^i can be adaptively estimated based upon the estimated measurement error according to the following approximation which is similar to that of Szollosi-Nagy (1976)

$$R_k^i = [(k-1)R_{k-1} + \nu_k^i(\nu_k^i)^T - H^T P_{k/k-1}^i H]/k \tag{40}$$

where

$$\nu_k^i = Z_k - H\hat{X}_{k/k-1}^i \tag{41}$$

Similarly, the system noise covariance Q_k^i can be estimated from the following approximation

$$Q_k^i = [(k-1)Q_{k-1} + \omega_k^i (\omega_k^i)^T + P_{k/k}^i - P_{k/k-1}^i]/k \tag{42}$$

where

$$\omega_k^i = K_k^i \nu_k^i \tag{43}$$

In addition, Q_k^i and R_k^i are assumed to be nonnegative definite. Hence, if they are not, an arbitrary small number λ can be artificially added to their diagonal entries, i.e.,

$$Q_k^i = Q_k^i + \lambda I \tag{44}$$

and

$$R_k^i = R_k^i + \lambda\ I \tag{45}$$

such that Q_k^i and R_k^i are nonnegative definite.

The adaptive procedures for Q_k and R_k are continued until convergence is reached.

NUMERICAL EXAMPLES

To test the proposed state-space formulation and estimation technique, applications are made to hypothetical cases for the chosen conceptual model. A real catchment is then used to demonstrate the performance of the algorithm.

Preliminary Procedures

In order to have a better performance of the Iterated Extended Kalman Filter, several preliminary procedures have to be carried out.

1. Sensitivity analysis

The purpose of a sensitivity analysis is to investigate the identifiability of the parameters. As pointed out by Dawdy and O'Donell (1965), the more sensitive the parameters are, the more rapidly the parameters will converge. The index of sensitivity used in this study is the covariance matrix of the estimated parameters, which can be shown (Bard, 1974 and Yeh and Yoon, 1976) as

$$COV(\hat{\theta}) = \frac{J(\hat{\theta})}{M-L}[A(\hat{\theta})]^{-1} \tag{46}$$

where $J(\hat{\theta})$ = summation of the squares of the differences between computed values (e.g. discharges) at $\hat{\theta}$ and observations,

$A(\hat{\theta})$ = $[J_a(\hat{\theta})]\ [J_a\ (\hat{\theta})]^T$,

$J_a(\hat{\theta})$ = the Jacobian matrix evaluated at $\hat{\theta}$,

$\hat{\theta}$ = the parameter estimates,

M = the number of observations,

L = the number of parameters.

The Jacobian matrix can be approximated by the influence coefficients (Becker and Yeh, 1972). The covariance matrix provides the information regarding the sensitivity of the estimated parameters. The larger the variance, the less sensitive the corresponding parameter will be. A correlation matrix (C) which would indicate the degree of interdependence among the parameters with respect to the objective function at the estimated parameters can also be obtained from the covariance matrix as

$$C(\hat{\theta}) = [c_{ij}] \tag{47}$$

and

$$c_{ij} = \frac{\sigma_{ij}}{\sqrt{\sigma_{ii}\sigma_{jj}}} \tag{48}$$

where c_{ij} = the entry of $C(\hat{\theta})$ at (i,j),

σ_{ij} = the entry of COV $(\hat{\theta})$ at (i,j).

2. <u>Generalized Least Squares Optimization (GLSO)</u>

In order to obtain better initial estimates of the parameters, the GLSO algorithm (Williams and Yeh, 1983) can be used to perform parameter estimations using historical observations.

3. <u>Determination of Q and R</u>

Although adaptive procedures do not require accurate initial estimates of Q and R, a better estimate of R (assuming that Q = 0) can be easily obtained from the result by the GLSO algorithm.

4. <u>Determination of initial state and its covariance matrix</u>

The initial state is assumed to be the same as the true observations at the starting point. The covariance matrix associated with the initial state can then be found by a trial-and-error procedure based upon some physical considerations about the state. The goodness of the assumed values can be verified by a post test of the satisfaction of the assumption, i.e., the whiteness of the residuals of the observations.

When the preliminary procedures have been carried out and the required information for the IEKF has been obtained, the IEKF can then be implemented.

<u>Application to Hypothetical Catchment</u>

A hypothetical catchment as shown in Fig. 3 is considered. The parameter values associated with the catchment are listed in Table 1. The catchment is subject to an arbitrarily chosen storm to generate discharges at specified stations. A storm with its duration of 20 hours and variable intensity, as indicated in Fig. 4, is considered. The resulting "true" hydrograph is also shown in Fig. 4.

Three ways of generating observations were used. The first case assumes that both system and measurement are subject to White Gaussian Noise (WGN). In this case, two sequences of random numbers were generated with zero mean, given variance (time invariant) and normal distribution. They were then added to the system response and observations which were simulated using true (known) parameter values. In the second case, the measurement error is assumed to be time variant. In other words, noise was added to each generated

