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The Two-Ion Hybrid Wave in an Inhomogeneous Plasma

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PREFACE

This is the fourth and last of a series of unclassified articles written for experimentalists to clarify the theoretical basis underlying the plasma separation process. The first paper, Task II-1359 (1978), showed how the plasma wave dispersion curves behaved in the region of the ion cyclotron and two-ion hybrid frequencies. The second paper, Task II-2185 (1979), concerned electrostatic drive and showed how the endplate sheath conditions affected the waves. This paper has been published in Phys. Fluids 22, 2346 (1979). The third paper, Task II-3224 (1980), concerned inductive drive and gave explicit formulas for the field enhancement effect. A short version of this paper, Task II-3552 (1981), was cleared for publication and submitted to Phys. Fluids. After a year's review and rebuttal, this paper has been rejected by three referees who cannot believe that the left-hand circularly polarized wave component can be finite inside the plasma at the ion cyclotron frequency. The manuscript has to be rewritten. The present paper deals with the question of space charge neutralization originally brought up by Ken MacKenzie. We show how standard plasma theory takes space charge effects into account, and we give explicit formulas for the frequency shifts caused by density gradients. There are no plans to publish either Part I or Part IV, which contain relatively trivial calculations.

The Two-Ion Hybrid Wave in an Inhomogeneous Plasma

Francis F. Chen

ABSTRACT

By extending the well-known theory of resistive drift waves to the case of a plasma with two ion species, we compute the frequency shift of two-ion hybrid waves caused by a density gradient. For reasonable experimental parameters, the shift is extremely small because it depends not only on the density gradient but also on the fractional concentration of the minor species and on the fractional mass difference between the two ion species. Specifically, for the two-ion hybrid resonance with $k_{||}^2/k_\perp^2 \ll 1$, $\alpha_2 = n_2/n_e \ll 1$, and $\Delta\Omega = \Omega_2 - \Omega_1 \ll \Omega$, we obtain

$$\mathrm{Re}(\Delta\omega) \simeq -\alpha_2 \frac{\omega_*}{\omega_B} \frac{\Delta\Omega}{\Omega} (\Delta\Omega - \alpha_2\Omega)$$

$$\mathrm{Im}\ (\Delta\omega) \simeq -2\alpha_2 \frac{\Delta\Omega}{\Omega} \left(\frac{k_{||}}{k_\perp}\right)^2 \omega_c \tau_{ei} \Delta\Omega,$$

where Ω and ω_c are the ion and electron cyclotron frequencies, ω_* is the eletron drift frequency, $\omega_B = k^2 D_B$ is the frequency for Bohm diffusion across a perpendicular wavelength, and τ_{ei} is resistive collision time.

We also show how drift-wave theory automatically accounts for MacKenzie space charge and how space charge neutralization is related to the turning points of the WKB problem.

I. FUNDAMENTAL EQUATIONS AND ASSUMPTIONS

Following the usual treatment of resistive drift waves,[1] we assume a slab geometry in which the equilibrium plasma density is $n_o = n_o(x)$ and consider electrostatic perturbations varying as $\exp i(\underline{k}\cdot\underline{r} - \omega t)$, with $\underline{k} = k_y\hat{y} + k_z\hat{z}$ $k_z^2 \ll k_y^2$. We treat the plasma as a fluid with $T_i = 0$ and $m/M = 0$, but with a finite resistivity η. For each species of ions, the equation of motion is then

$$Mn_i\left(\frac{\partial \underline{v}_i}{\partial t} + \underline{v}_i\cdot\nabla\underline{v}_i\right) = Z_i e n_i\left(-\nabla\phi + \underline{v}_i \times \underline{B}\right) - n_i^2 e^2 \eta\left(\underline{v}_i - \underline{v}_e\right). \tag{1}$$

For isothermal electrons, we have

$$mn_e\left(\frac{\partial \underline{v}_e}{\partial t} + \underline{v}_e\cdot\nabla\underline{v}_e\right) = -en_e\left(-\nabla\phi + \underline{v}_e \times \underline{B}\right) - KT_e\nabla n_e - n_e^2 e^2 \eta\left(\underline{v}_e - \underline{v}_i\right) \simeq 0. \tag{2}$$

The system of equations is closed by the condition of charge neutrality

$$n_e = \sum_i Z_i n_i \tag{3}$$

and the equations of continuity

$$\frac{\partial n_i}{\partial t} + \nabla\cdot(n_i\underline{v}_i) = \frac{\partial n_e}{\partial t} + \nabla\cdot(n_e\underline{v}_e) = 0. \tag{4}$$

The neglect of electron inertia in Eq. (2) is essential for keeping the treatment simple; it is a good approximation. Extension to cylindrical geometry[2] causes no problems unless the plasma rotates so fast that centrifugal and Coriolis forces become appreciable. Extension to finite T_i can also be done,[1] but the algebra becomes cumbersome because of finite Larmor radius effects. The consequences of $T_i \neq 0$ are discussed in Sec. V. Note that Eq. (1) allows the ion orbits to be large, but no spread in orbit sizes is included; furthermore, the orbits cannot span more than one wavelength because the ions are pushed only by the periodic E-field, and therefore no cyclotron harmonics are generated in this approximation.

In the equilibrium, we assume $\underline{E}_o = 0$ and $v_{oz} = 0$ and neglect the small "radial" velocity v_{ox} due to resistive diffusion. The ions then have $\underline{v}_o = 0$, and the electrons have only a diamagnetic drift

$$\underline{v}_{oe} = -\frac{KT_e}{eB}\frac{n_o'}{n_o}\hat{y} \equiv v_D\hat{y}, \tag{5}$$

where (') indicates $\partial/\partial x$ and $\underline{B}$ is a uniform field in the z direction (in esu).

II. DERIVATION OF THE DISPERSION EQUATION

Linearizing the electron equation (2) about this equilibrium and neglecting the effect resistivity $\eta_\perp$ on v_{ex} and v_{ey}, we obtain for the x, y, and z components of Eq. (2)

$$en_{oe}\phi_1' - en_{oe}v_{ey}B - en_{1e}v_D B - KT_e n_{1e}' = 0 \tag{6}$$

$$en_{oe}ik_y\phi + en_{oe}v_{ex}B - KT_e ik_y n_{1e} = 0 \tag{7}$$

$$en_{oe}ik_z\phi - KT_e ik_z n_{1e} - n_{oe}^2 e^2 \eta_{||} v_{ez} = 0. \tag{8}$$

The ion parallel velocity v_{iz} has been neglected; this is important only when k_z^2 is so large that ω/k_z becomes comparable to the ion acoustic speed. We now define the variables

$$\nu \equiv n_{1e}(x)/n_{oe}(x), \qquad \chi \equiv e\phi_1/KT_e \tag{9}$$

and use the relations

$$\omega_c = eB/m, \qquad \eta = m/n_o e^2 \tau_{ei}, \qquad D_B = KT_e/eB. \tag{10}$$

