Preliminary Results on the Interpretation of Low-Frequency Fluctuations Observed in the Reflex Arc

by

Francis F. Chen

MATT-249 January, 1964

PLASMA PHYSICS LABORATORY

Contract AT(30-1)—1238 with the
US Atomic Energy Commission

PRINCETON UNIVERSITY

PRINCETON, NEW JERSEY

Princeton University
Plasma Physics Laboratory
Princeton, New Jersey

Preliminary Results on the Interpretation of Low-Frequency Fluctuations Observed in the Reflex Arc

by

Francis F. Chen

MATT-249 January, 1964

AEC RESEARCH AND DEVELOPMENT REPORT

This work was supported under Contract AT(30-1)-1238 with the Atomic Energy Commission.

Because of recent progress in the theory of drift instabilities in partially ionized gases, we are now in a position to understand data taken in 1961-2 in collaboration with R. Bingham and A. W. Cooper on electrostatic fluctuations in reflex discharges. These measurements were carried out on two machines: L-1, a 2-foot long, 1/2-inch dia. discharge in 10 μ of He (B = 50 - 2000 G, pulsed to 8000 G); and L-2, a 11-foot long 2-inch dia. discharge in 5 μ of He (B = 1000 - 4000 G). The qualitative behavior of the oscillations is shown in Fig. 1; the data were taken in L-1, in which a stable discharge could be obtained over a wide range of B.

Below about 750 G, the nature of the oscillations changes in a complicated fashion; above this field, however, the state of turbulence is reproducible and changes only slowly with B up to the highest fields used. This is the region we have studied. It is characterized by the frequency spectrum (log scale) shown in Fig. 2. This spectrum has a low-frequency peak around 50 kc, which corresponds roughly to the rotation frequency of the plasma in the radial electric field, and a continuous background which is down $\sim$ 30 db at the ion cyclotron frequency ($\sim$ 1.5 Mc). This spectrum is unaffected by (a) the external impedance of the discharge circuit, (b) slight misalignment of the discharge tube relative to B, and (c) changing from potential measurements to density measurements with the Langmuir probe. A similar spectrum is observed in many other types of discharges subject to anomalous diffusion, including the Etude Stellarator.[1]

That the oscillations are not caused trivially by cathode surface effects has been established by operating the hot cathodes under space-charge-limited conditions[2] and by drastically altering their geometry.[3]

Figure 3 shows the floating potential signals on two probes in L-2 placed near either end of the discharge at the same azimuth. When the probes are at the same radii, both the fundamental frequency and the higher frequency fluctuations are in phase; when they are at different radii, the higher frequencies are less well correlated. Detailed correlation measurements[4] in L-1 have shown that the axial wavelengths are greater than the length of the machine. Transverse correlation measurements have indicated that the fundamental frequency ω_o is due to an $m = 1$ distortion of the plasma column and a rotation in the crossed electric and magnetic fields. There is no observable phase change of ω_o along a radius until one crosses the axis.

Several possibilities for the origin of these fluctuations, that is, for the linear instability which develops into the observed state of turbulence, have been rejected. That these are ion waves is ruled out by the axial wavelength, and hence velocity, measurements. That these are ion cyclotron waves perpendicular to B has been ruled out by the frequency spectrum. That these are ion waves generated at the cathode sheath[5] has further been ruled out on theoretical grounds.[6] We now consider the possibility, suggested by Simon[7] and by Hoh,[8] that these are drift instabilities driven by the radial electric field. The following treatment differs from those of

Simon and Hoh in two main respects: (1) the centrifugal force terms are retained, and (2) damping by conduction through the cathode sheaths is considered. The latter is necessitated by the observation that $\lambda_{\parallel}$ is greater than the machine length 2L, and by a computation by Bingham[9] that only $\lambda_{\parallel}$'s greater than 12 meters are unstable if the usual axial damping mechanism (via electron motion along B) is considered.

The equilibrium is assumed to be that described previously by the author.[2,10] This theory shows that the electric potential $\phi(r)$ is easily predictable in terms of L, R, B, p (pressure), T_i and T_e, although the function $\phi(r)$ cannot be given in closed form. The centrifugal force affects the equilibrium only in second order in Ω_o (see Eq. (2) below). The linearized equations of motion for the ion fluid in cylindrical coordinates are as follows:

$$Mn_o \left(\frac{\partial v_r}{\partial t} + v^{(o)} \cdot \nabla v_r + v \cdot \nabla v_r^{(o)} - \frac{2 v_\theta v_\theta^{(o)}}{r}\right) = qn_o(-\phi' + v_\theta B) -$$

$$- KT\left(n' - \frac{n}{n_o} n_o'\right) - \frac{Mn_o v_r}{\tau} \tag{1a}$$

$$Mn_o \left(\frac{\partial v_\theta}{\partial t} + v^{(o)} \cdot \nabla v_\theta + v \cdot \nabla v_\theta^{(o)} + \frac{v_\theta^{(o)} v_r}{r} + \frac{v_\theta v_r^{(o)}}{r}\right)$$

