MAE 150R
Rocket Propulsion Systems
Assignment 3: Due Monday, May 8, 2000
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1. a.) Show that the height of a rocket in a vertical trajectory at the time of burnout, t_b, is

$$h_{bo} = gI_{sp}t_b \left(\frac{m_f}{M_o - m_f} ln \left(\frac{m_f}{M_o} \right) + 1 \right) - \frac{1}{2} g t_b^2$$

or

$$h_{bo} = gI_{sp}t_b \left(\frac{1-k}{k} ln \left(1 - k \right) + 1 \right) - \frac{1}{2} g t_b^2$$

where the rocket's initial mass $M_o = m_i + m_p + m_{pl}$, with inert mass m_i, propellant mass m_p, and payload mass m_{pl}, and where $k = \frac{m_p}{M_o}$.

b.) If k is considered to be independent of t_b, show that the maximum height the rocket can attain for a given t_b is

$$h_{bo,max} = \frac{1}{2} g t_b^2$$

c.) If k is now considered to be dependent on t_b, show that the maximum height the rocket can attain for a given t_b is

$$h_{bo,max} = gI_{sp}^2 \left[(1-k) ln (1-k) + k - \frac{k^2}{2} \right]$$

2. Show that in an N-stage rocket, G, the liftoff-to-payload mass fraction, and S, the mission requirement fraction, defined by

$$S \equiv \frac{u_{bo,N} + g_e \tau_b}{g_e I_{sp}}$$

are related by

$$G = \left(\frac{(1-\epsilon)}{e^{-S/N} - \epsilon} \right)^N$$

where each stage has the same structural factor ϵ. Show also that, in the limit of an infinite number of stages,

$$\lim_{N \to \infty} G = e^{S/(1-\epsilon)}$$

3. Problem 10.2 in Hill and Peterson's text

4. Problem 10.6 in Hill and Peterson's text