Comments on "Generalized criterion for feasibility of controlled fusion and its application to nonideal D-D systems"

F. F. Chen, J. M. Dawson, and B. D. Fried

University of California, Los Angeles, California 90024

H. P. Furth

Plasma Physics Laboratory, Princeton University, Princeton, New Jersey 08540

M. N. Rosenbluth

Institute for Advanced Study, Princeton, New Jersey 08540

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The criticism of Lawson's $n\tau$ criterion in a recent paper is based on a misunderstanding of conventional definitions of the confinement time τ. In attempting to "generalize" the Lawson criterion, the authors introduce errors of physics that lead them to draw incorrect conclusions about the feasibility of controlled fusion and the relative advantages of various reactor concepts.

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A recent article[1] on simplified criteria for controlled fusion creates a number of false impressions regarding the physics of fusion reactors.[2]

1. *Fusion reactor criteria.* Reference 1 implies that conventional reactor studies have made improper use of Lawson's criterion[3] for energy break-even by neglecting important energy-loss processes. In fact, contemporary fusion reactor design studies are not based on simplified criteria such as Lawson's condition, but on direct computer simulations of the reacting plasma,[4] including all the relevant energy-loss processes. Simplified criteria are used only to provide a general overview.

When applying Lawson's condition on $n\tau$—i.e., on the product of plasma density n and "plasma replacement time" τ—the conventional way of taking into account energy transport terms other than radiative losses[5] is to identify Lawson's τ with the thermal *energy* replacement time τ_E of the plasma, rather than simply with the *particle* replacement time τ_p or the duration τ_r of the reaction period. In this way, all loss processes can be included realistically in a specific calculation of τ_E, without sacrificing the generality of Lawson's formulation. The case normally considered is the quasi-steady-state reactor regime, where $\tau_E \ll \tau_r$. An illustration of "modern" Lawson curves, for the case $T_e = T_i = T$, is shown in Fig. 1.

The procedure used in Ref. 1 is to apply Lawson's criterion in an alternative way: The quantity τ retains Lawson's original definition as the reaction period[6] τ_r. Various energy loss terms other than hydrogenic bremsstrahlung must then be added on explicitly—hence the "eighteen terms" alleged by Ref. 1 to be "missing from the energy balance of the Lawson criterion."

As regards the physics of the "eighteen terms," Ref. 1 introduces no new effects: The nontrivial portion of the material discussed is to be found in Ref. 7 and in more recent literature.[8,9] Several familiar topics are treated by Ref. 1 in an inappropriate manner. The most important items are as follows:

(i). *Ion "leaks" due to Coulomb scattering.* A misleading formalism is introduced here, which serves to understate scattering losses from open magnetic confinement systems such as that proposed in Ref. 10 by the authors of Ref. 1, and is irrelevant to losses from closed systems.[11] The ion loss time is defined by Ref. 1 as a single cumulative-small-angle 90°-scattering time τ_s, multiplied by a factor $1/(1-\eta_{c2})$. Values in the range 0.4—0.99 are assigned to the so-called "confinement efficiency," η_{c2}, in Tables I and II of Ref. 1, in connection with embodiments of the scheme of Ref. 10, implying that ions can be held in an open-ended system for times of $1.6\tau_s$ to $100\tau_s$. Straightforward Fokker-Planck calculations of scattering losses from such systems, however, lead to the familiar result[12] that the ion confinement time can never exceed τ_s substantially, so that, for realistic system parameters, η_{c2}

