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# Direct Measurement of Drift Wave Growth Rates

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Resistive drift waves in a $Q$ machine can be switched on and off by applying a potential to the aperture limiter. The exponential growth of the waves can be directly observed. The measured growth rates are slower than are predicted by the usual theories of collisional drift waves.

Because of their universal nature, drift waves are usually observable[1] only after they reach saturation amplitude; and, consequently, the theoretical linear growth rate cannot be checked directly. By measuring the change in damping rate in the stable regime, Wong and Rowberg[2] were able to infer the parametric dependences of the growth rate but could not check the magnitude of the growth rate itself. Although in principle one could switch collisional drift waves on with a sudden increase in magnetic field,[1] it is not convenient to do this because of the large inductance of the coils. We have found an easier method to switch the waves on in a time short compared with the growth time.

Figure 1 shows a schematic of the modified $Q$ machine. The vacuum chamber and the aperture limiters are insulated from the rest of the system and can be biased to an arbitrary potential $V_b$. Figure 2 shows that the plasma is quiescent for a certain range of $V_b$ and that sinusoidal drift waves set in when $V_b$ exceeds a threshold value, which varies with magnetic field $B$. To switch the waves on, one chooses a value of $V_0$ in the quiescent region and sets $V_b = V_0$ by means of a dc power supply. An increment $\Delta V$ is then suddenly added to $V_b$ by means of a square-wave current pulse applied to a 1 Ω resistor (Fig. 1). In this experiment we kept $\Delta V$ fixed and varied the distance of the unstable state from the threshold by varying either $V_0$ or $B$. In this paper we do not discuss the reason for the dependence of the oscillations on $V_b$, even though Fig. 2 shows that this dependence is not negligible. Instead, in making the theoretical calculations we ignore the effect of $V_b$, as was done in all previous work.[1,2] We then find, not surprisingly, that the calculated growth rates do not agree

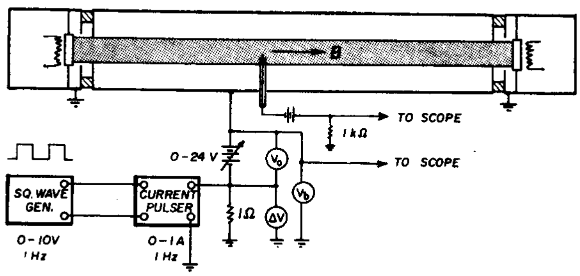


FIG. 1. Schematic of modified $Q$ machine with floating vacuum chamber and aperture limiters. The plasma column is 5 cm in diameter and 326 cm long. The uniform magnetic field varies from 1 to 4 kG, and the potassium plasma density varies from $10^{10}$ to $3 \times 10^{11}$ cm$^{-3}$.

with the measured ones. Note that the effect of $V_b$ cannot be eliminated by grounding the limiters because the latter have a work function different from that of the hot plates.

Figure 3 shows the envelope of the oscillations in ion current to a probe as the plasma is switched alternately between the stable and unstable states. The decay time of the oscillations is comparable to the rise time; we have analyzed only the rise times. Figure 4 shows expanded views of the growth of the wave (a) near threshold, when the growth is slow, and (b) farther from threshold, when the growth is faster. Figure 5 shows a case in which we were fortunate enough to catch the growth of a wave far from threshold, when the amplitude $e$-folds in a few periods.

From pictures such as Fig. 4, we can measure the $e$-folding time $\tau$ by plotting the logarithm of wave amplitude versus time. Such plots generally show a linear portion from which $\tau$ can be taken. We find that there is some scatter in the measured values of $\tau$, probably due to small fluctuations in the equilibrium state of the plasma. We have, therefore, averaged $\tau$ over 2–4 successive shots. Figure 6 shows measured values of the growth rate $\gamma \equiv \mathrm{Im}(\omega) \equiv \tau^{-1}$ as a function of $B$ with $V_0$ fixed, and as a function of $V_0$ with $B$ fixed. The error bars are an indication of the amount of shot-

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[1] H. W. Hendel, B. Coppi, F. Perkins, and P. A. Politzer, Phys. Rev. Letters 18, 439 (1967).
[2] A. Y. Wong and R. Rowberg, Phys. Rev. Letters 11, 390 (1967).