$$= - qn_o\left(\frac{\partial \phi}{r\partial\theta} + v_r B\right) - KT\frac{\partial n}{r\partial\theta} - \frac{Mn_o v_\theta}{\tau}\ , \tag{1b}$$

where B is constant, the prime denotes $\partial/\partial r$, τ is the ion-neutral collision time, $\underset{\sim}{v}$, ϕ, and n are first-order quantities, and the other symbols are obvious. The zero-order electric field has disappeared because we subtracted $(n_o + n)$ times the zero-order equations. We have neglected the viscosity tensor because very little shear is observed experimentally. In fact, we shall assume that $v_\theta^{(o)}/r \equiv \omega_o$ is a constant, so that

$$\psi \equiv \omega - m\omega_o \quad \text{and} \quad \Omega_o \equiv v_\theta^{(o)}/r\omega_c \tag{2}$$

are also constant. A similar set obtains for the electrons, but we shall neglect the inertial terms on the l. h. s. We have omitted the z-component because we shall assume no dependence on z, as is indicated by the observations. We consider perturbations of the form

$$\phi = \phi(r)\, e^{i(m\theta - \omega t)}$$

$$n = \nu(r)\, n_o(r)\, e^{i(m\theta - \omega t)} \quad , \tag{3}$$

and further simplify by assuming

$$v_r^{(o)} << v_\theta^{(o)} \quad , \quad v_\theta << v_r \quad , \quad \frac{\partial}{\partial r} << \frac{1}{r}\frac{\partial}{\partial \theta} \quad . \tag{4}$$

Defining the axial diffusion and mobility coefficients D and μ the usual way, we find for the solution of (1a) and (1b):

$$-[A_1 A_2 + (\alpha C)^2]\, v_r = A_2(\mu\phi' + D\nu') + i\gamma\alpha C(\mu\phi + D\nu) \tag{5a}$$

$$[A_1 A_2 + (\alpha C)^2]\, v_\theta = \alpha C(\mu\phi' + D\nu') - i\gamma A_1(\mu\phi + D\nu) \quad , \tag{5b}$$

where

$$A_1 = 1 - i\psi\tau + v_r^{(o)'}\tau$$

$$A_2 = 1 - i\psi\tau + r^{-1} v_r^{(o)}\tau \tag{6}$$

$$C = 1 + 2\Omega_o \ , \quad \alpha = \omega_c \tau$$

and similarly for the electrons.

For each species there is an equation of continuity of the form

$$Q = \frac{1}{r}\frac{\partial}{\partial r}(r n v_r) + \frac{1}{r}\frac{\partial}{\partial \theta}(n v_\theta) + \frac{\partial n}{\partial t} \ , \tag{7}$$

in which Q is a source term which includes volume ionization as well as loss or gain of particles from the ends of the machine. From the equilibrium theory,[2,10] we can write

$$Q_i = \frac{\gamma v_s}{2L} n \left[NM(\phi)\Gamma(\phi) - 2 \right] \tag{8a}$$

$$Q_e = \frac{n}{2L}\left[N\gamma v_s (1 + M)\Gamma(\phi) - v_e e^{-\chi} \right] \ , \tag{8b}$$

where $\chi = e\phi/KT_e$, N is the number of emitting cathodes, γ is a dimensionless parameter of order 0.2, v_s and v_e are sound and electron thermal velocities, M is the number of ion pairs created by each primary electron, and $\Gamma(\phi)$ is a function describing the number of primaries emitted under space-charge-limited conditions for a given sheath drop ϕ. The terms in M give the volume ionization. The second term in Q_i is the loss of ions by streaming to the cathodes. The exponential term in Q_e is the current of primary electrons in the Maxwellian tail which are able to escape to the cathode through the potential barrier. Finally, the first part of the term in Q_e represents the injection of the primary electrons into each tube of force.

The axial damping mechanism can now be described in terms of the sheath effects contained in Q. Suppose the potential ϕ is given a positive perturbation on a particular line of force. This will pull more primaries out of the cathode through an increase in Γ, thus tending to reverse the perturbation. It will also prevent the escape of more plasma electrons through an increase in χ, again tending to stabilize the perturbation. The volume ionization will also go up, because of an increase in $M\Gamma$, but this does not affect the charge balance. Whether or not the perturbation will grow will depend on the relative effectiveness of the driving force and these damping mechanisms.

We now linearize (7) and (8) to obtain

$$\chi \zeta_i = -i\omega\nu + \nabla_r v_{ri} + \lambda v_{ri} + i\frac{m}{r}(v_{\theta i}^{(o)}\nu + v_{\theta i}) + v_{ri}^{(o)}\nu' \qquad (9)$$

$$\chi(\zeta_e + \xi) = -i\omega\nu + \nabla_r v_{re} + \lambda v_{re} + i\frac{m}{r}(v_{\theta e}^{(o)}\nu + v_{\theta e}) + v_{re}^{(o)}\nu', \qquad (10)$$

where $$\lambda = n_o'/n_o \;, \qquad \xi = \frac{v_e}{2L}\, e^{-\chi_o}$$

$$\zeta_i = \frac{N}{2L}\gamma v_s \frac{\partial}{\partial\phi}(M\Gamma) \;, \qquad \zeta_e = \frac{N}{2L}\gamma v_s \frac{\partial}{\partial\phi}(\Gamma + M\Gamma) \;. \qquad (11)$$