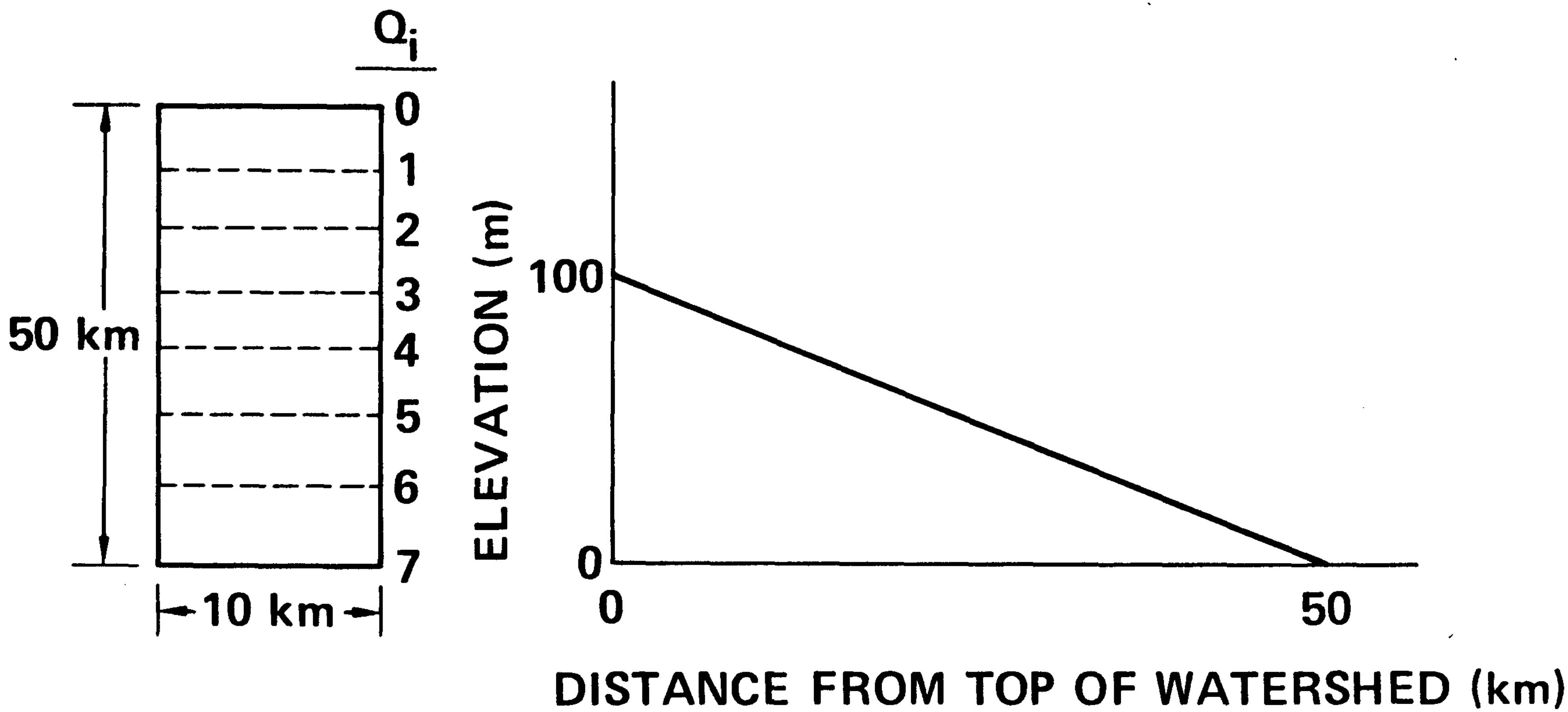

Fig. 3. Hypothetical Watershed

Table 1. Parameter Values for the Hypothetical Catchment

Parameters	True Value	Initial Estimates
Time of concentration, t_c (hrs)	15.0	12.0
Surface Water Storage Coefficient, K_s (hrs)	12.0	10.0
Groundwater Storage Coefficient, K_g (hrs)	40.0	50.0
Infiltration index, ϕ (mm/hr)	3.0	1.0

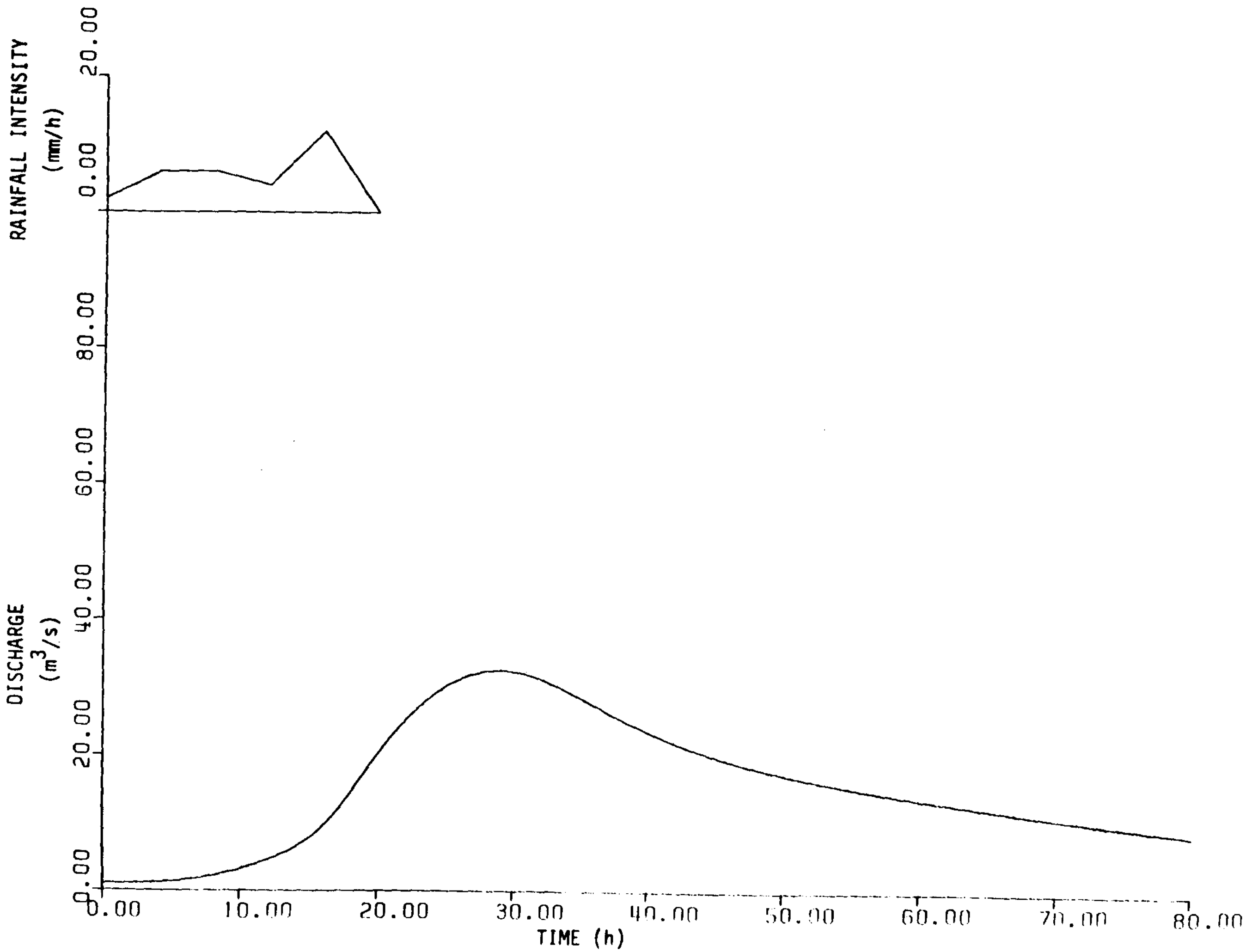

Fig. 4. **Input Hyetograph and "True" Hydrograph for the Hypothetical Catchment**

observation by an amount equal to the random number multiplied by 5% or 10% of the generated values. In the third case, the measurement error was generated by perturbing the parameter randomly at each time step (which can be considered as a noise with time-variant covariance). The system errors of cases 2 and 3 are assumed to be corrupted by a sequence of WGN which is generated by the same procedures as case 1. The generated measurements have a time base of 100 hours with a time interval of 0.2 hours.

In this hypothetical case study, the preliminary procedures were applied to the first 200 observations to obtain the required information for IEKF. The IEKF was then applied to the rest of the data points. In performing sensitivity analysis, true values of parameters were used. Since the variance of the measurement errors, $J(\hat{\theta})/(M-L)$, does not affect the relative relationship among the parameters, it is set to be unity for comparison purposes. The covariance and correlation matrices are shown in Tables 2 and 3. A similar result was obtained by Williams and Yeh (1983) and some physical interpretations were also illustrated. The true values of parameters, their estimates from GLSO and their estimates from the IEKF are listed in Table 4. The post analysis of verification of the whiteness of the residuals of the observations has been done. The lag-one cross correlation matrix of the residuals is shown in Table 5. The correlogram of the observations at the downstream station is also plotted in Figures 5a, 5b, 5c and 5d. Figures 6a, 6b, 6c and 6d show the observations, the one-step-ahead forecasts and the estimations of the discharges at the outlet of the watershed for three cases. The results show that the proposed algorithm provides a favorable performance of flood forecasting and parameter estimation. At each time step the IEKF takes at most 2 iterations after several time steps have been taken.

