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Editors: M.Q. Tran, M.L. Sawley - CRPP-EPFL, 21, av. des Bains
CH-1007 Lausanne / Switzerland

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ECOLE POLYTECHNIQUE FEDERALE DE LAUSANNE

WHY IS THE TWO-ION HYBRID RESONANCE OBSERVABLE?

Francis F. Chen

University of California, Los Angeles, California 90024

Abstract: The two-ion hybrid frequency ω_r can be seen in a plasma of finite length even though the finite-$k_{||}$ frequency shift should be large. A small amount of damping requires $\omega \approx \omega_r$ in order to satisfy sheath matching.

1. Use of the Two-Ion Hybrid as a Diagnostic

In a plasma with two ion species, electrostatic oscillations with $k_z = 0$ (that is, with phase velocity perpendicular to the magnetic field $B_0\hat{z}$) have the characteristic frequency ω_r given by[1]

$$\omega_r^2 = \Omega_1\Omega_2 \frac{\alpha_1\Omega_2 + \alpha_2\Omega_1}{\alpha_1\Omega_1 + \alpha_2\Omega_2}, \tag{1}$$

where $\Omega_{1,2}$ are the ion gyrofrequencies of species 1 and 2, and $\alpha_{1,2}$ are their weighted relative concentrations. Let species 2 be the "minor" or impurity species, so that $\alpha_2 << \alpha_1$; then Eq. (1) shows that $\omega_r = \Omega_2 + \Delta\omega$, and the frequency shift $\Delta\omega$ is a sensitive measure of the impurity concentration α_2. This is easily obtained by applying a frequency ω to the plasma and observing the peak in response at $\omega = \omega_r$. For instance, the contamination of H_2^+ ions in an H^+ plasma or of Ar^{++} ions in an Ar^+ plasma is often of interest. In the latter case, where $M_1 = M_2$ but $Z_1 \neq Z_2$, Eq. (1) reduces to

$$\omega_r^2 = \Omega_2^2 \left(1 + \frac{n_2}{n_1}\right)/\left(1 + \frac{Z_2^2 n_2}{Z_1^2 n_1}\right), \tag{2}$$

so that $\Delta\omega$ is sensitive to the density ratio n_2/n_1 because of the weighting factor $Z_2^2/Z_1^2 = 4$.

Dimonte et al.[2] have reported an experiment in an Ar-Xe plasma in which ω_r was observed to obey Eq. (1) accurately. However, the plasma column was limited in length by conducting endplates, and the value of k_z was necessarily finite. Any conventional treatment of the boundary conditions would lead to a value of $\Delta\omega$ larger than observed and well outside the experimental error. The unexpected validity of Eq. (1) proved to a difficult problem, which we have now solved.

2. The Electromagnetic Dispersion Relation

The resonance frequency ω_r is shifted by a small but finite value of k_z because rapid electron motion along $\underline{B}_0$ destroys the ion charge balance conditions leading to Eq. (1). The effect of finite k_z can be calculated by careful treatment of the electron dynamics, including inertia, collisions, Landau damping, and self-inductance. Consider waves in an infinite, uniform plasma with $T_i = 0$ and $k_x = 0$, so that $k^2 = k_y^2 + k_z^2$. From Maxwell's equations without displacement current, we obtain a wave equation whose components are

$$k^2E_x = (4\pi i\omega/c^2)j_x \tag{3}$$

$$k_z^2E_y - k_yk_zE_z = (4\pi i\omega/c^2)j_y \tag{4}$$

$$k_y^2E_z - k_zk_yE_y = (4\pi i\omega/c^2)j_z. \tag{5}$$

The perpendicular currents j_x and j_y are easily found from the electron $\underline{E}\times\underline{B}$ drifts and the cold-ion velocities. Eqs. (3) and (4) can then be written

$$(f + \kappa^2)E_x + igE_y = 0 \tag{6}$$

$$-igE_x + (f + \kappa_z^2)E_y - \kappa_y\kappa_zE_z = 0 \tag{7}$$

where

$$f(\Omega) = \frac{\alpha_1}{1 - (R\Omega)^{-2}} + \frac{\alpha_2 R}{1 - \Omega^{-2}} \tag{8}$$

$$g(\Omega) = R\Omega\left(\frac{\alpha_1}{R^2\Omega^2 - 1} + \frac{\alpha_2}{\Omega^2 - 1} + 1\right) \tag{9}$$

$$\alpha_j \equiv Z_jn_{oj}/n_{oe} \tag{10}$$

$$R \equiv \Omega_2/\Omega_1, \quad \Omega \equiv \omega/\Omega_2 \tag{11}$$

and

$$\kappa \equiv kL, \quad L^2 \equiv M_1c^2/4\pi Z_1n_{oe}e^2 = c^2/\Omega_{pi}^2. \tag{12}$$