Eqs. (6)-(8) respectively yield

$$v_{ey} = D_B(\chi - \nu)', \quad v_{ex} = -ik_y D_B(\chi - \nu), \quad v_{ez} = ik_z D_B \omega_c \tau_{ei}(\chi - \nu). \tag{11}$$

The linearized electron continuity equation is

$$-i\omega n_{e1} + n_{oe}(v_{ex}' + ik_y v_{ey} + ik_z v_{ez}) + v_{ex}n_o' + v_{eo}ik_y n_{1e} = 0. \tag{12}$$

Substituting Eq. (11) into Eq. (12), we find that the x-derivatives cancel out (a consequence of $m \to 0$) and that ν and χ are algebraically related by the simple expression

$$\nu = \chi \, \frac{\omega_* + ib\sigma_{||}}{\omega + ib\sigma_{||}}, \tag{13}$$

where

$$\omega_* \equiv k_y v_D \tag{14}$$

$$b \equiv k_y^2 a_i^2 \tag{15}$$

$$a_i^2 \equiv KT_e/M\Omega_c^2 = D_B/\Omega_c \tag{16}$$

and

$$\sigma_{||} \equiv (k_z/k_y)^2 \omega_c \tau_{ei} \Omega_c. \tag{17}$$

Note that $\nu = \chi$ is just the Boltzmann relation $n_e = n_o \exp(e\phi/KT_e)$ in linearized form; Eq. (13) expresses the deviation from Boltzmann when $\omega \neq \omega_*$ and $\sigma_{||} \neq \infty$. The quantity a_i is the ion Larmor radius evaluated with the electron temperature; and b is like the usual FLR parameter, but for cold ions. The quantity $b\sigma_{||}$ can be expressed as

$$b\sigma_{||} = k_z^2 \,(KT_e/m\nu_{ei}) = k_z^2\, D_{||e}, \tag{18}$$

which shows that $(b\sigma_{||})^{-1}$ is the time for electrons to diffuse a parallel wavelength against fixed ions. The ion mass does not appear in Eqs. (18) and (13); it appears only in the standard definitions (15)-(17), which will be needed later.

We next linearize the ion equation of motion (1) with the simplifications $\underline{v}_{oi} = 0$, $\eta_\perp = 0$, $v_{iz} = 0$. Solving for v_{ix} and v_{iy} in the usual fashion, we obtain for each ion species

$$\begin{aligned} iv_{xj} &= \left(\frac{k_y\phi}{B} - \frac{\omega}{\Omega_{cj}}\frac{\phi'}{B}\right)\left(1 - \frac{\omega^2}{\Omega_{cj}^2}\right)^{-1} \\ v_{yj} &= \left(\frac{\phi'}{B} - \frac{\omega}{\Omega_{cj}}\frac{k_y\phi}{B}\right)\left(1 - \frac{\omega^2}{\Omega_{cj}^2}\right)^{-1}, \end{aligned} \tag{19}$$

where

$$\Omega_{cj} \equiv Z_j eB/M_j \,. \tag{20}$$

The linearized ion continuity equation is in this case

$$-i\omega n_j + n_{oj}(v'_{xj} + ik_y v_{yj}) + v_{xj} n'_{oj} = 0. \tag{21}$$

Eqs. (19) and (21) yield

$$\frac{n_j}{n_{oj}} = \left[\phi'' + \frac{n'_{oj}}{n_{oj}}\phi' - \left(k_y^2 + k_y\frac{n'_{oj}}{n_{oj}}\frac{\Omega_{cj}}{\omega}\right)\phi\right](\Omega_{cj}B)^{-1}\left(1 - \frac{\omega^2}{\Omega_{cj}^2}\right)^{-1}. \tag{22}$$

We now specialize to the case of two ion species j = 1,2. Let

$$\alpha_j \equiv Z_j n_{oj}/n_{oe}, \text{ so that } \alpha_1 + \alpha_2 = 1, \tag{23}$$

and let

$$\delta(x) \equiv n'_{o1}/n_{o1} = n'_{o2}/n_{o2}. \tag{24}$$

The quasineutrality condition, Eq. (3), now reads

$$\nu = \sum_j \frac{\alpha_j}{B\Omega_{cj}}\left(1 - \frac{\omega^2}{\Omega_{cj}^2}\right)^{-1}\left[\phi'' + \delta\phi' - \left(k_y^2 + k_y\delta\frac{\Omega_{cj}}{\omega}\right)\phi\right]. \tag{25}$$

Upon normalizing ϕ to χ, the coefficient B^{-1} becomes D_B [Eq. (10)]. The rest of the coefficient has the dimensions of time and may be defined as

$$\tau_j \equiv \frac{\alpha_j}{\Omega_{cj}} \left(1 - \frac{\omega^2}{\Omega_{cj}^2} \right)^{-1} . \tag{26}$$

Finally, using the electron equation (13) to eliminate ν, we obtain

$$(\tau_1+\tau_2)(\chi''+\delta\chi') - \left[\tau_1\left(k^2+\delta\Omega_{c1}\frac{k}{\omega}\right)+\tau_2\left(k^2+\delta\Omega_{c2}\frac{k}{\omega}\right)+\frac{1}{D_B}\frac{\omega_*+ib\sigma_{\parallel}}{\omega+ib\sigma_{\parallel}}\right]\chi = 0. \tag{27}$$

This is the dispersion equation for drift-type waves in a two-ion-species plasma with a density gradient $\delta(x)$. For simplicity we have dropped the subscript on k_y, since k_y and total k are essentially the same.

III. RECOVERY OF FAMILIAR RESULTS

1. Resistive drift wave. Let $\alpha_1 = 1$, $\alpha_2 = 0$, and $\omega^2 \ll \Omega_c^2$, so that $\tau = \Omega_c^{-1}$. The "local" dispersion relation is obtained by setting $\chi'' = \chi' = 0$. Eq. (27) then becomes

$$D_B\frac{k^2}{\Omega_c} + D_B\frac{k\delta}{\omega} + \frac{\omega_* + ib\sigma_{||}}{\omega + ib\sigma_{||}} = 0. \tag{28}$$

From Eq. (16), the first term is b; and from Eqs. (14) and (5), the second term is $-\omega_*/\omega$. Clearing the denominators, we obtain

$$b\omega(\omega+ib\sigma_{\parallel}) - \omega_*(\omega+ib\sigma_{\parallel}) + \omega(\omega_*+ib\sigma_{\parallel}) = 0, \tag{29}$$

$$\omega^2 + ib\sigma_{\parallel}\omega + i\sigma_{\parallel}(\omega-\omega_*) = 0. \tag{30}$$