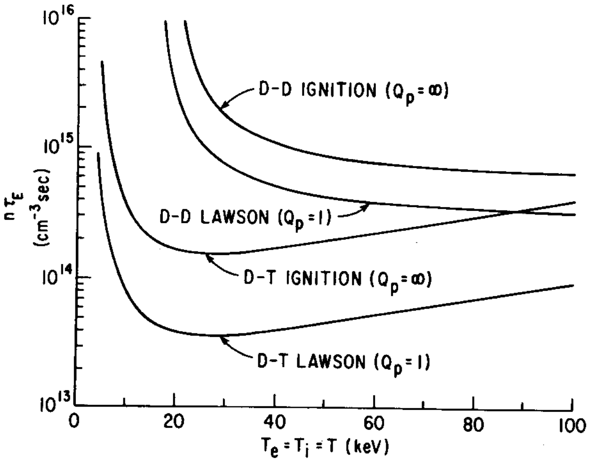

FIG. 1. Lawson and ignition curves for D-T and D-D. The power multiplication factor $Q_p = \eta \times$(fusion power/plasma input power) is calculated for $\eta = 1/3$ and includes the blanket reactions. The charged reaction products are allowed to thermalize with the plasma particles. In the D-D case, the reaction products T and ^{3}He are assumed to be recycled and burned. This figure is due to D.L. Jassby and H.H. Towner. (PPPL 763142).

will generally be quite small or negative. Moreover, actual experiments[13] resembling the scheme advocated in Ref. 10 have been found to lose their ions by collective scattering processes in times much shorter than τ_s, so that the "confinement efficiency" η_{c2} turned out to be a large negative number. The quantity η_{c2} is irrelevant to energy confinement in closed reactors[11]: The whole point in closing the magnetic field lines of a reactor configuration is to eliminate direct particle escape by scattering. Cross-field collisional diffusion is a relatively very slow process.

(ii) *Charge exchange.* Reference 1 implies that conventional thermonuclear plasmas depend on the achievement of unattainably low neutral atom densities to avoid excessive charge-exchange losses. The single-particle considerations of Ref. 1, however, are not relevant to charge-exchange losses from typical reactor plasmas, since they neglect the two dominant physical phenomena: burnout of neutral gas and reabsorption of escaping charge-exchange neutrals within the hot plasma region. (The authors note the existence of the burnout phenomenon; in Appendix F, however, they state that an ion-to-neutral ratio of $> 10^5$ is "many orders of magnitude away from the ionization that can be achieved with the means known at present." Actually, an ionization factor of 10^5 is reached even in small present-day tokamaks.[11] Properly calculated charge-exchange losses from a tokamak-type reactor are not found to be a significant element in the reactor energy balance.

(iii) *Heating.* The suggestion that electrical energy cannot be transformed nearly so efficiently into the random motion of thermal plasmas as into ordered ion motion is incorrect: This is evident from the consideration that bulk plasmas can be (and commonly are) heated by thermalization of an injected component of ordered ion motion.[14] Reactor engineering studies show that a number of alternative heating methods, such as wave heating, have a comparable potential; the low heating efficiencies cited in Ref. 1 are characteristic of small plasma research experiments, where the maximization of overall system efficiency is not an objective.

Reference 1 neglects the crucially important phenomenon of plasma heating by deposition of the energy of charged fusion reaction products (viz., in the case of D-T, the 3.5-MeV energy of the α particles). This term makes possible the achievement of extremely large energy multiplication factors in *closed* confinement systems, where the reaction products can thermalize without being lost. Fusion reactions can be maintained in steady state entirely by their own products, no external heating being required; this condition is variously called the "ignition" or "reactor equilibrium" condition (cf. Fig. 1). In an ignited plasma, the energy multiplication factor Q_p effectively becomes infinite.

(iv). *Synchrotron radiation.* The elementary single-particle formula cited in Ref. 1 is largely irrelevant to real reactor plasmas, since it neglects self-absorption and wall reflection. The correct treatment was given in 1958 by Trubnikov[15]; its practical application has been refined by more recent analyses.[16] Synchrotron radiation losses from the electrons of a typical D-T reactor in the $T_e \sim T_i \sim 10$ keV range do not constitute a significant element in the energy balance.