To evaluate j_z we neglect the ion v_z and solve the Vlasov equation with a Krook collision term to obtain the electron v_z:

$$\frac{\partial f_1}{\partial t} + v_z\frac{\partial f_1}{\partial z} - \frac{e}{m}E_z\frac{\partial f_o}{\partial v_z} = \nu_e\left(\frac{n_1}{n_o}f_o - f_1\right). \tag{13}$$

Using the electron continuity equation for n_1, we obtain

$$\frac{4\pi i\omega}{c^2}j_z = -\frac{\omega_p^2}{c^2}Z'(\zeta)\frac{\frac{\omega}{k_zv_{th}}\zeta E_z + \frac{\nu_e}{2\omega_c}\frac{k_y}{k_z}E_x}{1 - \frac{i\nu_e}{2\omega}Z'(\zeta)}, \tag{14}$$

where $Z'(\zeta)$ is the derivative of the plasma dispersion function[3], and

$$\zeta \equiv \frac{\omega + i\nu_e}{k_zv_{th}}, \qquad v_{th}^2 \equiv 2kT_e/m. \tag{15}$$

The term containing ν_e/ω_c in Eq. (14) can safely be neglected, whereupon Eq. (5) can be written

$$-\kappa_y\kappa_zE_y + (\kappa_y^2 + P)E_z = 0, \tag{16}$$

where

$$P \equiv \frac{M_1}{Z_1m}\frac{\zeta Re(\zeta)Z'(\zeta)}{1 - \frac{i}{2}\frac{\nu_e}{\omega}Z'(\zeta)} \tag{17}$$

The determinantal condition for the compatibility of Eqs. (6), (7), and (16) yields the dispersion relation

$$f(\Omega) = -\frac{P\kappa^2\kappa_z^2 + (f^2 - g^2)(P + \kappa_y^2)}{P(\kappa^2 + \kappa_z^2) + \kappa^2\kappa_y^2}. \tag{18}$$

For a cold, collisionless plasma, we may let $P \to \infty$ in Eq. (18) to obtain $(f + \kappa^2)(f + \kappa_z^2) = g^2$, which yields the usual electromagnetic ion cyclotron wave[4] when $\alpha_2 = 0$, $R = 1$. The electrostatic limit of Eq. (18) can be obtained by letting $c \to \infty$, so that $L, \kappa, \kappa_z \to \infty$:

$$f = -(k_z^2/k_y^2)P. \tag{19}$$

This is the two-ion hybrid dispersion relation with finite k_z. When $k_z = 0$, this becomes $f(\Omega) = 0$, which is identical to Eq. (1).

3. Sheath Boundary Conditions

Let the wave be excited by applying a voltage $V_p = V_m \cos(\omega t - k_yy)$ to a split endplate at $z = 0$, while the other endplate at $z = L$ is grounded. If we neglect the sheaths and naively assume that $\lambda_z = 4L$, $k_z = \pi/2L$, then Eq. (19) predicts an extremely large shift $\Delta\omega$ (dot-dash curve in Fig. 1). However, we have previously shown[5] that an ion sheath is an insulator, so that the density n_s and potential ϕ_s just outside the sheath on a conducting plate can fluctuate as long as the plasma can supply the additional electron current that penetrates the sheath Coulomb barrier because of the fluctuation in n_s or ϕ_s. The current j_z in the plasma depends on k_z, so that k_z is determined by a matching condition, which turns out to take the form

$$\frac{\partial\psi}{\partial z} + a\left(\frac{\partial}{\partial t} + \nu_e\right)\psi = 0 \qquad \text{at } z = L$$

$$\frac{\partial\psi}{\partial z} - a\left(\frac{\partial}{\partial t} + \nu_e\right)(\psi + \chi_p) = 0 \qquad \text{at } z = 0, \tag{20}$$