Table 2. Covariance Matrix of the True Parameter Values for the Hypothetical Catchment

	t_c	K_s	K_g	ϕ
t_c	0.54	-1.15	-0.11	0.15
K_s	-1.15	5.03	1.18	-1.02
K_g	-0.11	1.18	1.71	-0.33
ϕ	0.15	-1.02	-0.33	0.24

Table 3. Correlation Matrix of the True Parameter Values for the Hypothetical Catchment

	t_c	K_s	K_g	ϕ
t_c	1.00	-0.70	-0.12	0.43
K_s	-0.70	1.00	0.40	-0.94
K_g	-0.12	0.40	1.00	-0.52
ϕ	0.43	-0.94	-0.52	1.00

Table 4. Parameter Estimations on the Hypothetical Catchment

Case			True Parameter	Initial Guess	GLSO	IEKF
1	SD_1^{*} = 0.4**	t_c	15.0	12.0	14.998	15.023
	SD_2 = 0.4	k_s	12.0	10.0	12.016	11.979
		k_g	40.0	50.0	39.992	40.005
		ϕ	3.0	1.0	2.996	2.995
2-1	SD_1 = 0.4	t_c	15.0	12.0	15.109	15.087
	SD_2 = 5%***	k_s	12.0	10.0	11.625	11.625
		k_g	40.0	50.0	39.794	39.794
		ϕ	3.0	1.0	3.083	3.078
2-2	SD_1 = 0.4	t_c	15.0	12.0	15.235	15.201
	SD_2 = 10%	k_s	12.0	10.0	11.260	11.260
		k_g	40.0	50.0	39.825	39.823
		ϕ	3.0	1.0	3.148	3.143
3	SD_1 = 0.4	t_c	15.0	12.0	14.507	14.550
	SD_2 = 15%	k_s	12.0	10.0	12.614	12.612
		k_g	40.0	50.0	39.401	39.410
		ϕ	3.0	1.0	2.852	2.852

* SD_1 = System noise and SD_2 = measurement noise.

** 0.4 means (0.4)X(N(0,1))

*** 5% means (5%)X(magnitude)X(N(0,1))

Table 5. Lag 1 Cross Correlation Matrix of the Residuals for Case 2-1

	1	2	3	4	5	6	7
1	0.0517	0.0426	0.0784	0.1374	0.0585	0.1767	0.0373
2		0.0481	0.0141	0.0989	0.0325	0.118	-0.0135
3			-0.0124	0.1293	0.0938	0.0012	0.0636
4				0.0604	0.0953	0.0602	-0.1263
5	(sym)				-0.0068	-0.0365	-0.0487
6						-0.0491	0.0412
7							-0.0887

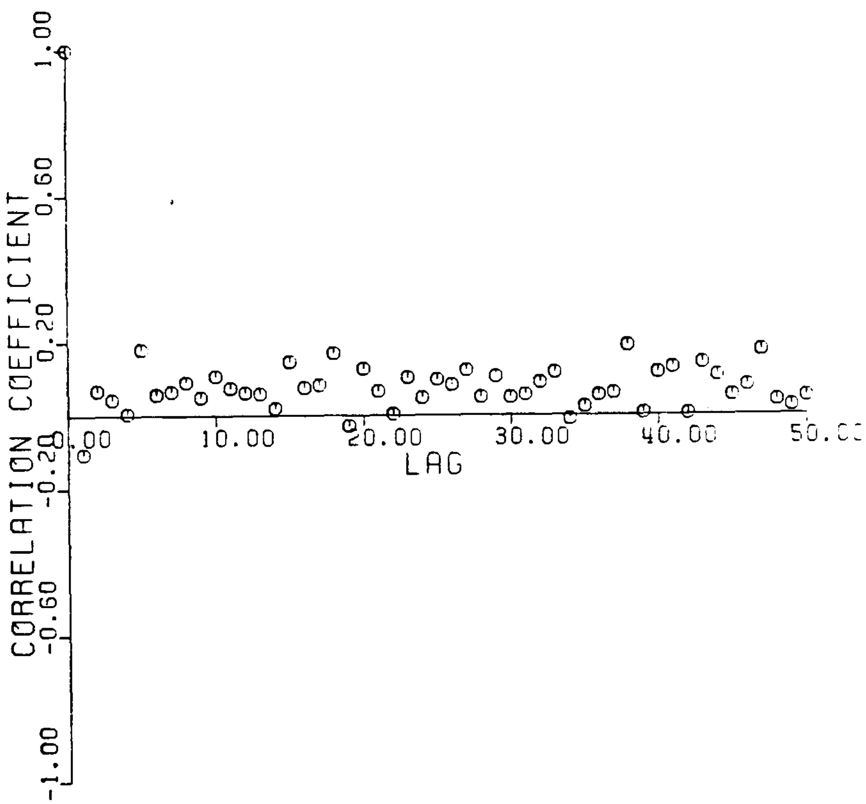

Fig. 5a. **Correlogram of the Residuals at Downstream Station of the Hypothetical Catchment - Case 1**

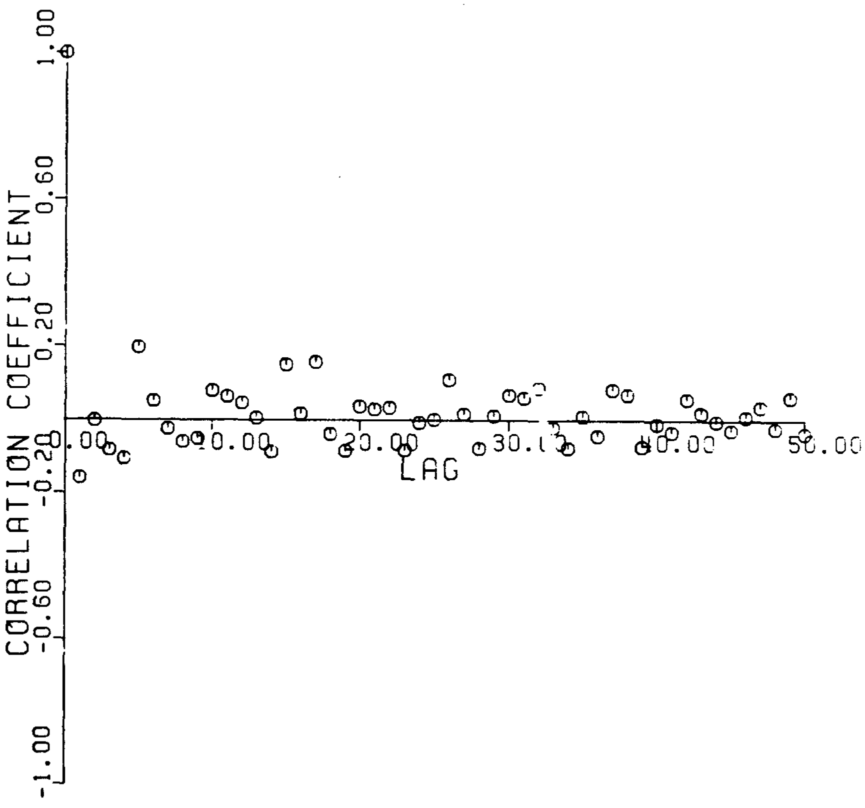

Fig. 5b. **Correlogram of the Residuals at Downstream Station of the Hypothetical Catchment - Case 2-1**