When the equations for $\underset{\sim}{v}$ are inserted into (9) and (10) these form two coupled differential equations (in r) for ν and χ. If the boundary conditions are given, the eigenvalues of ω give the dispersion relation $\omega(m)$. However, the coefficients in the equations contain complicated functions of r: the equilibrium electric field $\chi_o(r)$ and density distribution $\lambda(r)$. We make a great simplification by assuming that only a small error in ω will be committed if we take the density perturbation to have the same shape as n_o. Then $\nu' = 0$, and it can be shown that all other radial derivatives vanish also. We are thus reduced to two algebraic equations for ν and χ, and the condition that their determinant vanish gives the dispersion relation $\omega(m)$. We quote here only the approximate result, obtained after considerable algebra and neglect of small terms, corresponding to setting $A_1 = A_2 = 1$ in Eq. (6). Inclusion of the ψ terms will introduce another root for ω but will not greatly affect our conclusions.

$$\mathrm{Im}(\omega) \approx \frac{[-\beta D_{\perp i}\lambda\chi'_o(1+\alpha_i^2\Omega_o) - C^{-1}(\gamma^2 D_{\perp e} + \beta^{-1}\xi_1)](C^{-2}+\eta_2)}{C[(C^{-2}+\eta_2)^2 + \lambda^2\gamma^{-2}\alpha_i^{-2}(1+2\alpha_i^2\Omega_o)^2]} \tag{12}$$

$$\mathrm{Re}(\omega) = m\omega_o\Big[1 + \frac{C^{\frac{1}{2}}(C^{-2}-\eta_3)(\Omega_o+\alpha_i^{-2})}{C^{-2}+\eta_2} - \frac{C^{-1}\lambda\alpha_i D_{\perp i}(\beta\delta+\eta_2)}{v_o(C^{-2}+\eta_2)} \tag{13}$$

$$+ \frac{\mathrm{Im}(\omega)}{\gamma^2 v_o}\frac{\lambda}{\alpha_i}\frac{(1+2\alpha_i^2\Omega_o)}{C^{-2}+\eta_2}\Big] \quad , \text{ where}$$

$$C \equiv 1 + 2\Omega_o \qquad \lambda \equiv \frac{n_o'}{n_o} \qquad \chi \equiv \frac{e\phi}{KT_e} \qquad \beta \equiv \frac{T_e}{T_i} \qquad \gamma \equiv \frac{m}{r}$$

$$v_o = |v_{\theta i}^{(o)}| \quad , \quad \xi_1 = \zeta + \zeta_e \tag{14}$$

$$\eta_1 = \frac{\xi_1}{\beta\gamma^2 D_{\perp i}} \quad , \quad \eta_2 = \frac{\xi + \zeta_e - \zeta_i}{\beta\gamma^2 D_{\perp i}} \quad , \quad \eta_3 = \eta_1 - \eta_2 \quad .$$

Here γ is the azimuthal wave number, not the γ of Eq. (8); and $D_\perp$ is the usual transverse diffusion coefficient. In obtaining this result many reasonable approximations have been made, such as $\Omega_o << 1$, $\beta >> 1$, etc.

Let us first examine the bracket in the numerator of Im(ω). The term, in $(1 + \alpha_i^2 \Omega_o)$, is the unstable one, and the second term, which is always negative, represents the damping. The term "1" in the first term is the Simon-Hoh effect;[7,8] it is unstable whenever $\lambda\chi_o'$ is negative; that is, when the zero-order density gradient is in the same direction as the electric field. The term in $\alpha_i^2\Omega_o$ gives rise to an instability due to the centrifugal force. In our experiment, $\alpha_i^2\Omega_o$ is of order 40, so that this effect should completely mask the Simon-Hoh effect. Furthermore, the centrifugal instability occurs with either sign of χ_o', since Ω_o changes sign whenever χ_o' does. For a physical explanation of this phenomenon, the reader is referred to another paper.[11] Although the centrifugal force does not depend critically on collisions, the instability does, since the limit of $\alpha_i^2 D_{\perp i}$ vanishes for large τ.

We now consider the damping terms in Im(ω). The term in $D_{\perp e}$ simply represents transverse diffusion damping. The term ξ_1 is the dominant term arising from the axial damping mechanisms described above. In our experimental conditions ξ_1 is extremely small, which is to say that the cathode sheath is a good insulator. $D_{\perp e}$ is also small, of course, and therefore the total damping is negligible except for large values of γ. To be more quantitative, we refer to the data of Fig. 4, which shows the radial variation of n, ϕ, and KT_e, averaged over the fluctuations, for a typical discharge condition. The assumption of constant KT_e is apparently quite good. We do not know KT_i, but it is not critical. In the center of the discharge, λ is positive, so the discharge is stable. Near the edge λ is negative, and we can measure λ and X_o' to evaluate the unstable term in Eq. (12). This turns out to be so large that all γ's below 300, corresponding to frequencies below 37 Mc, are unstable. Note that the axial damping decreases with γ.