where $\psi = n_1/n_o - e\phi_1/KT_e$, $\chi_p = eV_p/KT_e$, and $a = \frac{1}{4}(KT_e/M)^{\frac{1}{2}}/(KT_e/m)$. Applying Eq. (20) to waves with real k and complex ω, we previously found the effective k_z to be[5] $k_z \approx (4/L)(a\nu_eL/2\pi)^{\frac{1}{4}}$, about an order of magnitude less than the naive estimate. Even this value, however, gives too large a frequency shift, as shown by the dashed curve in Fig. 1; the electromagnetic correction reduces the discrepancy by only 20%.

We have reexamined the problem recognizing that we have real ω and complex k here, so that it is appropriate to assume a perturbation ψ of the form $\psi = A(t) + B(t)e^{-\alpha z}\cos\beta z$, where $k_z = \beta + i\alpha$. The term $A(t)$ is needed to represent a uniform potential fluctuation of the whole plasma, which of course, is possible at any frequency. Since the excitation is of the form $\cos\phi$, where $\phi = \omega t - k_y y$, $A(t)$ and $B(t)$ will be of the form

$$A(t) = a_1\cos\phi + a_2\sin\phi, \quad B(t) = b_1\cos\phi + b_2\sin\phi. \quad (21)$$

Inserting this into Eqs. (20) and equating the coefficients of the $\sin\phi$ and $\cos\phi$ terms, we obtain four equations for the four coefficients a_1, a_2, b_1, and b_2, which are then expressed in terms of α, β, and ω; that is, in terms of ω and k_z (for fixed k_y). Since ω is also related to k_z by the dispersion relation (18), once ω is chosen one can compute a_1, a_2, b_1, and b_2 and hence $|\psi|^2$. The maximum in $|\psi|^2$ as ω is varied indicates the frequency that should be observed. The computations are simple enough to be done on a programmable hand calculator. For a wide range of parameters the eigenfrequency lies extremely close to the classical two-ion hybrid frequency ω_r. Apparently, the physical reason is that the damping constant $\alpha = \mathrm{Im}(k_z)$ is appreciable even for fairly weak collisions, and this gradient in $|\psi|^2$ causes an electron flow which the sheaths cannot accommodate. The only way to reduce the flow is for ω to be so close to ω_r that the space charge of one ion species is cancelled by that of the other ion species without the necessity for electron parallel conduction.

4. Comparison with Experiment

The quoted experiment[2] was done in an argon discharge with a controlled xenon impurity, whose fractional density α_2 was measured by the xenon partial pressure. The plasma parameters were $n_o = 2.2 \times 10^{11}\mathrm{cm}^{-3}$, $B_o = 10.23$ kG, $T_e = 3$ eV, $T_i = 0.3$ eV, $\nu_{ei} = 1.5 \times 10^6\ \mathrm{sec}^{-1}$, and $\nu_{eo} = 0.4 \times 10^6\mathrm{sec}^{-1}$. The 9-cm diam. plasma was bounded by endplates 150 cm apart, one of which was split to excite an m=2 mode with $k_y = 0.44\ \mathrm{cm}^{-1}$. The two estimates of k_z as described above were 10^{-2} and $9 \times 10^{-4}\mathrm{cm}^{-1}$. Since $|\zeta|$ was or order 20, P is accurately given by the asymptotic form of Eq. (17): $P = (M_1/Z_1 m)(1 + i\nu_e/\omega)^{-1}$. The resulting eigenfrequency is indistinguishable from ω_r, shown by the solid line in Fig. 1.

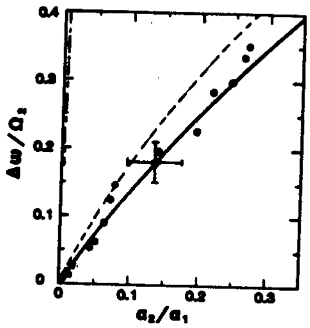

Fig. 1. Frequency shift $\Delta\omega/\Omega_2$ vs. relative xenon density α_2/α (points), compared with the two-ion resonance frequency ω_r (——), and with ω_r corrected for finite k_z effects according to previous calculations (-.-) and (---).

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