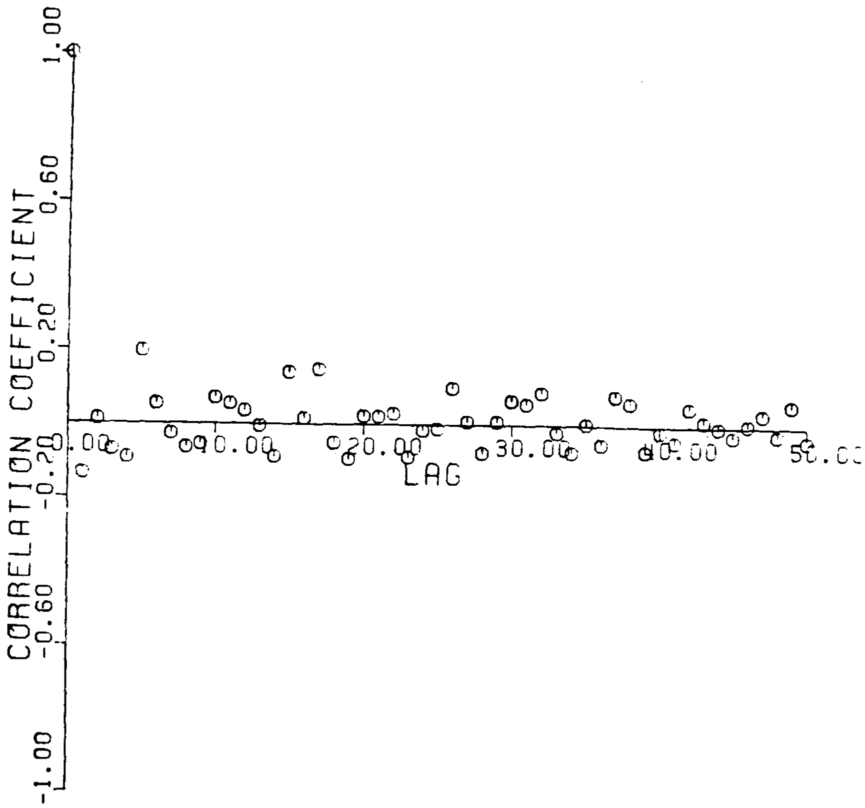

Fig. 5c. **Correlogram of the Residuals at Downstream Station of the Hypothetical Catchment - Case 2-2**

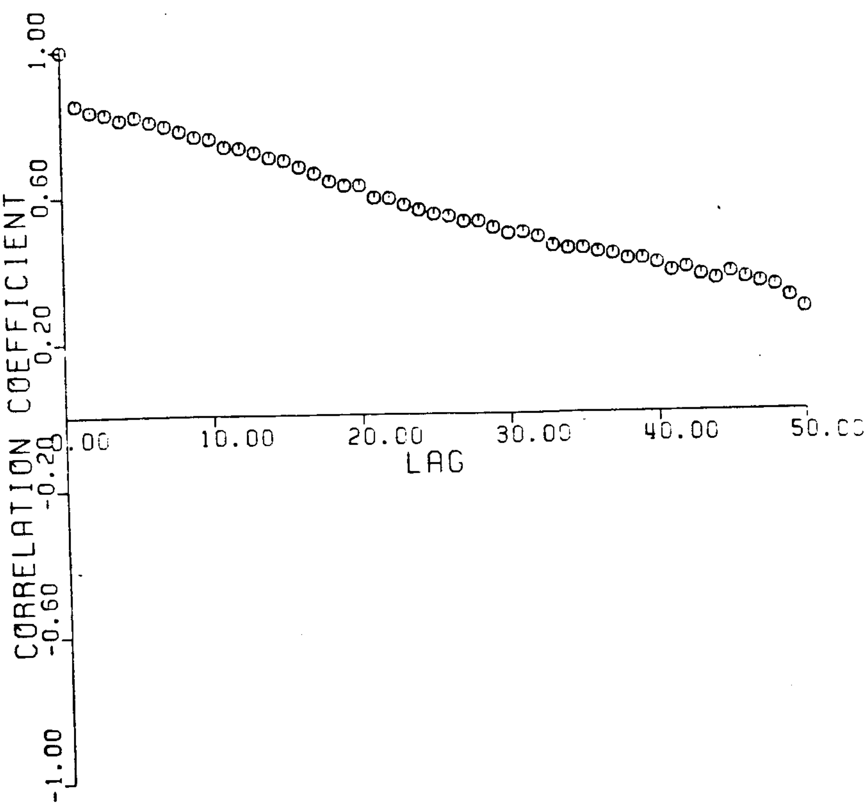

Fig. 5d. Correlogram of the Residuals at Downstream Station of the Hypothetical Catchment - Case 3