One root has Im $(\omega) \simeq -i\sigma_{||}$ and is heavily damped. The unstable root can be found most easily in the limit $\sigma_{||} \gg |\omega_*|$, so that $\omega - \omega_*$ must be small. Solving for $\omega - \omega_*$ and approximating ω by ω_* on the r.h.s., we obtain the usual expression for asymptotic growth rate of the resistive drift wave:

$$\omega - \omega_* \simeq -b\omega_* + i\omega_*^2/\sigma_{||}. \tag{31}$$

2. Electrostatic ion cyclotron wave. Let $\alpha_1 = 1$, $\alpha_2 = 0$ and make the local approximation $\chi'' = \chi' = 0$. Further, let k_z^2 be large enough that the electrons follow the Boltzmann relation, so that the fraction in the last term of

Eq. (27) is unity ($b\sigma_{||} \rightarrow \infty$). Eq. (27) then becomes, for $\omega \simeq \Omega_c$,

$$\frac{D_B}{\Omega_c}\left(1-\frac{\omega^2}{\Omega_c^2}\right)^{-1}\left(k^2+k\delta\frac{\Omega_c}{\omega}\right)+1=0, \tag{32}$$

$$\frac{KT_e}{M}\left(k^2+k\delta\frac{\Omega_c}{\omega}\right)=\omega^2-\Omega_c^2,$$

$$\omega^2 \simeq \Omega_c^2+k^2\frac{KT_e}{M}\left(1+\frac{\delta}{k}\right). \tag{33}$$

This is the usual formula for the ESIC wave except for the addition of the δ/k term for an inhomogeneous plasma. This correction term comes from the charge separation caused by the $E_y \times B$ drift in the x direction along $\underline{\nabla} n_o$.

3. Drift-cyclotron instability. We now make the same approximations as in the previous case for the e.s.i.c. wave, but we allow $b\sigma_{||}$ to be finite. Eq. (33) then becomes (for $\omega \simeq \Omega_c$)

$$\omega^2=\Omega_c^2+k^2c_s^2\left(1+\frac{\delta}{k}\right)\frac{\Omega_c+ib\sigma_{11}}{\omega_*+ib\sigma_{11}}. \tag{34}$$

We may drop the δ/k term, which gives a numerical correction of no interest to us. We might expect drift excitation of the cyclotron wave when there is a matching of the frequencies ω_* and Ω_c. It is clear that the threshold occurs at $\omega_* = \Omega_c$, when ω^2 is real. Rationalizing the denominator of Eq. (34), we obtain

$$\omega^2=\Omega_c^2+k^2c_s^2\,\frac{\Omega_c\omega_*+b^2\sigma_{11}^2+ib\sigma_{11}(\omega_*-\Omega_c)}{\omega_*^2+b^2\sigma_{11}^2}. \tag{35}$$

It is clear that there is instability only if $\omega_* > \Omega_c$. From Eqs. (5), (14), and (16), we see that this implies

$$-\,kD_B\delta/\Omega_c > 1, \text{ or } |ka_i||\delta a_i| > 1. \tag{36}$$

The Larmor radius has to be large with respect to both the wavelength and the density scale length, and this formulation cannot be expected to give accurate results for the drift-cyclotron instability.

4. Two-ion hybrid resonance. The standard two-ion hybrid resonance frequency ω_r can be recovered from Eq. (27) by assuming uniform n_o ($\delta = \omega_* = 0$),

perpendicular propagation ($k_z = \sigma_{||} = 0$), and an infinite plasma ($\chi'' = \chi' = 0$). Eqs. (26) and (27) then give $\tau_1 + \tau_2 = 0$, or

$$\frac{\alpha_1}{\Omega_1}\left(1 - \frac{\omega^2}{\Omega_2^2}\right) + \frac{\alpha_2}{\Omega_2}\left(1 - \frac{\omega^2}{\Omega_1^2}\right) = 0, \tag{35}$$

where the subscript c in Ω_{cj} has been suppressed. Thus

$$\omega^2\left(\frac{\alpha_1}{\Omega_1\Omega_2^2} + \frac{\alpha_2}{\Omega_2\Omega_1^2}\right) = \frac{\alpha_1}{\Omega_1} + \frac{\alpha_2}{\Omega_2},$$

$$\omega^2 = \Omega_1\Omega_2\,\frac{\alpha_1\Omega_2 + \alpha_2\Omega_1}{\alpha_1\Omega_1 + \alpha_2\Omega_2} \equiv \omega_r. \tag{36}$$

IV. EFFECT OF THE DENSITY GRADIENT

We now make the local approximation on Eq. (27) to evaluate the effect of the density inhomogeneity on the modes in a two-ion-species plasma. Defining

$$\varepsilon_1 = (\delta/k)\ (\Omega_1/\omega) \text{ and } \varepsilon_2 = (\delta/k)\ (\Omega_2/\omega), \tag{37}$$

we can write Eq. (27) as

$$\tau_1(1+\epsilon_1) + \tau_2(1+\epsilon_2) + \frac{1}{k^2 D_B}\,\frac{\omega_* + ib\sigma_{11}}{\omega + ib\sigma_{11}} = 0. \tag{38}$$

Inserting the definition of τ_j from Eq. (26), we have

$$\frac{\omega_* + ib\sigma_{11}}{\omega + ib\sigma_{11}}(\omega^2-\Omega_1^2)(\omega^2-\Omega_2^2) = k^2 D_B\left[\alpha_1\Omega_1(1+\epsilon_1)(\omega^2-\Omega_2^2) + \right.$$
$$\left. + \alpha_2\Omega_2(1+\epsilon_2)(\omega^2-\Omega_1^2)\right]. \tag{39}$$

Note that $D_B\Omega_c = KT_e/M = c_s^2$, so that we can define

$$\omega_{sj} = kc_{sj} \tag{40}$$

and write Eq. (39) in the alternate form

$$\frac{\omega_* + ib\sigma_{11}}{\omega + ib\sigma_{11}}(\omega^2-\Omega_1^2)(\omega^2-\Omega_2^2) = \alpha_1\omega_{s1}^2(1+\epsilon_1)(\omega^2-\Omega_2^2) + \alpha_2\omega_{s2}^2(1+\epsilon_2)(\omega^2-\Omega_1^2). \tag{41}$$

It is not possible to find a simple expression for the frequency shift for arbitrary values of $b\sigma_{||}$ covering the transition from the two-ion hybrid to the two-ion electrostatic ion cyclotron waves. We must treat the small $b\sigma_{||}$ and large $b\sigma_{||}$ cases separately.