We thus arrive at the following picture. Instabilities are generated whenever the density gradient is inward. The nature of the oscillations is such that they tend to wipe out the density gradient. They succeed in doing so in the body of the discharge, where the gradient is zero and in fact is reversed because of loss by streaming along the axis, the potential barrier for electrons being very small there. At the edge of the discharge, there must be a layer where the gradient is negative, and the fluctuations continue to be generated here. The reason oscillations are detected in the stable region is that there must be a small component of $\underset{\sim}{k}$ along r, and the oscillations propagate inward from the generation region. One can compute an anomalous transport coefficient from the theoretical correlation between v_r and n; this turns out to be very large.

If we neglect the centrifugal force, we find that the Simon-Hoh effect is sufficient to cause the instability under these conditions. What evidence we have that it is the former effect that we are observing comes from consideration of Re(ω). In Eq. (13) we see that Re(ω) is approximately $m\omega_o$, the ion rotation frequency. The first two (small) correction terms increase this while the last term decreases it if Im(ω) is positive and λ negative. In Fig. 5a we see the profiles for another discharge condition (smaller L) for which we have sufficient data for the following analysis. In Fig. 5b are shown the measured dominant frequencies as well as the values of f_o computed from the measured λ and χ_o' at two radii where the instability might arise. Figure 6 shows more data of the same type. It is seen that f is always below f_o, although it sometimes comes within the experimental error. In order for this to happen, the term in Im(ω) of Eq. (13) must be sufficiently large. If centrifugal force is neglected, Im(ω) is not large enough to explain the discrepancy. If it is not neglected, Im(ω) is too large: it predicts a lower frequency than is observed. This is not unreasonable, since the local value of λ, and hence of Im(ω), must be smaller than that predicted by average values of λ and χ_o'.

We wish to point out that D. Morse[12] has proposed a different mechanism which also will lead to instability with either sign of χ_0'. This mechanism depends on a radial variation of the phase of the oscillation. Since no such dependence was observed in our experiments, we feel that this effect must play a minor role.

In summary, our primary aims in this work have been the following: (1) to show how the frequently-observed low-frequency fluctuations in partially-ionized gas in a magnetic field can arise; (2) to point out the importance of ion inertia in such instabilities; (3) to show how to treat axial damping properly when $\lambda_{\parallel}$ is larger than L. We are grateful to R. Bingham for his help with the experiments and analysis.