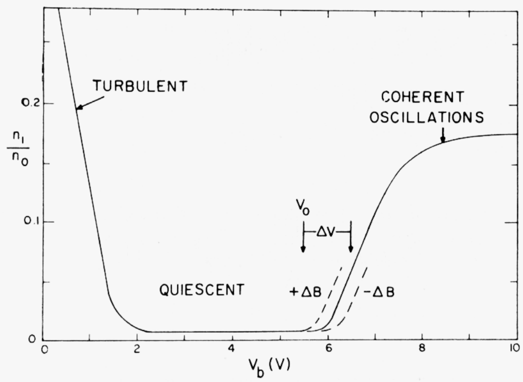
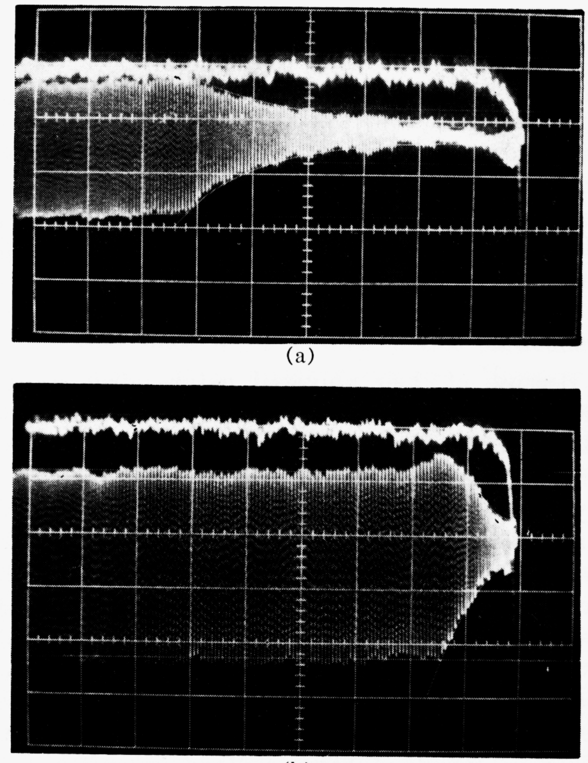



FIG. 2. Schematic of behavior of oscillation amplitude $n_1/n_0$ with chamber bias $V_b$, showing regions of turbulent fluctuations, quiescence, and coherent drift-wave oscillations. The threshold value of $V_b$ changes with magnetic field in the manner shown by the dashed curves. The absolute value of $V_b$ at threshold varies typically from 1 V to 8 V, depending on the plasma density, magnetic field, and temperature distribution of the end plates. The range of the applied jump in $V_b$ is shown.

to-shot scatter. It is seen that the growth rate varies linearly with the distance from threshold in each case.

Since it takes time for the plasma to relax to its new equilibrium state when $V_b$ is suddenly changed, there is an upper limit to the growth rates which are measurable. The average density to a probe can be used as a measure of the relaxation time of the plasma. We have found that the chamber bias $V_b$, shown on an expanded scale, is an equally good monitor; and this is shown in Figs. 3 and 4. As the plasma relaxes, the current to the limiters reaches a new equilibrium value, and this shows up on the $V_b$ trace because of the 1-Ω series resistor (Fig. 1). It is seen that the $V_b$ trace has a relaxation time of 2–4 msec, and we cannot measure values of $\tau$ smaller than this. Far from threshold, the oscillation quickly grows to a large enough amplitude to cause appreciable anomalous transport; then, the relaxation time is controlled by the oscillation itself.

In Fig. 4(b), one can see that the oscillation envelope grows symmetrically at first, but then there is a downward shift of the envelope at later times. We interpret this to be a decrease in plasma



(a)

(b)

FIG. 4. Sweep speed: 10 msec/cm. (a) $V_0 = 5.16$ V. (b) $V_0 = 5.60$ V. Other data same as in Fig. 3.