1. Damping of the two-ion hybrid. We first take the case $b\sigma_{||} \lesssim O(\omega_*) \ll \omega_r \simeq \Omega_c$ to examine the effect of ∇n_o on the imaginary part of ω. Let

$$\omega = \omega_r + \Delta\omega, \qquad \Delta\omega \ll \omega_r, \qquad \omega^2 \simeq \omega_r^2 + 2\omega_r\Delta\omega, \tag{42}$$

where the homogeneous two-ion hybrid frequency ω_r is given by Eqs. (35) and (36), which we repeat here for convenience:

$$\omega_r^2 - \Omega_2^2 = -\frac{\alpha_2\Omega_2}{\alpha_1\Omega_1}\left(\omega_r^2 - \Omega_1^2\right) \tag{43}$$

$$\omega_r^2 = \Omega_1\Omega_2\frac{\alpha_1\Omega_2 + \alpha_2\Omega_1}{\alpha_1\Omega_1 + \alpha_2\Omega_2}. \tag{44}$$

Since $|\omega_*| \ll \omega_r$ and $b\sigma_{||}$ is also small, the l.h.s. of Eq. (41) is small, and we can replace ω by ω_r there. We also set $\omega = \omega_r$ in ε_1 and ε_2 [Eq. (37)]. Eq. (39) then becomes

$$\frac{\omega_* + ib\sigma_{||}}{\omega + ib\sigma_{||}}(\omega_r^2 - \Omega_1^2)(\omega_r^2 - \Omega_2^2) = k^2D_B\left[\alpha_1\Omega_1(\omega_r^2 - \Omega_2^2) + \alpha_2\Omega_2(\omega_r^2 - \Omega_1^2) + \right.$$

$$\left. + 2\omega_r\Delta\omega(\alpha_1\Omega_1 + \alpha_2\Omega_2) + \alpha_1\Omega_1\epsilon_1(\omega_r^2 - \Omega_2^2) + \alpha_2\Omega_2\epsilon_2(\omega_r^2 - \Omega_1^2)\right] + O(\epsilon\Delta\omega). \tag{45}$$

The first two terms on the l.h.s. cancel by virtue of Eq. (43). Substituting for $\omega_r^2 - \Omega_2^2$ from Eq. (43) in the remaining terms, we obtain

$$2\omega_r(\alpha_1\Omega_1 + \alpha_2\Omega_2)\Delta\omega = \alpha_2\Omega_2(\omega_r^2 - \Omega_1^2)(\epsilon_1 - \epsilon_2) - \frac{\alpha_2\Omega_2}{\alpha_1\Omega_1}\frac{(\omega_r^2 - \Omega_1^2)^2}{k^2D_B}\frac{\omega_* + ib\sigma_{||}}{\omega + ib\sigma_{||}}. \tag{46}$$

Separating into real and imaginary parts and noting that $|\omega_*| \simeq b\sigma_{||} \ll \omega_r$, we have

$$\frac{\omega_* + ib\sigma_{||}}{\omega + ib\sigma_{||}} \simeq \frac{\omega_*}{\omega_r} + i\frac{b\sigma_{||}}{\omega_r}\frac{\omega_r - \omega_*}{\omega_r}. \tag{47}$$

The frequency shift $\Delta\omega$ is then approxmately

$$\Delta\omega = \frac{\alpha_2\Omega_2(\omega_r^2-\Omega_1^2)(\epsilon_1-\epsilon_2) - \frac{\alpha_2\Omega_2}{\alpha_1\Omega_1}\frac{(\omega_r^2-\Omega_1^2)^2}{k^2D_B}\frac{\omega_*}{\omega_r} - \frac{\alpha_2\Omega_2}{\alpha_1\Omega_1}\frac{(\omega_r^2-\Omega_1^2)^2}{k^2D_B}\frac{ib\sigma_{11}}{\omega_r}}{2\omega_r(\alpha_1\Omega_1+\alpha_2\Omega_2)} . \qquad (48)$$

The damping rate is given by

$$-\mathrm{Im}(\omega) = b\sigma_{11}\frac{\alpha_2\Omega_2}{\alpha_1\Omega_1}\frac{(\omega_r^2-\Omega_1^2)^2}{2k^2D_B\omega_r^2}\frac{1}{\alpha_1\Omega_1+\alpha_2\Omega_2} = \frac{b\sigma_{11}}{2k^2D_B}\frac{\alpha_2}{\alpha_1}\frac{(\omega_r^2-\Omega_1^2)^2}{\Omega_1^2(\alpha_1\Omega_2+\alpha_2\Omega_1)} , \qquad (49)$$

where we have used Eq. (44).

From Eq. (18), we see that

$$\frac{b\sigma_{11}}{k^2D_B} = \frac{k_z^2KT_e}{k^2m\nu_{ei}}\frac{eB}{KT_e} = \frac{k_z^2}{k^2}\omega_c\tau_{ei} , \qquad (50)$$

so that

$$-\mathrm{Im}(\omega) = \frac{1}{2}\frac{\alpha_2}{\alpha_1}\frac{k_z^2}{k^2}\frac{(\omega_r^2-\Omega_1^2)^2}{\Omega_1^2(\alpha_1\Omega_2+\alpha_2\Omega_1)}\omega_c\tau_{ei} . \qquad (51)$$

In particular, for a small concentration of the species 2, so that $\alpha_2 << \alpha_1$, and $\omega_r \approx \Omega_2$, we have

$$-\mathrm{Im}(\omega) \simeq \frac{\alpha_2}{2}\frac{k_z^2}{k^2}\frac{(\Omega_2^2-\Omega_1^2)^2}{\Omega_2\Omega_1^2}\omega_c\tau_{ei} . \qquad (52)$$

The drift terms do not appear here, since resistive damping dominates over drift-wave growth. This is not surprising, since we found in the treatment of the drift-cyclotron wave that instability required $\omega_* > \omega$. We see the dependence on $\omega - \omega_*$ again in Eq. (47). There is a simple physical reason for $\mathrm{Im}\,(\omega) \propto \omega_* - \omega$. In the physical picture of how a drift wave is destabilized[1], a phase shift between the density oscillation ν and the potential oscillation χ must exist such that ν leads χ. Let the phase of χ be zero so that $\chi = \bar{\chi}\exp i(ky-\omega t)$

and $\nu = \bar{\nu} \exp i(ky - \omega t + \delta)$. For ν to lead χ, we must have $\delta < 0$, or $\mathrm{Im}\,(\nu/\chi) < 0$. The modified Boltzmann relation, Eq. (13), and Eq. (47) then tell us that $\omega_* > \omega_r$ is required for instability. When ν lags χ, as occurs for $|\omega_*| < \omega_r \simeq \Omega_c$, the drift terms are stabilizing.