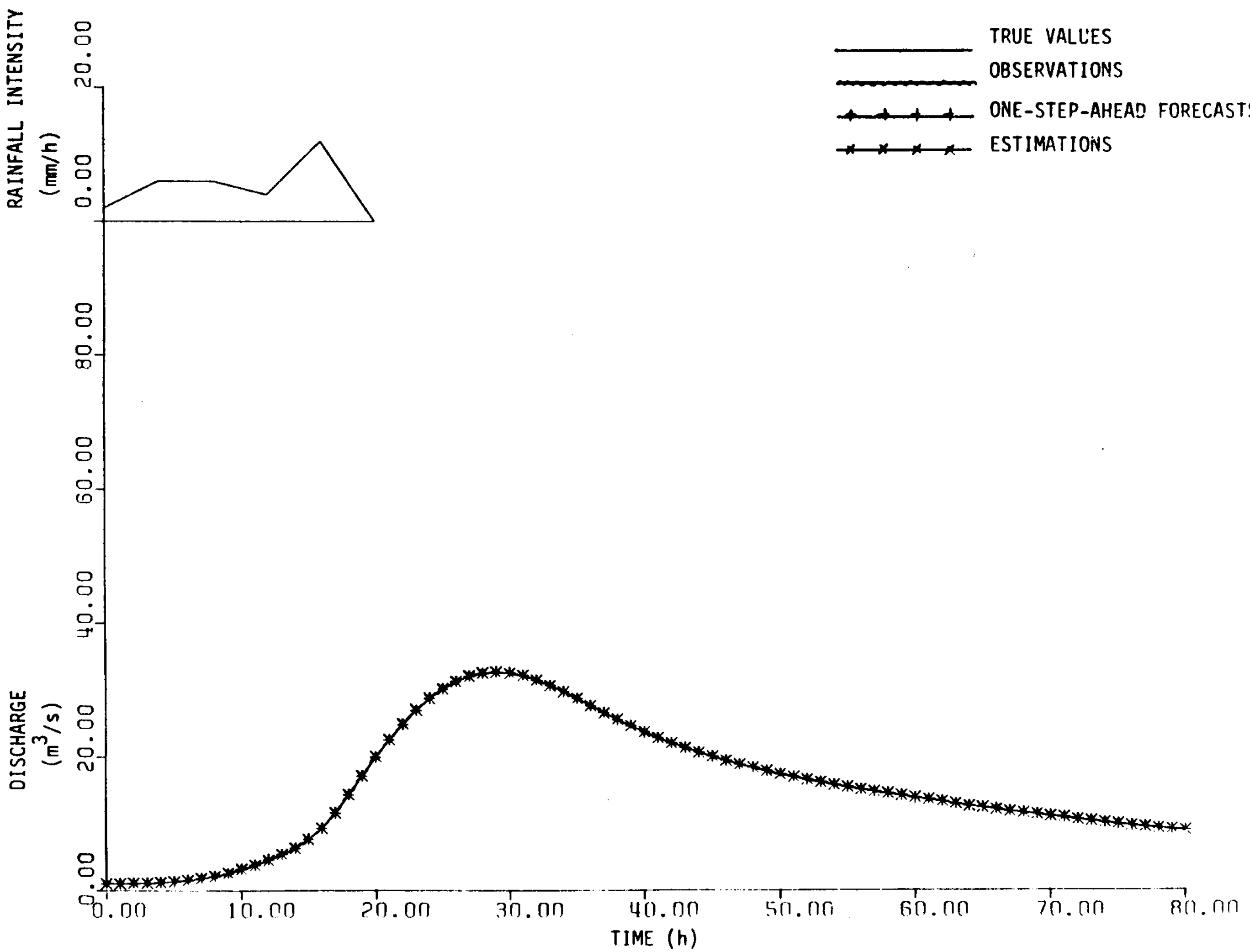

Fig. 6a. Observed, One-Step-Ahead-Forecasted, and Estimated Hydrographs of the Conceptual Model - Case 1

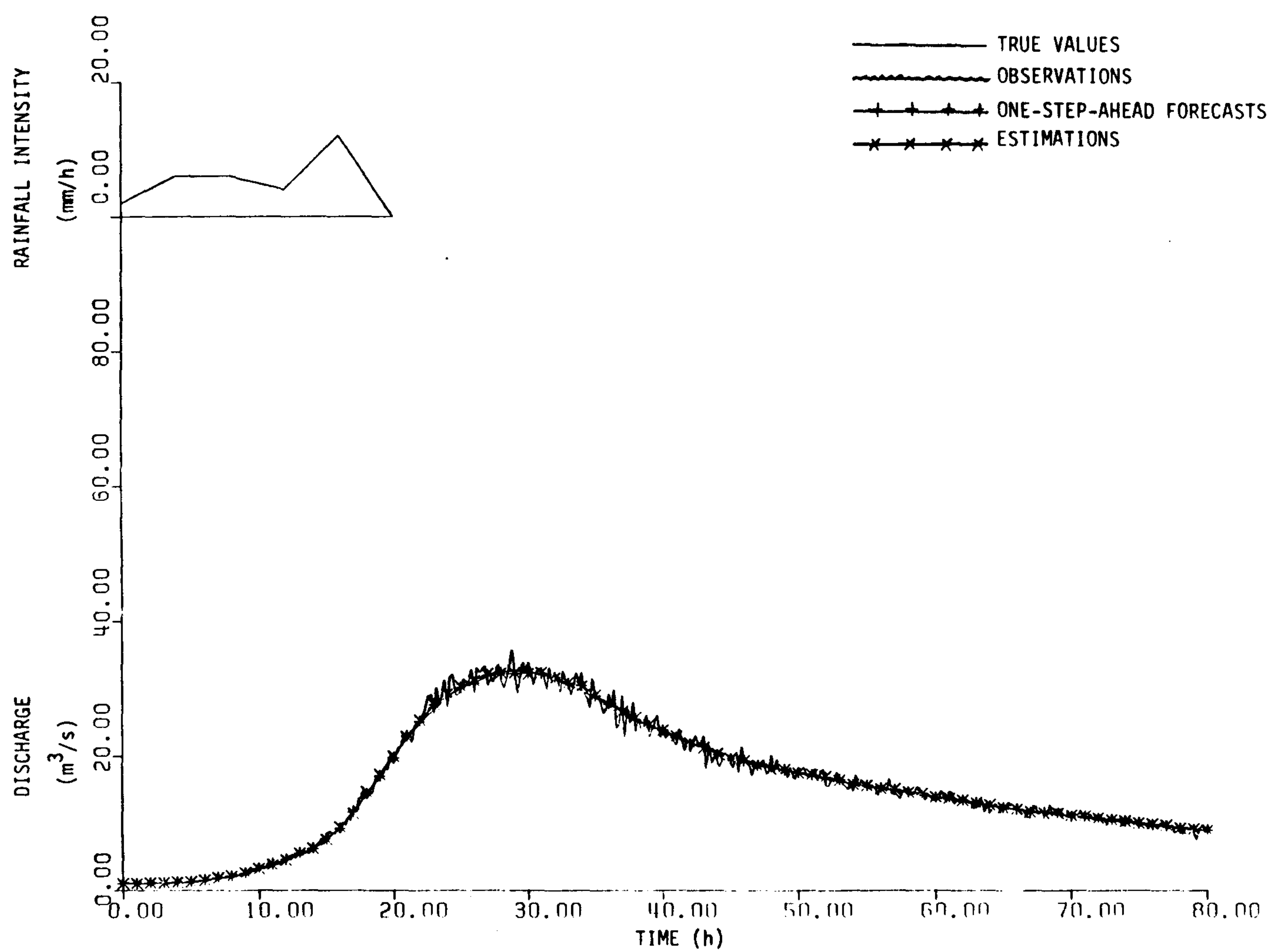

Fig. 6b. **Observed, One-Step-Ahead-Forecasted, and Estimated Hydrographs of the Conceptual Model - Case 2-1**