References

1. K. Bol, MATT-224 (1963); submitted to Physics of Fluids

2. R. Bingham, F. Chen, and W. Harries, MATT-63 (1962)

3. R. Bingham and F. Chen, Bull. Am. Phys. Soc. 7, 401 (1962)

4. F. Chen and A. W. Cooper, Phys. Rev. Letters 9, 333 (1962)

5. F. Chen, Phys. Fluids 4, 1448 (1961)

6. F. Chen, Nuovo Cim. 26, 698 (1962)

7. A. Simon, Phys. Fluids 6, 382 (1963)

8. F. C. Hoh, Phys. Fluids 6, 1184 (1963)

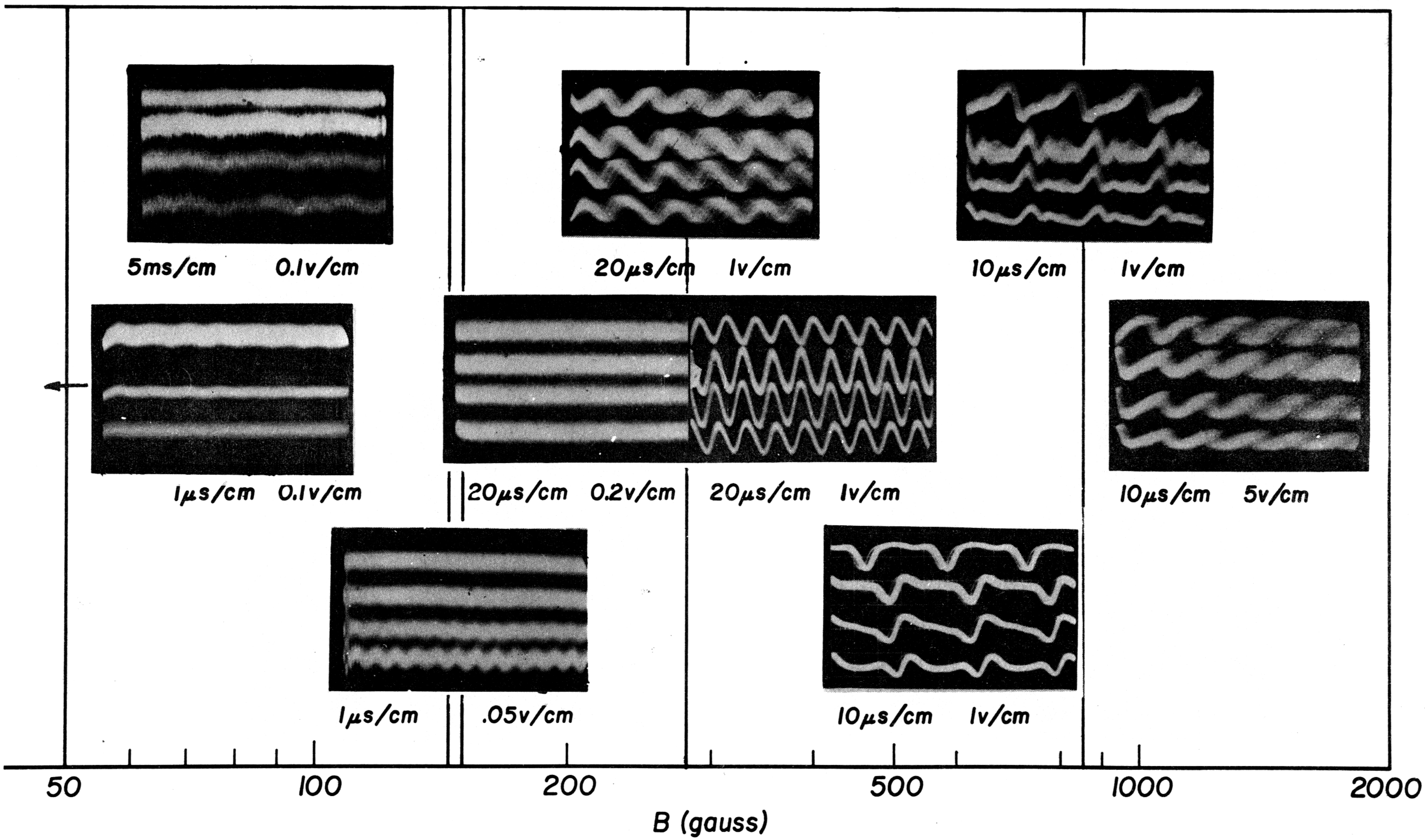
9. R. Bingham, MATT-198 (1963)

10. F. Chen, Phys. Rev. Letters 8, 234 (1962)

11. F. Chen, Proceedings of the Sixth International Conference on Ionization Phenomena in Gases, Paris, 1963; MATT-214 (1963)

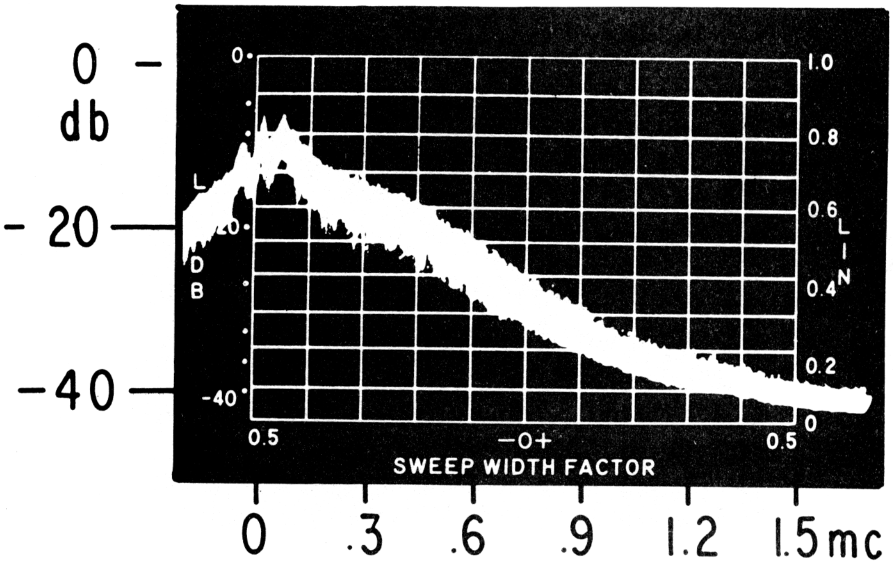
12. D. Morse, Proceedings of the Meeting of the Plasma Physics Division, A. P. S., San Diego, 1963