2. Numerical estimates. Consider a cylindrical plasma column with radius $a = 10$ cm and length $L = 100$ cm, immersed in a 20 kG magnetic field. The plasma has $n_e = 10^{12}$ cm^{-3}, $T_e = 2$ eV, $T_i \simeq 0$. Let the local disperison relation be evaluated at $r = a/2$, where the density scale length is $\Lambda = r = 5$ cm, and let the mode have $\lambda_{||} = 2L = 200$ cm and $\lambda_\perp = 3.9$ cm, corresponding to azimuthal mode number $m = 8$. The two ion species have $M_1 = 238\ M_H$ and $M_2 = 235\ M_H$. The plasma parameters are then as follows:

$$k = m/r = 1.6\ \mathrm{cm}^{-1},\quad k_z = 2\pi/\lambda_{||} = 3.142 \times 10^{-2}\ \mathrm{cm}^{-1}$$

$$\delta = -1/\Lambda = -\ 0.2\ \mathrm{cm}^{-1},\quad \delta/k = -\ 0.125$$

$$c_s = (KT_e/M_1)^{1/2} = 8.97 \times 10^4\ \mathrm{cm/sec}$$

$$a_i = c_s/\Omega_c = 0.111\ \mathrm{cm}$$

$$b = k^2 a_i^2 = 3.18 \times 10^{-2}$$

$$v_{th} = (KT_e/m)^{1/2} = 5.93 \times 10^7\ \mathrm{cm/sec}$$

$$\omega_c = 3.516 \times 10^{11}\ \mathrm{sec}^{-1}$$

$$\nu_{ei} = 1.5 \times 10^{-6}\ Zn \ln \Lambda / T_{eV}^{3/2} = 5.3 \times 10^6\ \mathrm{sec}^{-1}$$

$$\omega_c \tau_{ei} = 6.6 \times 10^4$$

$$D_B = 10^8\ T_{eV}/B_G = 10^4\ \mathrm{cm}^2/\mathrm{sec}$$

The relevant frequencies are then ordered as follows (in rad/sec):

$$\omega_* = -\ k\delta D_B = 3.2 \times 10^3$$

$$k^2 D_B = 2.56 \times 10^4$$

$$kc_s = 1.436 \times 10^5$$

$$b\sigma_{||} = k_z^2\ v_{th}^2 \tau_{ei} = 6.55 \times 10^5$$

$$\Omega_1 = 8.051 \times 10^5,\quad \Omega_2 = 8.154 \times 10^5$$

$$\nu_{ei} = 5.3 \times 10^6$$

$$\omega_c = 3.5 \times 10^{11}$$

Frequency ratios are:

$$(\Omega_2 - \Omega_1) = \Delta\Omega/\Omega_1 = 1.28 \times 10^{-2}$$

$$\Delta \equiv (\Omega_2^2 - \Omega_1^2)/\Omega_1^2 = 2.575 \times 10^{-2}$$

$$\omega_*/\Omega_c = 4.0 \times 10^{-3}$$

$$\omega_*/b\sigma_{||} = 4.9 \times 10^{-3}$$

$$b\sigma_{||}/\Omega_c = 0.81$$

Using these numbers in the damping rate of Eq. (52) for $\alpha_2 << \alpha_1$, we find

$$-\frac{\mathrm{Im}(\omega)}{\omega_r} \simeq \frac{\alpha_2}{2} \frac{k_z^2}{k^2} \left(\frac{\Omega_1}{\Omega_2}\right)^2 \Delta^2 \omega_c \tau_{ei} = 8.2 \times 10^{-3} \alpha_2 , \tag{53}$$

which is entirely negligible.

3. <u>Frequency shift of the two-ion hybrid</u>. The real part of Eq. (48) gives for the frequency shift

$$\Delta\omega = \frac{\alpha_2\Omega_2(\omega_r^2-\Omega_1^2)(\epsilon_1-\epsilon_2) - \frac{\alpha_2\Omega_2}{\alpha_1\Omega_1}\frac{\omega_*}{\omega_r}\frac{(\omega_r^2-\Omega_1^2)^2}{k^2 D_B}}{2\omega_r(\alpha_1\Omega_1+\alpha_2\Omega_2)}$$

$$= \frac{\alpha_2\Omega_2(\omega_r^2-\Omega_1^2)}{2\omega_r(\alpha_1\Omega_1+\alpha_2\Omega_2)}\left[\frac{\sigma}{k}\frac{\Omega_1-\Omega_2}{\omega_r} - \frac{\omega_r^2-\Omega_1^2}{\alpha_1\Omega_1 k^2 D_B}\frac{\omega_*}{\omega_r}\right]$$

$$= \frac{\alpha_2\Omega_2(\omega_r^2-\Omega_1^2)}{2\omega_r^2(\alpha_1\Omega_1+\alpha_2\Omega_2)}\frac{\delta}{k}\left[\Omega_1-\Omega_2+\frac{\omega_r^2-\Omega_1^2}{\alpha_1\Omega_1}\right], \tag{54}$$

where we have used $\omega_* = -k\delta D_B$. Replacing ω_r^2 in the denominator by Eq. (144), we obtain

$$\Delta\omega = \frac{1}{2}\frac{\alpha_2}{\alpha_1}\frac{\delta}{k}\frac{\omega_r^2-\Omega_1^2}{\Omega_1^2}\frac{\alpha_1\Omega_1(\Omega_1-\Omega_2)+\omega_r^2-\Omega_1^2}{\alpha_1\Omega_2+\alpha_2\Omega_1}$$

$$\Delta\omega = \frac{1}{2}\frac{\alpha_2}{\alpha_1}\frac{\delta}{k}\frac{\omega_r^2-\Omega_1^2}{\Omega_1^2}\;\frac{\omega_r^2-\alpha_1\Omega_1\Omega_2-\Omega_1^2(1-\alpha_1)}{\alpha_1\Omega_2-\alpha_2\Omega_1} . \tag{55}$$

Specializing to the case $\alpha_2 \ll \alpha_1$, we may replace α_1 by 1 and ω_r by Ω_2, obtaining

$$\Delta\omega = \frac{\alpha_2}{2}\frac{\delta}{k}\frac{\Delta}{\Omega_2}(\,\Omega_2^2 - \Omega_1\Omega_2 - \alpha_2\Omega_1^2\,). \tag{56}$$

If Δ is also small, let $\Delta\Omega = \Omega_2 - \Omega_1$ and $\Delta \simeq 2\Delta\Omega/\Omega$. We then have

$$\Delta\omega \simeq \alpha_2\frac{\delta}{k}\Delta\Omega(\,\frac{\Delta\Omega}{\Omega} - \alpha_2). \tag{57}$$

The sign of the shift depends on the relative size of α_2 and $\Delta\Omega/\Omega$. For the parameters of Sec. IV-2 and $\alpha_2 < 1\%$, we have

$$\frac{\Delta\omega}{\Delta\Omega} \simeq -1.6 \times 10^{-3}\,\alpha_2, \tag{58}$$

which is extremely small. However, for $\alpha_2 \gg .01$, we have

$$\frac{\Delta\omega}{\Delta\Omega} \simeq \frac{\alpha_2^2}{8}, \tag{59}$$

which may be appreciable. Our choices of $\Lambda = a/2$ and $m = 8$ were quite arbitrary. For $m = 1$ or 2 and Λ smaller than $a/2$, $\Delta\omega$ can be an order of magnitude larger than in Eq. (59).