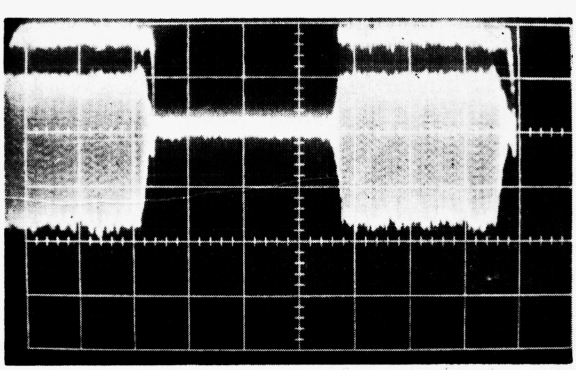


FIG. 3. Lower trace: probe ion current, 1 $\mu$A/cm. Upper trace: $V_b$, 20 mV/cm, with 1-kHz low-pass filter. Both traces dc coupled with suppressed zero. Time goes from right to left, 100 msec/cm. Probe: 0.05 mm diam, 1 mm long, shielded, 7 mm from axis. $B = 2.16$ kG, $V_0 = 5.22$ V, $\Delta V = 0.95$ V. Average ion current: 14 $\mu$A, corresponding to $n = 2.6 \times 10^{11}$ cm$^{-3}$.

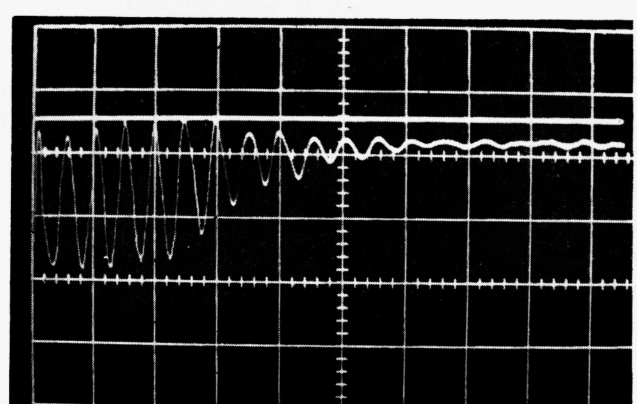


FIG. 5. Sweep speed: 2 msec/cm. $B = 1755$ G, $V_0 = 1.94$ V, $\Delta V = 0.25$ V, $n_1/n_0 = 0.25$, $n_0 \approx 10^{11}$ cm$^{-3}$. Data taken on a different run from Fig. 4; here the radial electric field had the opposite sign ($E_r < 0$) from that in Fig. 4.

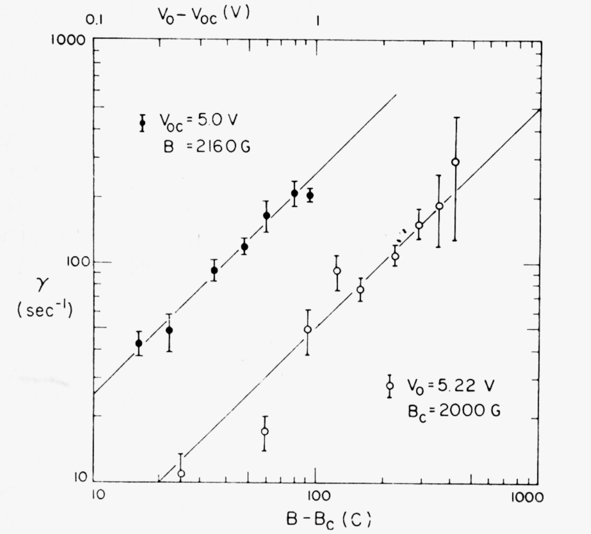



FIG. 6. Growth rate $\gamma$ as a function of $B - B_c$ with $V_0$ fixed (open points) and as a function of $V_0 - V_{0c}$ with $B$ fixed (solid points). $B_c$ and $V_{0c}$ are, respectively, the threshold values of $B$ and of $V_0$.

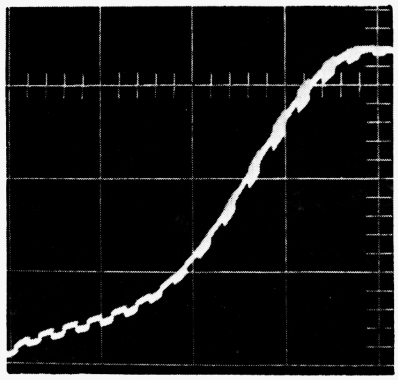


FIG. 8. Radial profile of plasma density (probe current), with the oscillations switched on and off at 1-sec intervals during the scan. Peak density (at right) was $2.9 \times 10^{11}$ cm$^{-3}$. Abscissa: 6.3 mm/div.