2. *Reactor temperature considerations.* Reference 1 suggests that the reactor ion energy range of 100 keV–10 MeV has substantial advantages. We note, first of all, that only those reactor schemes that are insensitive to Coulomb scattering losses[11] have the *option* of operating at ion temperatures below 100 keV. For reactors where the option exists, the preferred operating temperature is generally in the 10-keV range. There are two major reasons for avoiding high ion temperature.

(i). *Fusion power density.* The maximum fusion power density per unit plasma pressure is obtained for D-T reactions at 15 keV. At higher energies, and/or for reactions other than D-T, the power density decreases sharply.[8] (At 100 keV, it is down by an order of magnitude for D-T, and by two orders of magnitude for D-D.) The achievement of a local ion density concentration by means of intersecting orbits[10] is not of substantial help here, since three-dimensional small-angle Coulomb scattering imposes sharp upper limits on the steady-state density concentration factor.

(ii). *Synchrotron radiation.* For reactors operating at high ion temperature, the rate of ion energy flow to the electrons goes up in proportion to T_i. The net effect is a ratio of fusion power to single-particle synchrotron radiation power that resembles the power-to-pressure ratio in regard to T_i dependence (see above).

3. *Ordered* vs *random ion motion.* References 1 and 10 imply that fusion reactors designed for highly ordered ion motion are advantageous relative to reactors designed for more nearly random motion. In fact, because of Coulomb scattering, any confinement system that requires highly ordered motion is incapable of reaching substantial energy multiplication factors. This is true over the entire ion energy range 10 keV–10 MeV. Reactor schemes of this sort depend on high ratios of circulating power to output power, and are correspondingly vulnerable to small inefficiencies and to mild collective loss processes. In addition, the proven tendency toward instability inherent in highly ordered motion[13,17] reduces the probability that collective losses can be neglected.

4. *Advanced fuels.* Fusion power has the desirable feature that reactors of increasing environmental attractiveness should become feasible as a result of advancing technological and plasma physics expertise. The easiest plasma physics task is to breed fissile fuel with the neutrons of the D-T reaction; this solution also offers the smallest environmental improvement over a fission breeder economy—though it could offer some. The pure D-T reactor is more difficult to achieve, since it requires higher energy multiplication; however, it eliminates the generation of fission waste products. The engineering problems associated with the fast neutrons of the D-T reaction have been considered in detail and are expected to be manageable.[8] The D-D reactor eliminates the need to breed tritium in a lithium blanket and provides a modest reduction in the ratio of neutron power to total power. The plasma physics

problems are greatly increased: The D-D ignition temperature is so high as to make synchrotron radiation a severe obstacle, and the peak ratio of fusion power density to plasma pressure is only 10^{-2} times that for D-T. The D-^{3}He reactor is similar in power density and environmentally more attractive than the D-D reactor, since its power output goes predominantly into charged reaction products. One should keep in mind, however, that the fuel ^{3}He can be bred economically only from *other* fusion reactors that burn D-T or D-D. An "environmentally perfect" fusion reactor, producing only charged reaction products, could use p-^{11}B or several other reactions, but the scientific problems involved are formidable.

The intimation of Ref. 1 that "advanced fuel" reactors are near at hand, while D-T reactors suffer from relatively severe technical difficulties, is not based on serious arguments. The existence of the advanced-fuel option constitutes an important long-range asset of fusion research, and one may hope that ultimately practical reactor solutions will be found.

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[2] Further documentation of the material covered in the present comment, omitted because of space restrictions, can be found in F.F. Chen, J.M. Dawson, B.D. Fried, H.P. Furth, and M.N. Rosenbluth, Princeton Plasma Physics Laboratory Report MATT-1237, 1976 (unpublished).

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[4] See, for example, R.W. Conn and J. Kesner, Nucl. Fusion **15**, 775 (1975).

[5] See, for example, D.M. Meade, Nucl. Fusion **14**, 289 (1974).

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