Note that $\Delta\omega$ is independent of KT_e, and its sign is not necessarily related to that of ω_*. The two terms in Eq. (57) do not have simple physical interpretations, since they are combinations of other terms. In Eq. (54), however, one can trace the first term $(\Omega_1 - \Omega_2)$ back to the charge separation due to the difference of ion E_y/B drifts along $\underline{\nabla} n_o$, and the last term to the electron E_y/B drift along $\underline{\nabla} n_o$. To lowest order in ω/Ω_c, the electron and ion E_y/B drifts of course are equal, so that there is no net charge separation. When there are two ion species, the electron charge cannot cancel the ion charge for all ion mixtures; this is probably why the second term in Eq. (57) depends on α_2.

4. <u>Frequency shift of the ESIC waves</u>. The regime of electrostatic ion cyclotron waves is obtained in the limit $b\sigma_{||} \gg \omega \gg \omega_*$, when the Boltzmann relation holds and

$$\frac{\omega_* + ib\sigma_{||}}{\omega + ib\sigma_{||}} \simeq 1. \tag{60}$$

The dispersion relation Eq. (41) is then approximately

$$(\omega^2-\Omega_1^2)(\omega^2-\Omega_2^2)=\alpha_1\omega_{s1}^2(1+\epsilon_1)(\omega^2-\Omega_2^2)+\alpha_2\omega_{s2}^2(1+\epsilon_2)(\omega^2-\Omega_1^2). \tag{61}$$

Let ω_o be the solution of the homogeneous-plasma equation

$$\frac{\alpha_1\omega_{s1}^2}{\omega_o^2-\Omega_1^2}+\frac{\alpha_2\omega_{s2}^2}{\omega_o^2-\Omega_2^2}=1. \tag{62}$$

We replace ω by ω_o in the small terms containing ϵ_j and assume a small shift $\Delta\omega$ such that

$$\omega=\omega_o+\Delta\omega \qquad \omega^2=\omega_o^2+2\omega_o\Delta\omega. \tag{63}$$

The first-order part of Eq. (61) then reads

$$2\omega_o\Delta\omega(\omega_o^2-\Omega_1^2+\omega_o^2-\Omega_2^2)=2\omega_o\Delta\omega(\alpha_1\omega_{s1}^2+\alpha_2\omega_{s2}^2)+ \\ +\alpha_1\omega_{s1}^2\epsilon_1(\omega_o^2-\Omega_2^2)+\alpha_2\omega_{s2}^2\epsilon_2(\omega_o^2-\Omega_1^2), \tag{64}$$

or

$$\Delta\omega=\frac{1}{2\omega_o}\;\frac{\alpha_1\omega_{s1}^2\epsilon_1(\omega_o^2-\Omega_2^2)+\alpha_2\omega_{s2}^2\epsilon_2(\omega^2-\Omega_1^2)}{2\omega_o^2-\Omega_1^2-\Omega_2^2-\alpha_1\omega_{s1}^2-\alpha_2\omega_{s2}^2}. \tag{65}$$

Call the denominator D, and add and subtract a term in the numerator:

$$\Delta\omega=\frac{(\omega_o^2-\Omega_1^2)(\omega_o^2-\Omega_2^2)}{2\omega_o D}\left[\frac{\alpha_1\omega_{s1}^2}{\omega_o^2-\Omega_1^2}\epsilon_1+\frac{\alpha_2\omega_{s2}^2}{\omega_o^2-\Omega_2^2}\epsilon_2+\frac{\alpha_2\omega_{s2}^2}{\omega_o^2-\Omega_2^2}\epsilon_1-\frac{\alpha_2\omega_{s2}^2}{\omega_o^2-\Omega_2^2}\epsilon_1\right].$$

The first and third terms cancel by virtue of Eq. (62). Thus,

$$\Delta\omega=\frac{\alpha_2\omega_{s2}^2}{2\omega_o D}(\omega^2-\Omega_1^2)(\epsilon_2-\epsilon_1). \tag{66}$$

Since $\epsilon_2-\epsilon_1=(\delta/k)(\Delta\Omega/\omega_o)$ and $\omega_{s2}^2=k^2c_{s2}^2=k^2D_B\Omega_2=-(k/\delta)\omega_*\Omega_2$, we have the frequency shift

$$\Delta\omega=-\frac{\alpha_2}{2}\,\frac{\omega_*\Delta\Omega\,\Omega_2}{\omega_o^2}\;\frac{\omega_o^2-\Omega_1^2}{[\omega_o^2-(\Omega_1^2+\alpha_1\omega_{s1}^2)]+[\omega_o^2-(\Omega_2^2+\alpha_2\omega_{s2}^2)]}. \tag{67}$$

For the particular case $\alpha_2 << \alpha_1$, the root of Eq. (62) that is close to Ω_2 is given by

$$\omega_0^2 - \Omega_2^2 \simeq \alpha_2 \omega_{s2}^2 \left(1 - \frac{\omega_{s1}^2}{\Omega_2^2 - \Omega_1^2}\right)^{-1} . \tag{68}$$

The last fraction is equal to $k^2 c_{s1}^2/\Omega_1^2\Delta = k^2 D_B/\Omega_1\Delta \approx 1.2$ for the numerical parameters of Sec. IV-2. Unless there is a fortuitous coincidence between the acoustic shift ω_{s1}^2 and the frequency difference $\Omega_2^2 - \Omega_1^2$, the denominator of Eq. (68) is of order unity, and ω_o differs from Ω_2 only by a term of order α_2. With $\omega_o \approx \Omega_2$, Eq. (67) becomes

$$\Delta\omega \simeq -\frac{\alpha_2}{2}\frac{\omega_*}{\Omega_2}\Delta\Omega \; \frac{\Omega_2^2 - \Omega_1^2}{\Omega_2^2 - \Omega_1^2 - \omega_{s1}^2}$$

$$\simeq -\frac{\alpha_2}{2}\frac{\omega_*}{\Omega_2}\Delta\Omega \left(1 - \frac{k^2 D_B}{\Omega_1 \Delta}\right)^{-1} . \tag{69}$$

Again, there is a possible resonant denominator, and the general case requires a more accurate approximation for ω_o. However, we have taken the m = 8 mode; for smaller m-numbers, the $k^2 D_B/\Omega_1\Delta$ term would be negligibly small. We then have the simple formula

$$\Delta\omega \simeq -\frac{\alpha_2}{2}\frac{\omega_*}{\Omega}\Delta\Omega . \tag{70}$$

V. EFFECTS WE HAVE NEGLECTED

1. Radial eigenfunctions. In making the "local" approximation, we dropped the terms $(\tau_1 + \tau_2)$ $(\chi'' + \delta\chi')$ in Eq. (27). These terms are related to the radial electric field E_r which must exist in order for ϕ to satisfy a boundary condition. The $\delta\chi'$ term represents the ion space charge due to polarization drift along $\underline{\nabla} n_o$. The χ'' term is due to the nonuniformity of this drift if E_r varies radially; this effect would exist in a finite cylinder even if $\underline{\nabla} n_o = 0$. If there is a strong shear in equilibrium quantities such as B_o, E_o, or n_o', the modes would be localized between closely spaced turning points, and the radial wave equation must be solved carefully. However, in a non-sheared system, the radial (x) gradients are generally smaller than the azimuthal (y) gradients unless m is as small as 1 or 2. In any case, since the frequency shifts are so small, it is not worthwhile to evaluate them more accurately by removing the local approximation.