L-I OSCILLATIONS

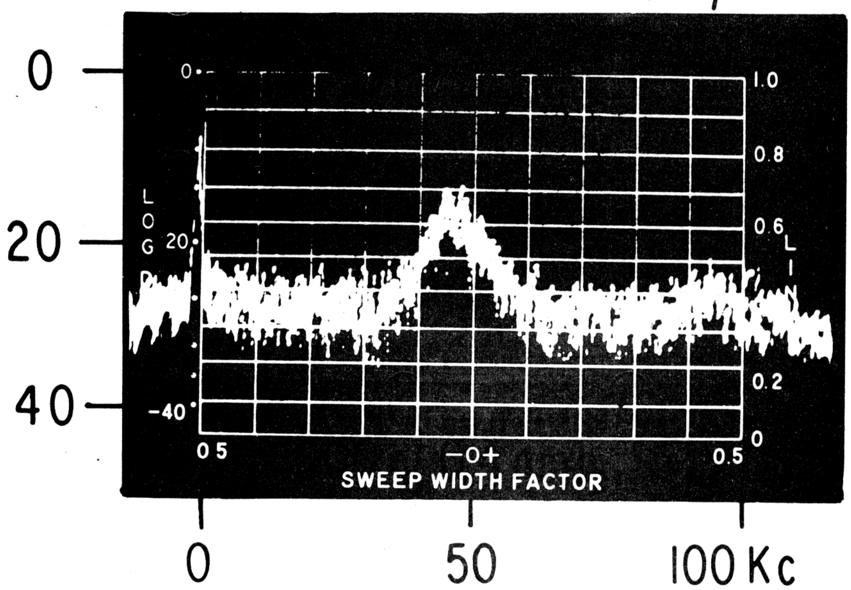


633569

2000 g, 0.4 amp., 5 μ

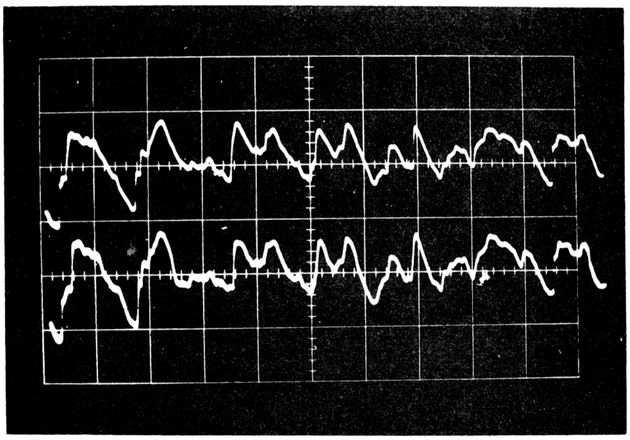


2000 g, 0.2 amp., 10 μ



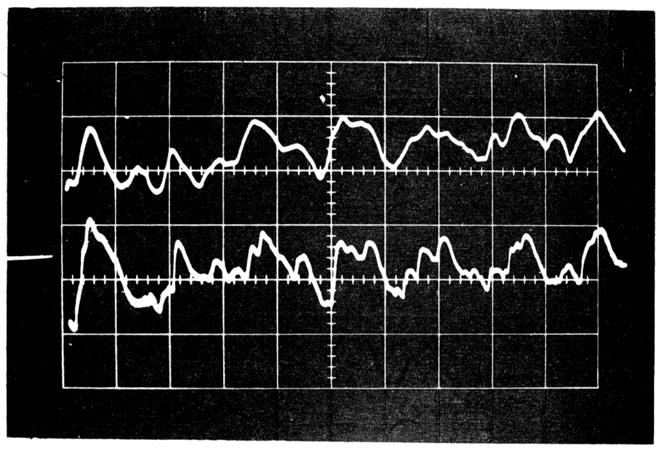
1 OUT

2 OUT



