SPACE CHARGE IN RADIAL ENERGY ANALYZERS

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We wish to calculate the maximum density at which a radial energy analyzer (REA) can operate without space charge problems. In its simplest form, an REA consists of a tube and a positively biased collector plate, as shown in Figure 1.

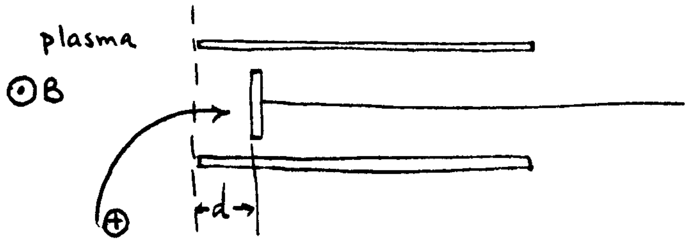

Figure 1

Electrons are scraped off by the shield tube; and fast ions enter the gap with velocity v_o, are slowed down by the retarding field, and are collected if $\frac{1}{2} Mv_o^2 > eV_c$, where V_c is the collector potential relative to the plasma.

We consider a one-dimensional gap in which ions are emitted at $x=o$, $V=0$ with velocity v_o and flux $\Gamma = n_o v_o$. Poisson's equation is

$$\frac{d^2V}{dx^2} = -4\pi \, Zen \tag{1}$$

Continuity of flux gives

$$\Gamma = n_o v_o = nv, \quad n = n_o v_o / v \tag{2}$$

Energy conservation gives

$$\frac{1}{2} M v_o^{\,2} = \frac{1}{2} M v^2 + eV(x),$$

or

$$v = (v_o^2 - 2eV/M)^{1/2} \tag{3}$$

Substituting into Eq. (1) for Z = 1 yields

$$\frac{d^2V}{dx^2} = - \frac{4\pi e n_o}{\left(1 - \frac{2eV}{MV_o^{\,2}}\right)^{1/2}}$$

In terms of the variables

$$\chi \equiv \frac{eV}{\frac{1}{2} M v_o^{\,2}} = \frac{eV}{\mathcal{E}_o} \quad \text{and} \tag{4}$$

$$\xi = \left(\frac{4\pi n_o e^2}{\frac{1}{2} M v_o^{\,2}} \right)^{1/2} x, \tag{5}$$

this becomes

$$\frac{d^2\chi}{d\xi^2} = -(1-\chi)^{-1/2} \tag{6}$$

Because the potential is retarding, the curvature of $\chi(\xi)$ is opposite to that in the Child-Langmuir problem. Figure 2 shows the behavior of $V(x)$ (or $\chi(\xi)$) as n_o is increased.

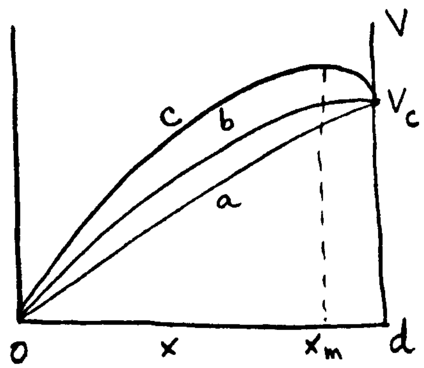

Figure 2

For small n_0, the potential has only a slight curvature (curve a). As n_0 is increased, space charge eventually causes the electric field $E = -dV/dx$ to vanish at the collector (curve b). If n_0 is further increased, a potential maximum is created in front of the collector at x_m. If $eV(x_m)$ should become as large as $\frac{1}{2} Mv_0^2$, the ions cannot reach the collector. Thus, the critical condition at which the monoenergetic ion flux is reflected is

$$d = x_m, \text{ where } V(x_m) = \mathcal{E}_0/e,$$

or $$\xi_d = \xi_m, \text{ where } \chi(\xi_m) = 1 \text{ and } \chi'(\xi_m) = 0 . \tag{7}$$

We note that the finite electric field at x=0 is usually too large to match the transverse field in the plasma. The matching is done in the scrape-off boundary layer, where electrons and low-energy ions contribute a net negative charge to decrease the slope of V(x) near x=0.

Multiplying Eq. (6) by $\chi' = d\chi/d\xi$, we obtain

$$\chi'\chi'' = \frac{1}{2}(\chi'^2)' = -(1-\chi)^{-\frac{1}{2}}\chi' \qquad (8)$$

$$\frac{1}{2}\chi'^2 = 2(1-\chi)^{1/2} + C_1 \qquad (9)$$

Condition (7) states that $\chi' = 0$ when $\chi = 1$, so that $C_1 = 0$.

Thus we have

$$\chi' = 2(1-\chi)^{1/4} \qquad (10)$$

$$(1-\chi)^{-1/4}\chi' = 2$$

$$-\frac{4}{3}(1-\chi)^{3/4} = 2\xi + C_2 \ . \qquad (11)$$

Since $\chi = 1$ at $\xi = \xi_m$, we have

$$0 = 2\xi_m + C_2, \ C_2 = -2\xi_m$$

$$(1-\chi)^{3/4} = -\frac{3}{2}(\xi - \xi_m)$$

$$(1-\chi)^{3/2} = \frac{9}{4}(\xi_m - \xi)^2 \qquad (12)$$

This gives the potential distribution $\chi(\xi)$. The critical space charge condition is given by the boundary condition $\chi(0) = 0$. This yields

$$1 = \frac{9}{4}\xi_m^2 \qquad (13)$$

From Eqs. (5) and (7) we have

$$\frac{4}{9} = \frac{4\pi n_o e^2}{\frac{1}{2}Mv_o^2} d^2 \ . \qquad (14)$$

In terms of the analyzer current density

$$J = n_o e v_o, \tag{15}$$

this becomes

$$J = \frac{M v_o^3}{18\pi e d^2} \text{ esu}, \tag{16}$$

or

$$J = \frac{\mathcal{E}_o^{3/2} (2/M)^{1/2}}{9\pi e d^2} \frac{10}{c} \quad \text{A/cm}^2$$

$$= 5.44 \times 10^{-8} E_{ev}^{3/2}/d^2 A^{1/2} \quad \text{A/cm}^2, \tag{17}$$

Where A is the atomic number of the collected ions and E_{ev} is their initial energy in eV.

In terms of the density n_o of the collected species, Eqs. (15) and (16) yield

$$n_o = \frac{M v_o^2}{18\pi e^2 d^2} = \frac{\mathcal{E}_o}{9\pi e^2 d^2} \tag{18}$$

If n_o is measured in units of 10^{10} cm^{-3}, $\mathcal{E}_o$ in keV, and d in mm, this becomes

$$\boxed{n_{10} = 2.5 \, E_{kev}/d_{mm}^2} \tag{19}$$

This is the critical density of the collected species in the plasma above which a virtual anode forms and space charge effects impede the collection of the REA.

If the velocity distribution of the collected species were known, this calculation should be redone taking the spread in velocities into account. A conservative estimate can be obtained by using for E_{kev} the minimum energy collected, namely, eV_c, and for n_0 the total density of the collected species, including those below the energy cutoff.