2. Finite ion temperature. In fluid theory, finite T_i has two main

effects: there is a Doppler shift due to the ion diamagnetic drift, and there is a charge separation because the ions E x B drift more slowly than electrons, due to the ions' finite Larmor radius. The latter effect can be taken into account by adding the viscosity term $\underline{\nabla} \cdot \underline{\underline{\pi}}$ to the equation of motion.[1] The effect is to increase the pressure-driven effects proportional to ω_* by a factor $(1 + T_i/T_e)$. Collisional viscosity can also be included in $\underline{\underline{\pi}}$. The algebra becomes much more cumbersome, even with the local approximation, and no new effects or order-of-magnitude changes are expected. In kinetic theory, finite T_i introduces Ω_c harmonics, and there will be new physical effects.

3. Finite radial electric Field E_o. If E_o is uniform, there is a simple Doppler shift of the frequency, unless E_o is so large that the centrifugal force[2] in a cylindrical system causes a Rayleigh-Taylor instability. If E_o is nonuniform, the azimuthal $\underline{E} \times \underline{B}$ drift is sheared, and there are Kelvin-Helmholtz instabilities that can be excited. The treatment of K-H instabilities in an inhomogeneous plasma, including finite Larmor radius effects, has been treated in only a few papers.[3,4] The analysis is necessarily complicated, because no local approximation can be made; the K-H instability depends on the radial gradients of first-order quantities. If nonuniform E-fields are present, it would not pay to attempt an analytic treatment.

VI. HOW DRIFT WAVE THEORY ACCOUNTS FOR MACKENZIE SPACE CHARGE

Some years ago, Ken MacKenzie suggested that ion oscillations near Ω_c could never become large in an inhomogeneous plasma, because there is no adequate mechanism for neutralizing the space charge as ions gyrated into regions of different density. In the $k_{||} = 0$ case, neutralizing electron currents must come from a modulation of the endplate currents, and these are limited in magnitude to the ion losses at the acoustic velocity. By exciting finite-$k_{||}$ modes, one is able to provide neutralizing currents by allowing electrons to flow along B between wave crests and troughs within the plasma. Actually, there are many other mechanisms for generating neutralizing currents, and drift wave theory takes all these into account, though generally only to order $k_\perp^2 r_L^2$. In particular the ions themselves can effect charge neutralization by changing the wave amplitude profile. We shall show how this mechanism is related to the location of the turning points in the WKB problem.

With the help of Eqs. (5), (14), (16), and (24), the radial wave equation (27) for a single-ion-species plasma with $T_i = 0$ can be written as follows:

$$\chi'' + \left(\frac{1}{r} + \frac{n_o'}{n_o}\right)\chi' - \frac{1}{a_i^2}\left[\frac{m^2}{r^2}a_i^2 - \frac{\omega_*}{\omega} + \left(1 - \frac{\omega^2}{\Omega_c^2}\right)\left(\frac{\omega_* + ib\sigma_{||}}{\omega + ib\sigma_{||}}\right)\right]\chi = 0 \qquad (71)$$

Here, we have converted from plane geometry to cylindrical geometry, and the prime indicates $\partial/\partial r$. Eq. (71) is an equation of continuity, and each term can be traced back to its origin in the equations of motion to find the physical effect that caused a separation of charge. The last term is easily recognized as coming from the modified electron Boltzmann equation (13), the $b\sigma_{||}$ part representing electron diffusion along B, and the ω_* part representing $\underline{E}_\theta \times \underline{B}$ drift along $\underline{\nabla}n_o$. The factor $(1 - \omega^2/\Omega_c^2)$ causes the electron contribution to space charge to be negligible when $\omega \simeq \Omega_c$, because then all the other terms (which are due to ion space charge) become large. The $-\omega_*/\omega$ term is the ion accumulation due to $\underline{E}_\theta \times \underline{B}$ drift along $\underline{\nabla}n_o$. Since this is equal to the electron $\underline{E} \times \underline{B}$ drift at low frequencies, it normally cancels the ω_*/ω part of the last term when $k_{||} = 0$. The $(m/r)^2 a_i^2$ term is from the polarization drift of the ions along E_θ. There is a divergence of ions from azimuthal locations where electrons accumulate because of $E_\theta \times B$ drift; hence, the polarization term adds to the electron space charge.

The remaining terms in Eq. (71) depend on the radial variation of the potential χ and are neglected in the local approximation. The $(n_o'/n_o)\chi'$ term is due to the E_r-driven polarization drift alone $\underline{\nabla}n_o$, and the $\chi'' + \chi'/r$ terms come from the radial inhomogeneity of this drift, which causes ion space charge to pile up. The $n_o'\chi'$ term is the lowest-order MacKenzie space charge effect. It causes ions to "stick out" radially past the electrons when the ions are pulled out by the radial electric field.

If electron currents are insufficient to cancel ion space charge when χ is constant with radius, then a radial variation in χ develops so that the E_r-driven ion currents can provide neutralization. The more severe the charge imbalance, the faster χ must vary with radius. This implies either a high-order radial mode or closely spaced turning points.

To see how space charge imbalance is removed by adjustment of the turning points, let us examine the particularly simple case of constant ω_*. The radial density profile is then a Gaussian:

$$n_o = n_{oo} e^{-r^2/a^2} . \qquad (72)$$

We further consider low frequencies such that $\omega^2 << \Omega_c^2$, and k_z^2 large enough that $b\sigma_{||} >> \omega, \omega_*$. Eq. (71) then simplifies to

$$\chi'' + \left(\frac{1}{r}\frac{n_o'}{n_o}\right)\chi' - \frac{1}{a_i^2}\left(a_i^2\frac{m^2}{r^2} - \frac{\omega_*}{\omega} + 1\right)\chi = 0. \tag{73}$$

In terms of the dimensionless variables

$$s = r/a, \qquad \lambda = a_i/a, \tag{74}$$

this can be written

$$\lambda^2\left(\chi_{ss} + (\frac{1}{s} - 2s)\chi_s\right) - \left(1 - \frac{\omega_*}{\omega} + \lambda^2\frac{m^2}{s^2}\right)\chi = 0. \tag{75}$$

The first derivative χ_s can be transformed away with the substitution

$$W(s) = \sqrt{s}\, e^{-s^2/2}\,\chi, \tag{76}$$

giving the WKB equation

$$\frac{d^2W}{ds^2} + \left(2 - s^2 + \frac{\frac{1}{4} - m^2}{s^2} - \frac{1}{\lambda^2}(1 - \frac{\omega_*}{\omega})\right) W = 0. \tag{77}$$

The turning points are determined by

$$Q(s_o) = \frac{\frac{1}{4} - m^2}{s_o^2} + 2 - \frac{1}{\lambda^2}(1 - \frac{\omega_*}{\omega}) - s_o^2 = 0. \tag{78}$$

Since $\omega_* - \omega = O(\lambda^2\omega_*)$, let

$$\omega_* - \omega = 2\lambda^2\omega_* f, \tag{79}$$

where $f = O(1)$. Then Eq. (78) becomes, approximately,

$$s_o^4 - 2(1 + f)s_o^2 + m^2 - \frac{1}{4} = 0. \tag{80}$$

This quadratic is easily solved to give the turning point locations s_o as a function of frequency shift f and azimuthal mode number m, as shown in Fig. 1.