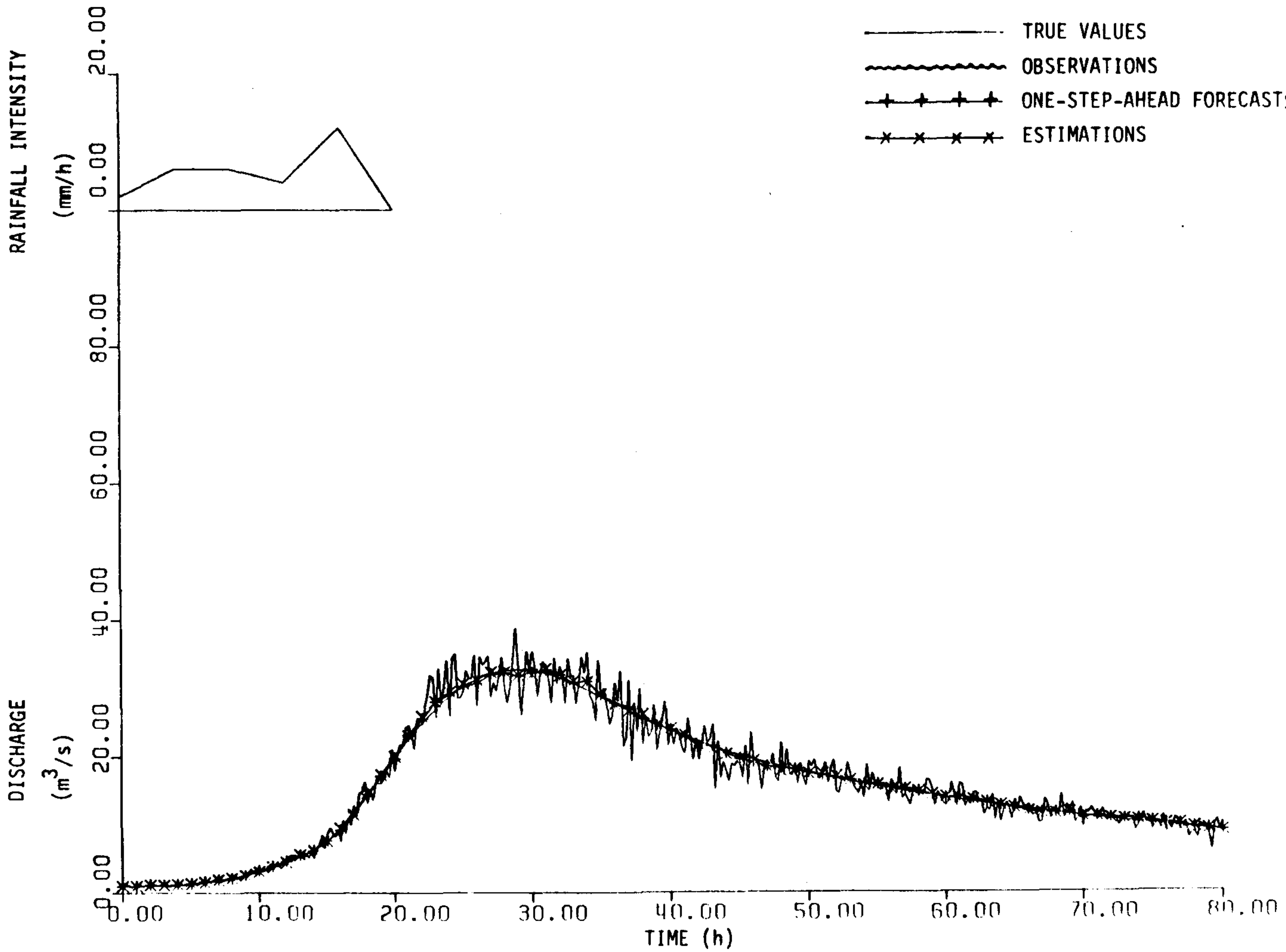

Fig. 6c. **Observed, One-Step-Ahead-Forecasted, and Estimated Hydrographs of the Conceptual Model - Case 2-2**

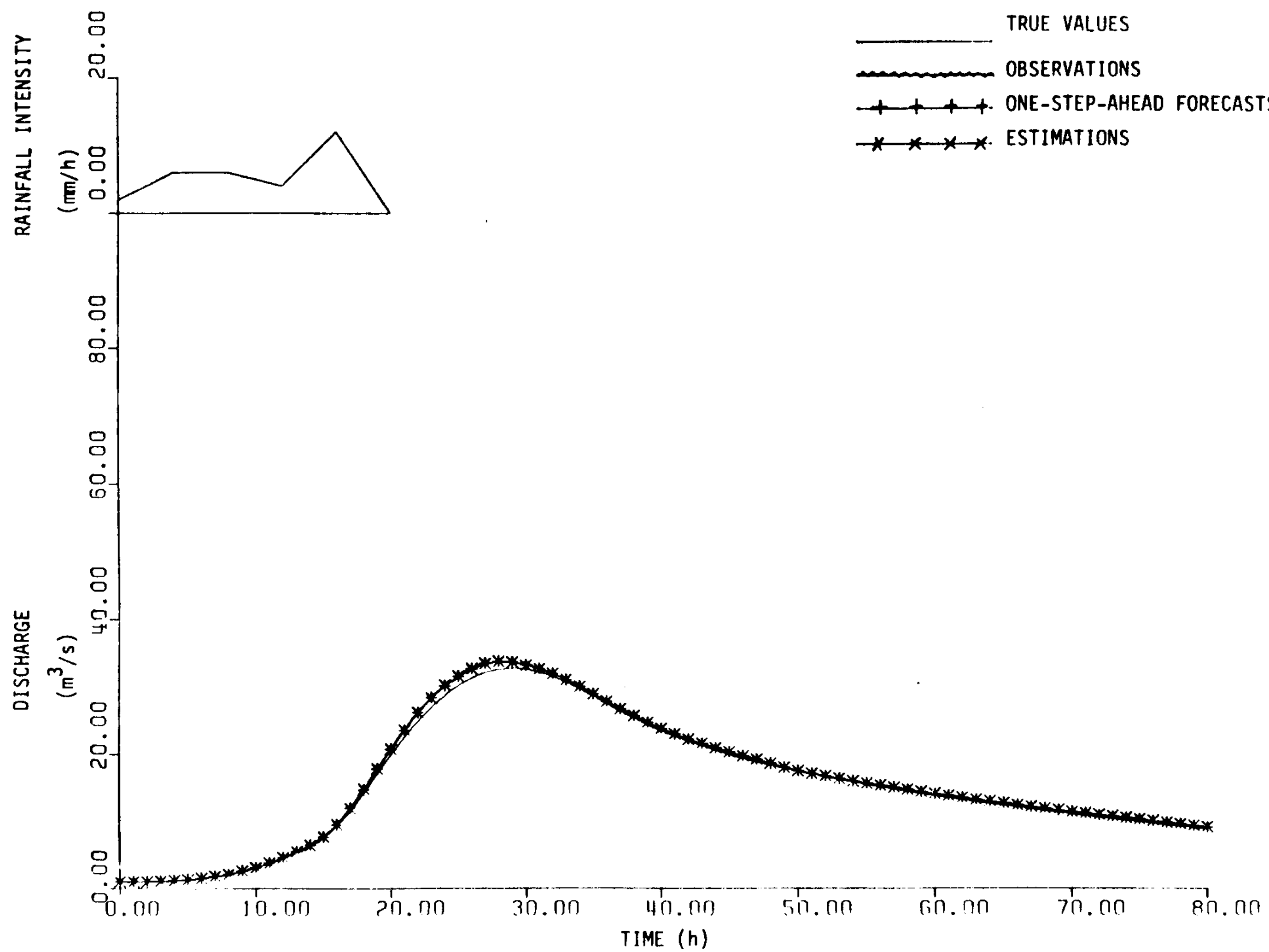

Fig. 6d. **Observed, One-Step-Ahead-Forecasted, and Estimated Hydrographs of the Conceptual Model - Case 3**

Application to Real Catchment

The catchment of the Williams River in the Hunter Valley in N.S.W., Australia (Fig. 7) was selected for study. The storm of March 1976 was chosen to test the proposed algorithm. The hyetograph with a duration of 25 hours and its corresponding hydrograph at the outlet of the catchment with a time base of 138 hours are shown in Fig. 8. Fifty-two (52) data points of the hyetograph and 46 data points of the hydrograph were recorded.