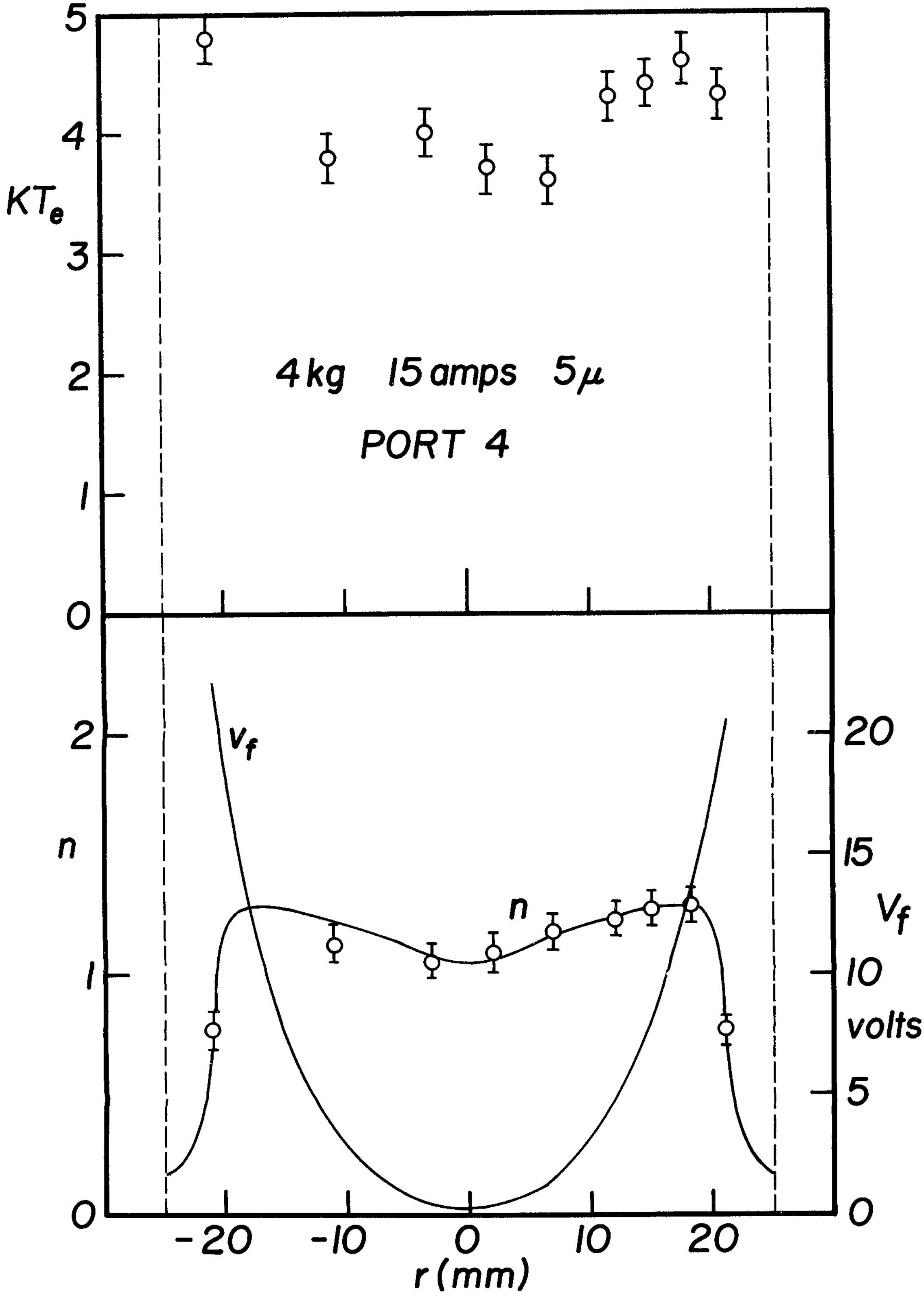
1 OUT

2 IN



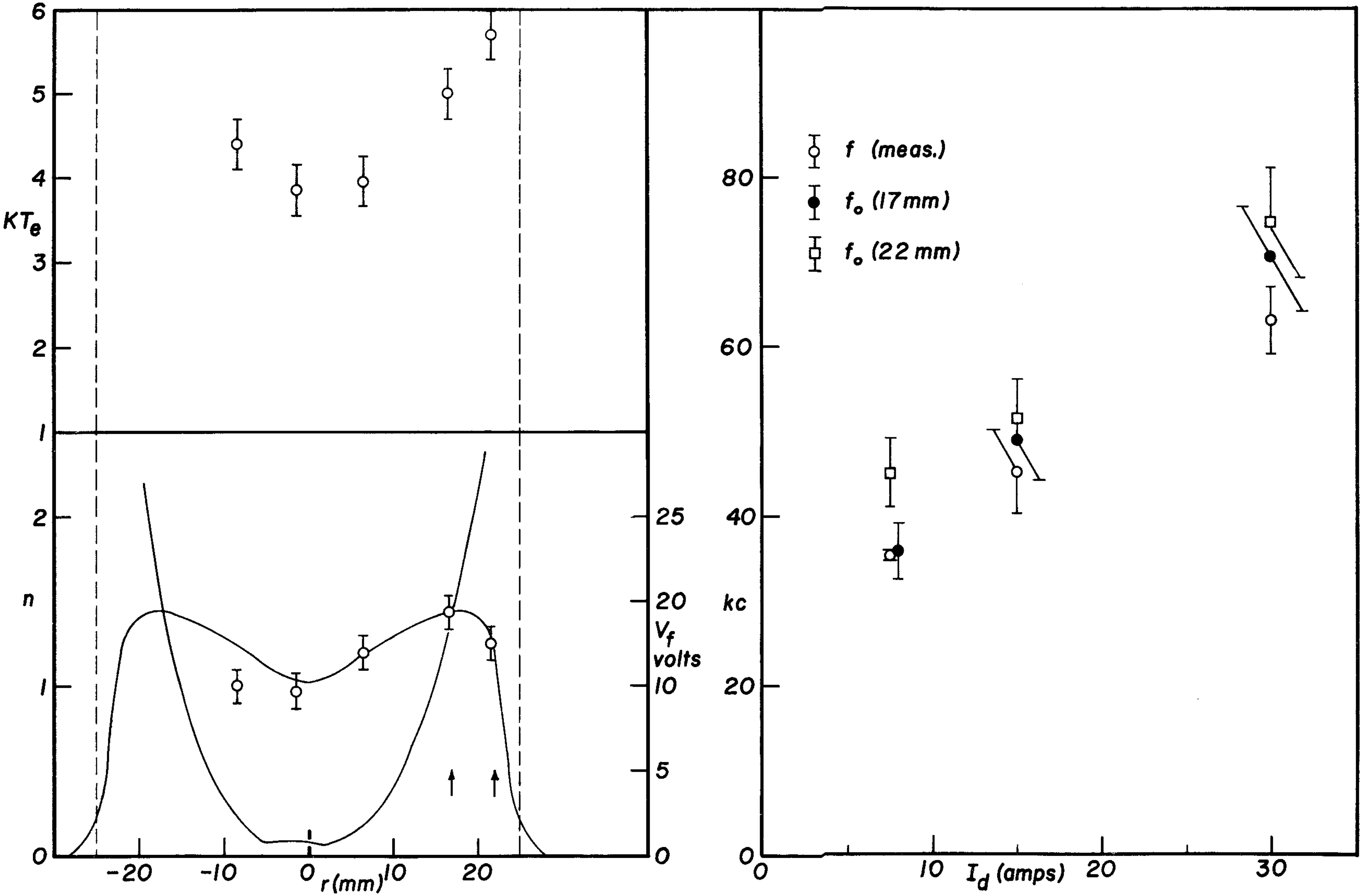
PROBES 2.5 METERS APART

633407

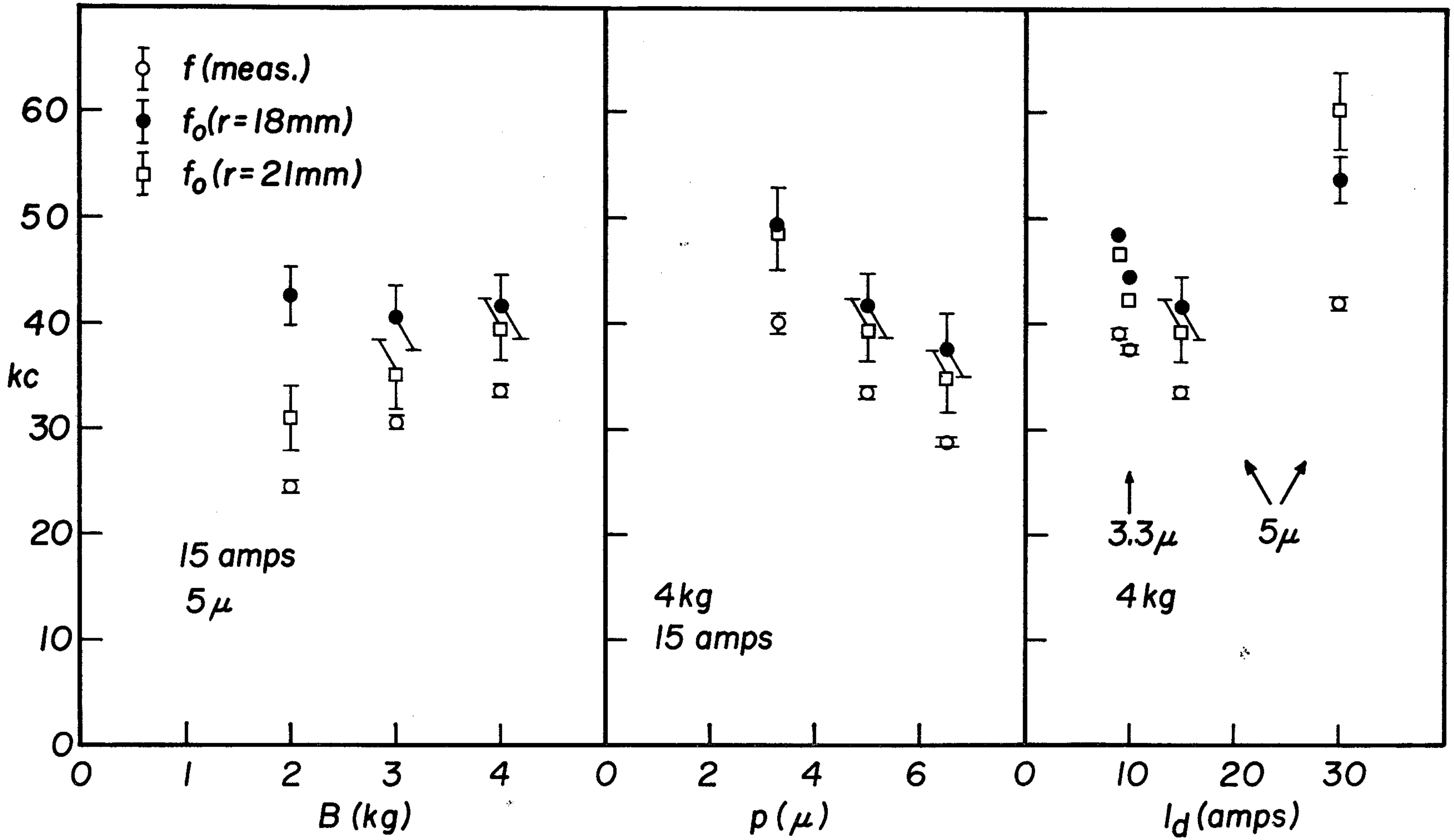


5
4
KT_e
3
2
1
0
4 kg 15 amps 5 μ
PORT 4
2
v_f
n
n
1
0
20
15
V_f
10
volts
5
0
-20
-10
0
10
20
r (mm)

633552



633568



f (meas.)
f_o(r=18mm)
f_o(r=21mm)
60
50
40
kc
30
20
10
0
15 amps
5μ
4kg
15 amps
3.3μ
5μ
4kg
0
1
2
3
4
0
2
4
6
0
10
20
30
B (kg)
p (μ)
I_d (amps)

633557