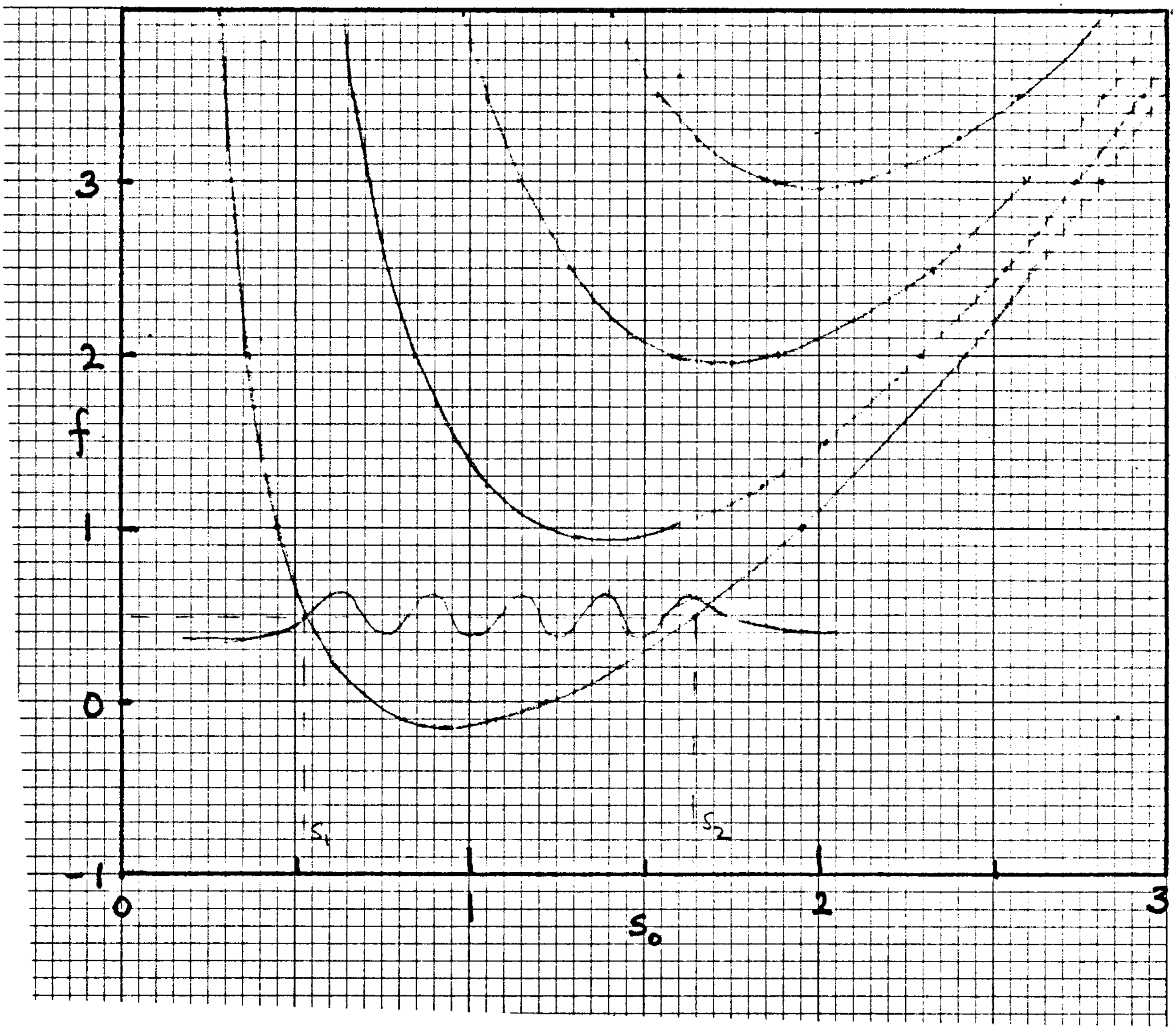

FIG. 1

For a given value of m and f, there are two turning points, s_1 and s_2, between which W(s) oscillates, and beyond which W(s) exponentially decays. [Inclusion of the $b\sigma_{||}$ terms in Eq. (71) would give rise to complex turning points, which simply means that the interior solution is not a pure sine wave and the exterior solution not a pure exponential.] As f is varied, the ratios among the various polarization and $\underline{E} \times \underline{B}$ drifts change, and a charge imbalance can occur. Moving the turning points changes the drifts dependent on χ' and χ'' and therefore adjusts the charge accumulations due to these drifts. However, charge neutrality cannot be achieved for all values of f, since the WKB quantization condition must also be satisfied:

$$\int_{s_2}^{s_1} Q^{\frac{1}{2}}(s)\, ds = (n + \tfrac{1}{2})\pi. \tag{81}$$

This gives the eigenvalues of f (or ω) for which charge balance can occur everywhere. These are entirely analogous to energy levels in a potential well, but here the levels are set by space charge. There is usually one value of f for each set of mode numbers (m,n). For other frequencies, the radial E-field cannot be adjusted so as to cancel all space charges arising from unequal ion and electron drifts.

In practice, the second turning points in Fig. 1 would lie well outside the plasma. The radial eigenmode is in this case determined by the boundary condition at the wall plus the inner turning point. When ω_* is not constant, the shear in $\omega_*(s)$ would bring the turning points closer together. In that case, s_2 can lie inside the plasma, and the radial mode profile would be insensitive to the wall boundary condition. In tokamaks the shear in $\underline{B}$ produces a large shear in $k_{||}$ so that $b\sigma_{||}$ is a function of s_ℓ, and it is this effect that localizes the drift wave. Thus we see that standard drift wave theory takes MacKenzie space charge into account, though the way it is done is often obscured by the mathematical details. When the plasma has a sharp boundary, the wave function contains many radial harmonics, and analytic solutions are not useful because the short radial wavelengths involve very large finite Larmor radius corrections. The plasma column then behaves like a crankshaft, and this has been described in computer models.

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