For the conceptual model, the first 20 discharges and the parameter values estimated by Field and Williams (1982) were used to estimate the initial values of parameters for the IEKF algorithm. A sensitivity analysis was made prior to the GLSO procedures. The results were similar to those of Williams and Yeh (1983) in that the groundwater storage coefficient is not sensitive. Therefore, it has been set to be a fixed value in this study. Tables 6 and 7 show the covariance matrix and correlation matrix of the estimates of the parameters. The parameter estimates by Field and Williams, by GLSO, and by IEKF are listed in Table 8. The proposed IEKF algorithm has been carried out to obtain the parameter estimates and the flood forecasts. Figure 11 shows the observations, the one-step ahead forecasts and the estimations of the discharges at the outlet of the watershed.

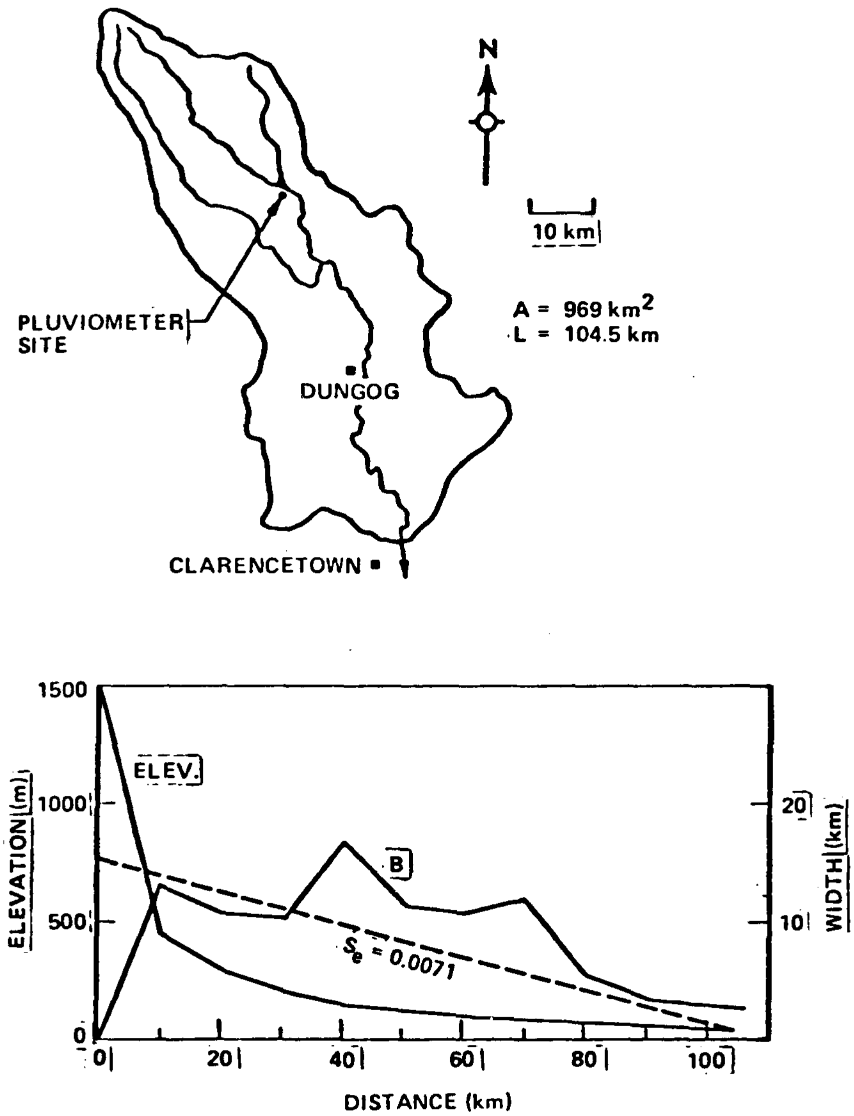

Fig. 7. **Catchment Details of Williams River**

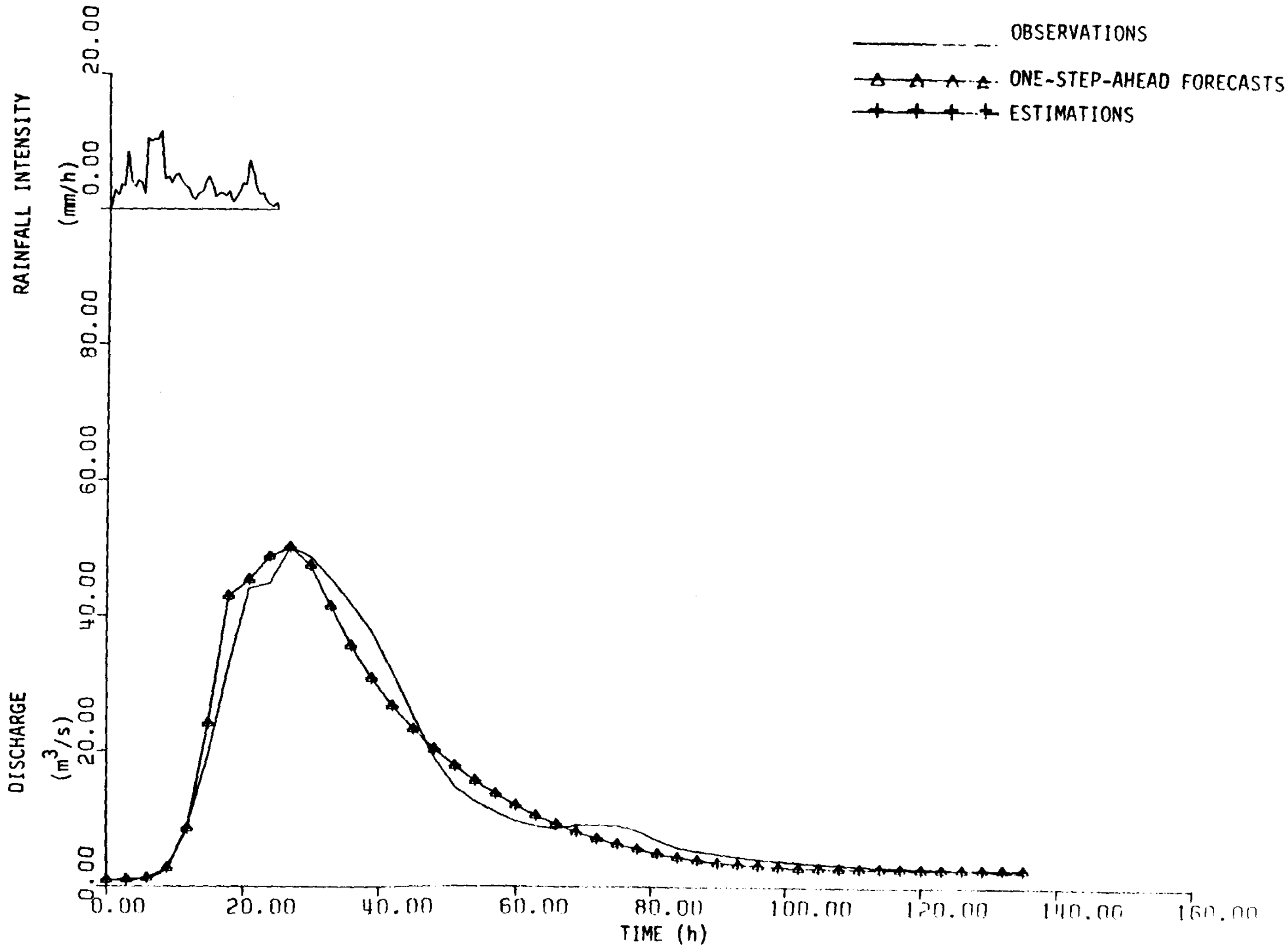

Fig. 8. Observed (3/1976), One-Step-Ahead Forecasted, and Estimated Hydrographs of Williams River from the Conceptual Model

