SPECTRUM OF LOW-β PLASMA TURBULENCE*

Francis F. Chen
Plasma Physics Laboratory, Princeton University, Princeton, New Jersey
(Received 4 June 1965)

The theory of drift waves[1] in an inhomogeneous plasma has greatly elucidated the mechanism of anomalous transport of plasma across a magnetic field B via electrostatic-potential fluctuations. These waves are in qualitative agreement with experiment in regard to the predominance of low frequencies, the phase velocities perpendicular to B $[\omega/k_\perp \approx \pm(KT_e/eB)(n_0'/n_0)]$, and the phase velocities parallel to B $(v_i \ll \omega/k_\parallel \ll v_e)$, v_i and v_e being ion and electron thermal velocities. In this paper we appeal to yet another experimental observation—the universality of the phenomenon—to explain the shape of the frequency spectrum.

In Figs. 1 and 2 are shown typical power spectra, taken with probes, of the electrostatic oscillations in two entirely different devices: a hot-cathode, partially ionized reflex discharge[2] in a uniform magnetic field, and a highly ionized, ohmically heated discharge in a stellarator.[3] The spectra are strikingly similar, as are the oscilloscope traces of the probe signals themselves. We have also seen similar spectra in a lithium arc,[4] which is a fully ionized reflex discharge, and in the plasma created by a beam-plasma interaction[5] when an electron beam is shot into a neutral gas. In all these cases anomalous transport was present. The universality of the spectrum under such diverse conditions suggests that the final turbulent state is independent of the excitation mechanism, provided that there is sufficient time for the instability involved to reach its limiting amplitude.

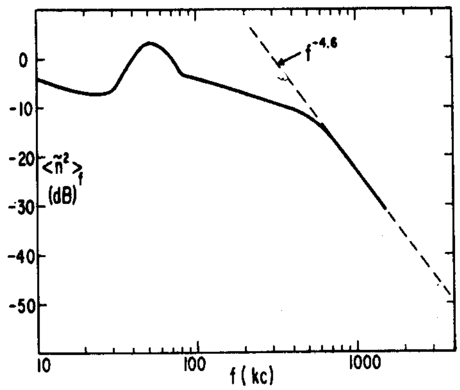

FIG. 1. Spectrum of density fluctuations observed by Chen and Bingham (reference 2) in a hot-cathode reflex arc operating in 2000 G and 5 μ of He. At 0.3 A/cm^2 the plasma density was about 10^{12} cm^{-3}.

When the plasma lies in a curved magnetic field or is rotating because of a radial electric field, drift waves are excited by the equivalent gravitational field the plasma feels. In this case the drift instability is identical to the Rayleigh-Taylor instability with finite Larmor-radius stabilization. When an electric field parallel to B can exist in virtue of finite resistivity η, electron inertia, or resonant electrons, then $k_\parallel$ can be finite; and "universal" instabilities, driven only by the plasma pressure, can also occur, while the gravitational instabilities are slowed down. Except

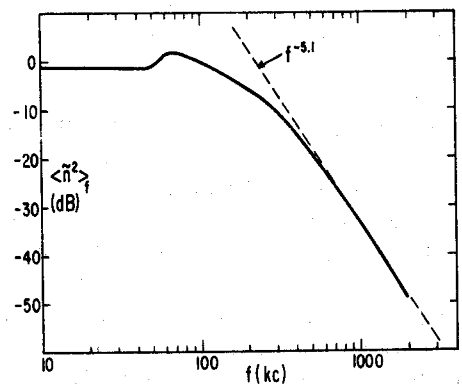

FIG. 2. Spectrum of density fluctuations published by Bol (reference 3) and taken in the Etude stellarator operating at 6700 G in He. With 440 A of ohmic heating current the plasma density was about 10^{13} cm^{-3}. The low-frequency end of such spectra generally contains peaks corresponding to the lowest azimuthal modes, and hence the shape of the curve there depends on the dimensions of the system. The high-frequency portion, however, is nearly linear on such a plot.

for resonant electron phenomena, a zero-order current parallel to B does not excite these drift waves. We do not believe resonant electrons or electron inertia to be important in the dense plasmas under consideration. In this case, it can be shown[6-8] that all these drift modes are correctly described by the simple fluid equations for $m_e/m_i \to 0, \beta \equiv 8\pi nKT/B^2 \to 0$, provided that the finite Larmor-radius terms in the ion stress tensor are retained.

For reasons we cannot present here, we believe that drift waves are the sole cause of anomalous transport in dense, low-β plasmas. At any rate, if we make this hypothesis and the further one that the turbulent state is independent of the excitation mechanism, then to describe the turbulent state we may eliminate the universal instabilities and consider only the gravitational ones. This allows us to set $k_\parallel = \eta = 0$ and consider only motions perpendicular to B. If our hypotheses are correct, the description of "Bohm diffusion" is contained in the nonlinear solution in two dimensions of the following simple set of equations:

$$m_i n\left(\frac{\partial \vec{v}_i}{\partial t}+\vec{v}_i\cdot\vec{\nabla}\vec{v}_i\right)-en(-\vec{\nabla}\varphi+\vec{v}_i\times\vec{B}) = -\vec{\nabla}\cdot \mathrm{P}_i + m_i n\vec{g} - m_i n\vec{v}_i/\tau_{i0}, \qquad (1)$$

$$en(-\vec{\nabla}\varphi+\vec{v}_e\times\vec{B}) = -KT_e\vec{\nabla}n, \qquad (2)$$

$$\frac{\partial n}{\partial t}+\vec{\nabla}\cdot(n\vec{v}_i) = \frac{\partial n}{\partial t}+\vec{\nabla}\cdot(n\vec{v}_e) = 0, \qquad (3)$$

where $\vec{g}$ is an effective gravitational force, τ_{i0} is the ion-neutral collision time, and we have assumed isothermal electrons. The right-hand side of Eq. (1) contains the parameters T_i (in P_i), the radius of curvature of B (in $\vec{g}$), and τ_{i0}. These parameters affect the growth rates but cannot greatly affect the final state because the spectrum does not seem to be sensitive to wide variations in these parameters. In what follows we shall neglect the right-hand side of Eq. (1).