density at the position of the probe due to wave-induced losses. This change in density $\delta n$ is plotted as a function of $B$ for fixed $V_0$ in Fig. 7 together with the final oscillation amplitude $n_1/n_0$. It is seen that the two curves behave similarly. Note that $-\delta n$ can be negative; there is apparently a small change in the density profile when $V_b$ is changed, such that the local density at the probe is increased. In Fig. 7 we have also shown for comparison the curve for $\gamma^{1/2}$ taken from the line through the points in Fig. 6. It is seen that $\gamma^{1/2}$ is roughly proportional to $n_1/n_0$, as assumed by Chu *et al.*[3]; but the data are not good enough to distinguish between a $\gamma^{1/2}$ and a $\gamma$ dependence.

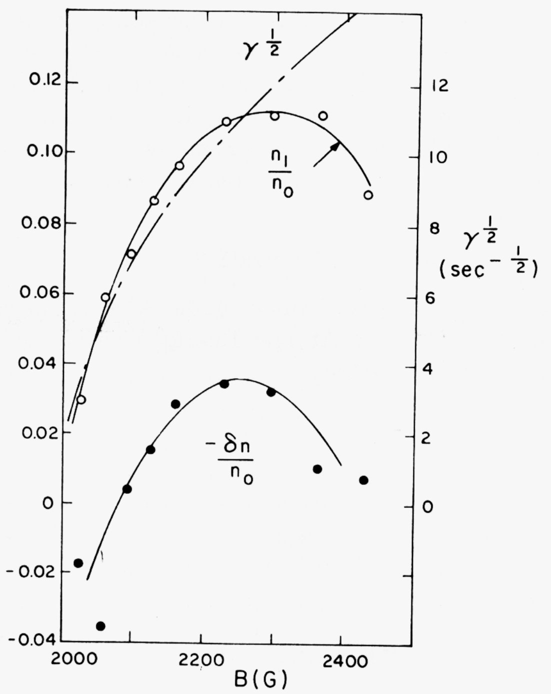



FIG. 7. The relative change in average density $\delta n/n_0$, the saturated oscillation amplitude $n_1/n_0$, and the measured growth rate $\gamma$ as a function of $B$ for $V_0 = 5.22$ V.

The effect of the drift wave on the plasma profile can be seen in Fig. 8, which shows a slow radial scan of plasma density with the wave alternately switched on and off. Near the center of the plasma, the wave broadens the trace and lowers the time-averaged density. Near the outside, the density is increased during the periods when the wave is on. This type of behavior has previously been reported by Chu *et al.*[4]

Finally, we wish to compare the absolute magnitude of $\gamma$ with theory for a typical case. The oscillation is the $m = 1$, $k_{\parallel} = \pi/L$ mode at $B = 2160$ G, $n_{\text{peak}} = 2.85 \times 10^{11}$ cm$^{-3}$ shown in Figs. 3, 4, 6, and 7. Unfortunately, no single theoretical paper is available which gives usable results including the following effects: (a) cylindrical geometry, (b) ion viscosity, (c) radial variation of eigenfunction, (d) plasma rotation in uniform electric field, and (e) nonuniform electric field. Chu *et al.*[5] have treated all but (e), but they give computations for only one case. Chen[6] has given numerical computations, but has neglected both (b) and (e). We shall use the results of Ref. 6 and

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[3] T. K. Chu, H. W. Hendel, R. W. Motley, F. Perkins, P. A. Politzer, T. H. Stix, and S. von Goeler, in *Plasma Physics and Controlled Nuclear Fusion Research* (International Atomic Energy Agency, Vienna, 1969), Vol. 1, p. 611.

[4] T. K. Chu, H. W. Hendel, and P. A. Politzer, Phys. Rev. Letters **19**, 1110 (1967).

[5] T. K. Chu, B. Coppi, H. W. Hendel, and F. W. Perkins, Phys. Fluids **12**, 203 (1969).