Table 6. Covariance Matrix of Estimated Parameters

	t_c	K_s	K_g	ϕ
t_c	2.961×10^{-5}	-9.847×10^{-5}	-1.241×10^{1}	1.498×10^{-5}
K_s		4.761×10^{-4}	6.864×10^{-1}	-8.013×10^{-5}
K_g		(sym)	1.186×10^{3}	-1.323×10^{-1}
ϕ				1.508×10^{-5}

Table 7. Correlation Matrix of Estimated Parameters

	t_c	K_s	K_g	ϕ
t_c	1.0	-0.829	-0.662	0.709
K_s		1.0	0.913	-0.946
K_g		(sym)	1.0	-9.893
ϕ				1.0

Table 8. Parameter Estimations on Real Catchment

Method	t_c	K_s	K_g^*	ϕ
Initial Guess (Field & Williams)	8.0	23.0	500.0	0.5
GLSO	8.45	23.97	500.0	0.447
IEKF	8.45	23.97	500.0	0.447

* K_g is fixed to be 500.0

CONCLUSIONS

In terms of the hypothetical data which have been examined, the Iterated Extended Kalman Filter seems to be an effective technique for parameter estimation and flood forecasting in nonlinear conceptual models.

In the application of the IEKF on the chosen catchment, the performance of the conceptual model is fairly good in that the peaks for both observations and one-step ahead prediction are very close. Unfortunately, the post analysis of the prediction error could not be done due to the lack of data.

The proposed approach has the advantage that practical nonlinear conceptual models can be used, in conjunction with the IEKF for real-time parameter estimation and flood forecasting.

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LIST OF SYMBOLS

$A(\hat{\theta})$	=	$[J_a(\hat{\theta})]\ [J_a(\hat{\theta})^T]$;
ARMAX	=	AutoRegressive Moving-Average with exogenous input;
a	=	the dimensional coefficient in the relationship between p velocity, v and depth, y;
B	=	width of the elemental strip (Fig. 1);
b	=	the constant in the relationship between storage volume, h and discharge, s;
$C(\hat{\theta})$	=	correlation matrix about parameter $\hat{\theta}$;
$COV(\hat{\theta})$	=	covariance matrix about parameter $\hat{\theta}$;
c_{ij}	=	the entry of $C(\hat{\theta})$ at (i,j);
$\frac{d}{dt}$	=	differential with respect to t;
E	=	the statistical expectation;
$F(X_k,U_k)$	=	a vector function which corresponds to the watershed simulation model in this paper;
F'	=	the Jacobian of $F(X_k,U_k)$;
f_1	=	N-vector function;
f_2, f_3	=	scalar functions;
G	=	$a(n_X \times n_U)$ matrix;
GLSO	=	Generalized Least Squares Optimization;
H	=	observation matrix of dimension $n_Z \times n_X$ with entries of values 0 and 1;
$H(X_k)$	=	a vector function;
h	=	volume per unit area;
I	=	the identity matrix;
i	=	superscript which represents the ith iteration;
$J(\hat{\theta})$	=	summation of the squares of the differences between computed values (e.g. discharges) at $\hat{\theta}$ and observations;
$J_a(\hat{\theta})$	=	the Jacobian matrix evaluated at $\hat{\theta}$;
j	=	indices of increments of spatial variable, x;
K	=	the gain matrix in Kalman filter;

K_c = storage coefficient which could be surface water storage coefficient, K_s or groundwater storage coefficient, K_g;

K_g = groundwater storage coefficient;

K_s = surface water storage coefficient;

$k-1/k$ = subscript representing the value at time k-1 based on the information from the initial stage up to time k;

L = the number of parameters;

M = the number of observations;

m = the exponent in the relationship between channel velocity, v and depth, y;

N = number of increments in the main channel;

n_U = number of inputs to the system;

n_V = dimension of the measurement noise;

n_W = dimension of the system noise;

n_X = number of states of the system;

n_Z = number of observation stations;

P = covariance matrix used in Kalman filter;

p = precipitation or rainfall intensity;

p^{k+1} , p^k = rainfall intensity at time k+1 and k respectively;

p_e = a characteristic rainfall intensity, which can be taken as the average intensity for the storm duration;

p_1^{k-1} , p_2^{k-1} = the rainfall inputs at time k-1 to subsystems 1 and 2 respectively;

Q_k = $E[W_k, W_k]$, covariance matrix of W_k,

q = discharge;

q_k = the runoff (discharge) at time k,

q_*^k = $[q_1^k , q_2^k ,\ldots, q_N^k]^T$;

q_j^k = discharge a time k at the jth spatial point in the main channel;

R_k = $E[V_k, V_k]$, covariance matrix of V_k;

s = the volumetric rate of total supply from surface storage and groundwater per unit area to channel flow;

s_g = groundwater suply (discharge per unit area);

s_g^k , s_g^{k-1} = groundwater supply at time k and k-1 respectively;

s_s = surface supply (discharge per unit area);

s_s^k , s_s^{k-1} = surface water supply at time k and k-1 respectively;

s_* = supply which could be surface supply or groundwater supply;

T = symbol denoting transpose of a matrix;

t = temporal variable;

U_k = the n_U-vector of the deterministic inputs;

V_k = the n_V-vector of the measurement noise which is zero mean, independent white Gaussian processes, i.e., V_k - $N(0,R_k)$ and $E[V_k , V_k]=0$ for $k \neq i$;

v = velocity;

W = the n_w-vector of the system noise which is zero mean, independent white Gaussian process, i.e., W_k - $N(0,Q_k)$ and $E[W_k,W_k] = 0$ for $k \neq i$;

X_k = the n_X-vector of the states of the system at discrete time k, k=0, 1, 2, ...;

X_k^* = any estimate of state X_k;

x = spatial variable;

Y_k = $[Z_1 , Z_2 , \ldots , Z_k]^T$;

y = water depth in the main channel;

Z_k = the n-vector of the measurements on the system at discrete time k, k=1, 2, 3, ...;

ξ , η = certain functionals characterizing the system.

Φ = a $(n_X \times n_X)$ matrix;

ϕ = infiltration index which is set to be p when $\phi > p$ and $s_s = 0$;

Γ = a $(n_X \times n_W)$ matrix;

$\Gamma(X_k)$	=	a vector function;
γ	=	the constant in the relationship between storage volume, h and discharge, s;
$\hat{}$	=	symbol denoting estimated value;
λ	=	an arbitrarily small positive number;
θ	=	parameters;
$\hat{\theta}$	=	estimates of θ;
Δx	=	difference step in x - direction;
Δt	=	difference step in t - direction;
σ_{ij}	=	the entry of $COV(\hat{\theta})$ at (i,j);
ν	=	innovation, i.e., $\nu_k = Z-H\hat{X}^i_{k/k-1}$;
ω	=	$K\nu$, and
/	=	conditioning notation.