In ordinary turbulence Kolmogoroff[9] predicted on dimensional grounds that the energy spectrum $E(k)$ should vary as $k^{-5/3}$ in the limit of small viscosity or large Reynolds number. We wish to apply similar arguments to plasma turbulence. With the right-hand side of Eq. (1) neglected, it can be seen that Eqs. (1)-(3) contain only four-dimensional parameters:

L522 2-3

m_i, $v_s = (KT_e/m_i)^{1/2}$, $\omega_c = eB/m_i$, and, implicitly, R, the radius of the plasma. Note that the equations are homogeneous in the density n, and therefore the absolute value of n and its dimensions are irrelevant. Let us consider the spectrum $E(k)$ of fluctuations $\tilde{\varphi}$ in plasma potential, defined as follows:

$$\langle \tilde{\varphi}^2 \rangle = \int_0^\infty E(k) dk, \tag{4}$$

where $k = |k_\perp|$ and the $\langle\ \rangle$ denotes an average over a time long compared with the oscillation periods but short compared to the plasma lifetime. Since φ has dimensions ML^2/eT^2, $E(k)$ has dimensions $(M/e)^2(L^5/T^4)$. The small-k end of the spectrum should depend on the dimensions R of the system; and the large-k end on the effective Larmor radius $a \equiv v_s/\omega_c$, which determines the shortest possible wavelengths for drift waves. If $a/R \ll 1$, one may be able to find a region of the spectrum which is independent of both R and a. This condition is analogous to the large Reynolds-number requirement in aerodynamic turbulence. Since a is the only length in the problem if k is sufficiently large, and ω_c^{-1} the only time, the dimensions of $E(k)$ require that it be of the form

$$E(k) \propto (m_i/e)^2(a^5\omega_c^4)\Phi(ka), \tag{5}$$

where Φ is a universal function of its argument. If $E(k)$ is independent of a for $a \to 0$, Φ must be of the form $\Phi \to (ka)^{-5}$. Hence in an intermediate range of k, $E(k)$ must be proportional to k^{-5}.

For experimental reasons of frequency response and output impedance, what was measured in Figs. 1 and 2 was not $\langle \tilde{\varphi}^2 \rangle$ but $\langle \tilde{n}^2 \rangle$, as given by the saturation-probe ion current. For the reason given above a dimensional analysis cannot be given for $\langle \tilde{n}^2 \rangle$, but the spectra of $\langle \tilde{n}^2 \rangle$ and $\langle \tilde{\varphi}^2 \rangle$ are very nearly the same. This is because a small but finite $k_\parallel$ existed in the experiments so that electrons flowing along B preserved the approximate relation $n \approx n_0 \exp(e\varphi/KT_e)$. Expansion of the exponential for $e\varphi < KT_e$ then proves the equivalence of the spectra. The abscissa in Figs. 1 and 2 is $\log f$ rather than $\log k$. This does not affect the slope if f and k are related by $2\pi f = kv_0$, with constant v_0. In Fig. 1, v_0 is essentially the plasma rotation velocity under a radial electric field and is constant. In Fig. 2, v_0 was measured by Bol[3] and found to be approximately constant for $f \gtrsim 300$ Kc/sec. Note that the absolute value of f depends on the observer's frame and is therefore insignificant. Within the experimental accuracy Figs. 1 and 2 show a linear portion of the spectrum at large f with a slope in good agreement with the f^{-5} dependence predicted above. Some caution must be exercised in interpreting probe data at the high-frequency end of the spectrum, not only because of the frequency response of the probe and associated circuitry, but also because the wavelength can become comparable to the probe diameter. In the data shown, these effects are believed to be negligible.

If a diffusion coefficient D independent of R can be defined, this dimensional analysis shows that it must be proportional to $v_s^2/\omega_c = KT_e/eB$, which is Bohm's formula. An interesting question which remains is whether the coefficient in Bohm's formula depends on the growth rates, and hence on the parameters on the right-hand side of Eq. (1), or whether this coefficient is also universal.

---

*Work supported by the U. S. Atomic Energy Commission.

[1]A. A. Galeev, S. S. Moiseev, and R. Z. Sagdeev, J. Nucl. Energy, Pt. C, 6, 645 (1964).

[2]F. F. Chen and R. Bingham, unpublished. In our haste to reach an understanding of "Bohm diffusion" we have put off writing up a series of experiments performed in 1960-1962. However, it is our intent eventually to publish these data.

[3]K. Bol, Phys. Fluids 7, 1855 (1964).

[4]F. Bottiglioni, M. Fumelli, and F. Prevot, in Proceedings of the Sixth International Conference on Ionization Phenomena in Gases, Paris, 1963, edited by P. Hubert (S.E.R.M.A., Paris, 1964), Vol. II, p. 427.

[5]C. Etievant, Nucl. Fusion Suppl., Pt. 3, 1025 (1962).

[6]K. V. Roberts and J. B. Taylor, Phys. Rev. Letters 8, 197 (1962).

[7]T. E. Stringer, Princeton Plasma Physics Laboratory Report No. MATT-320, 1965 (unpublished).

[8]F. F. Chen, to be published.

[9]Kolmogoroff's arguments are neatly presented by S. Chandrasekhar, J. Madras Univ. B27, 251 (1957).