[6] F. F. Chen, Phys. Fluids **10**, 1647 (1967).

treat viscosity as a correction. The measured density profile is fitted to a Gaussian, and the radial electric field is assumed to be that corresponding to isothermal end plates ($s = -2$ in the notation of Ref. 6). These data and the curves of Ref. 6 then yield $\omega^* = 9.5$ kHz, $\omega - \omega_{0i} = 1.93\,\omega^*$, $\gamma = 0.22\,\omega^*$. The effect of viscosity in plane geometry has been computed by Chen.[7] Expanding Eq. (76) of Ref. 7 for large $(Y + W_1)/|\delta k|$ and neglecting $W_2$, $\rho$, and $T_i - T_e$, we obtain, in the notation of Ref. 7,

$$\mathrm{Im}\,(\psi) \approx 2\delta^2\kappa^2 Y(Y + W_1)^{-3}[Y - W_1 - (Y + W_1)^2 W_1 \delta^{-2}]. \qquad (1)$$

If the last term in [ ] is neglected,[8] the stability condition is seen to be $Y = W_1$. Apart from notation $Y = W_1$ is identical to the stability criterion found later by Hendel *et al.*[1]

We have evaluated $W_1 \equiv k_y^2 r_L^2/8\omega_{ci}\tau_{ii}$ from the experimental data and have taken the radial dependence roughly into account, as in Ref. 1, by replacing $k_y^2$ by $k_x^2 + k_y^2$, where $k_x \approx \pi/R$. Equation (1) then predicts that the viscosity-free growth rate $\mathrm{Im}\,\psi = 2\delta^2\kappa^2/Y$ should be decreased by a factor 0.46, so that $\gamma = 0.10$, $\omega^* = 950\ \mathrm{sec}^{-1}$. As seen from Fig. 6, this is a factor of 3 faster than the fastest growth rate we were able to measure. Apparently, the effect of $V_b$ is to decrease the growth rate in a manner not yet accounted for in the theory—perhaps through effect (e) listed above. The smallness of $\gamma$ is consistent with the fact that the saturation amplitude is also smaller than expected.

The effect of viscosity on the real part of the frequency is given by

$$\mathrm{Re}\,(\psi) \approx -2\delta\kappa Y/(Y + W_1). \qquad (2)$$

This gives $\omega - \omega_{0i} = 1.64\,\omega^*$, or $\omega = -3.5$ kHz for $\omega_{0i} = -2\omega^*$. We observed $f = -1.6$ kHz, or $\omega = -10$ kHz (propagation in the ion diamagnetic drift direction). The discrepancy is not surprising, since the electric-field Doppler shift is opposite to $\omega^*/k_\perp$; and $\omega$, being the difference of two large numbers, is sensitive to the viscosity correction and to the exact value of the electric field. In the experiment, $|E_r|$ was actually larger than the ideal one we assumed, and this would account for the major part of the discrepancy.

Note that since $W_1/Y$ is 0.18 in this particular case, viscosity is a minor correction. However, since $W_1/Y$ is proportional to $B^{-4}$, we can satisfy the stability criterion $W_1/Y = 1$ by rescaling $B$ downward by a factor of 1.5. This is the same factor needed by Hendel *et al.*[1] to explain their measurements. In our experiment, rescaling $B$ by 0.67 brings not only Im $(\omega)$ but also Re $(\omega)$ into much better agreement with observations. It is inconceivable, however, that $B$ is in error by as much as 1%; the collision frequency at threshold disagrees with theory not by 50% but by $(1.5)^4 \cong (0.18)^{-1} \cong 5.5$. This disagreement is not caused by effects (a)–(d) listed above: we have taken these into account. A possible cause of error is the neglect of the $W_2$ terms given in Ref. 7 but neglected in Eq. (1) and by Hendel *et al.*[1] The $W_2$ terms arise from the $\pi_{xx}$ and $\pi_{yy}$ components of the viscosity tensor and are not negligible for intermediate values of $\omega_{ci}\tau_{ii}$, such as the value 2.3 pertaining to this experiment. A second possible cause is the nonuniformity of the zero-order electric field. A third possible cause is a gradient in ion temperature due to the differential acceleration of ions in the electron sheaths at the end plates. These effects are under investigation.

## ACKNOWLEDGMENT

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[7] F. F. Chen, Phys. Fluids 8, 1323 (1965).

[8] This term leads to the short-$\lambda_\parallel$ stabilization criterion discussed by H. W. Hendel, T. K. Chu, and P. A. Politzer [Phys. Fluids **11**, 2426 (1968)].