ELECTRON RUNAWAY IN RF DISCHARGES

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ABSTRACT

The critical electric field is computed as a function of pressure and starting energy for electrons to run away to high energies in moderate pressure discharges. The runaway conditions depend critically on the shape of the elastic cross section vs. energy curve. Computations are made for H, H₂, and He gases, and it is shown that runaway occurs much more readily in atomic hydrogen than in the other gases. The values of the runaway fields are larger than would normally occur in dc discharges, where large voltages would lead to arc formation. However, in rf discharges such electric fields can be sustained over times long compared to electron transit times but short compared to ion transit times.

I. INTRODUCTION

Recent experiments by Sakawa and Shoji (1) have confirmed the report by Schneider and Handel (2) of the striking appearance of pairs of bright rings in hydrogen and deuterium rf discharges at pressures of 0.1 to 30 torr. The radiofrequency power of several hundred watts at around 1 MHz is capacitively coupled to the plasma by two external ring electrodes about 10 cm apart. The spacing between ring pairs decreases, and hence the number of pairs increases, as the pressure is increased. Except for the sharpness of the rings, the phenomenon is reminiscent of stationary and moving striations in glow discharges. In rf discharges, the striations must of course be stationary, since there is no preferred direction.

To emit brightly, an atom in one of the rings must be excited by at least a small population of electrons with energies above about 10 eV. On the other hand, a thermal electron at, say, 2 eV will gain only 2-4 eV under typical pressure and electric field conditions before making an elastic collision with a neutral atom and losing its forward momentum. A few electrons in the tail of the distribution can, however, undergo the runaway process, well known in toroidal fusion devices, in which the electron gains so much energy between collisions that it enters a region of negligibly small cross section.

At the location of the striations or rings, the plasma potential usually shows some structure, such as a dip. These modulations are usually only a few volts in magnitude and are probably the result, not the cause, of the primary excitation process. Our working model is that a group of runaway electrons

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gains energy until it exceeds the excitation and ionization potential. There the runaways suffer inelastic collisions and lose most of their energy. The resulting increase in electron density, together with increased ionization, causes a potential dip. The fast electrons are then re-accelerated, starting from a higher than thermal energy. The fact that the rings are double is probably that there are two sets of rings, one from each electrode, and that these are excited in alternate half-cycles of the rf. It would be quite fortuitous for the two sets to coincide, since the electrode spacing is arbitrary.

In making this short study, we had an ulterior motive: to understand the ionization mechanism in noble gas discharges. It is well known that argon is more easily ionized than other gases, even neon, xenon, and krypton. These gases have an deep Ramsauer minimum in their cross sections, and it is possible that the runaway process is sensitive to the exact shape of these curves.

II. ATOMIC HYDROGEN

To follow an electron as it gains energy from the electric field and loses energy in neutral collisions, one must have an accurate, smooth curve for the elastic collision cross section as a function of energy. Unfortunately, cross section data are sketchy, differing from theory and also between authors. For hydrogen, we used the comparatively recent data by Neynaber et al. (3). These data are compared with old data from Brown's book (4) in Fig. 1. Though the two references agree in absolute magnitude, the new data show that the cross section continues to increase at low energies. It is this behavior that makes hydrogen different from the other gases. The physical reason for this increase is the dipole interaction between the spins of the electrons and the protons and the effect of electron exchange between the free and bound electrons (5). The old data missed this increase probably because of poor angular resolution. From the standpoint of electron runaway, however, it is not clear what angular range should be used. In fact, an accurate calculation should integrate over the differential cross section, if such data can be trusted.

The high-energy part of the Neynaber curve can be fitted well by an exponential of the form $A \exp(-BW)$, and Fig. 2 shows the six points we used to fix the constants A and B. Since the data extended only to 12 eV, and no theoretical expression exists for the behavior at higher energies W (5), we were forced to continue the exponential to 100 eV and higher. These points could also be fitted by a polynomial, but extrapolating the polynomial to higher energies gave unreasonable results. To fit the lower energy points, we added to the same exponential a fourth-order polynomial. The resulting fit is shown in Fig. 3.

Using the analytic fit to the cross section data, we next computed the mean free path for electron-neutral collisions as a function of electron energy and neutral pressure:

$$\lambda_m = (n_0 \sigma^-)^{-1}$$  \hspace{1cm} (1)
The result is shown in Fig. 4. The same curves are shown in Fig. 5 together with lines representing the energy gained per mean free path for various values of the electric field $E$:

$$\Delta W = E\lambda_m.$$ (2)

Fig. 5 shows the possibility of runaway. Consider an electron starting at 7 eV, for instance, in a 200 V/cm E-field in 1 torr of H. From the solid squares, one sees that the mean free path is slightly above 0.4 mm. Scanning horizontally, one sees that the 0.4 mm line intersects the 200 V/cm line at 8 eV. Thus, 7-eV electrons will have, on the average, 15 eV by the next collision. Since a 15-eV electron will have a mean free path of 1 mm and gain 20 eV before the next collision, the energy will continue to increase. Thus, there is a possibility of runaway whenever the E-field line lies to the right of the appropriate mfp line. Because of the curvature of the mfp lines, it is possible for the E-field line to lie at the right at some energies and to the left at others. This creates the possibility of oscillations in energy, in which an electron, after gaining energy, finds itself in a region of larger cross section, and so loses more energy than it gained. It then finds itself in a region of smaller cross section and so stands to gain more energy again. These oscillations will be seen in the succeeding graphs, particularly those for H₂ and He.

To calculate the runaway threshold more accurately, we have traced the energy history of each electron, taking into consideration the change of cross section at each step. The energy (in eV) gained in a length $dx$ (in cm) is $dW = Edx$, where $E$ is measured in V/cm. The energy lost by collisions in $dx$ is, on average, $-dW = 2W(dx/\lambda_m)$, where $W$ is the energy of the electron at that point and $\lambda_m(W)$ is the mean free path for that energy. Using Eq. (1), we can write

$$dW/dx = E - 2W_0\sigma(W).$$ (3)

The factor of 2 accounts for the fact that we are using a momentum exchange cross section in an energy equation. If $\sigma(W)$ were a simple function, Eq. (3) could be solved by integral equation techniques. In the present case, we have chosen to follow the energy of an electron in 0.1 mm steps on a PC, using the smooth functional fit to $\sigma(W)$. The initial energy is $W_0$, and the energy at the nth step is

$$W_n = W_{n-1} + [E - 2W_0W_{n-1}\sigma(W_{n-1})]dx
= W_{n-1} + [E - (2)(3.3)pW_{n-1}\sigma_{16}(W_{n-1})]dx.\quad (4)$$

Here, $p$ is in torr, and $\sigma_{16}$ in units of $10^{-16}$ cm².

Because of the shape of the hydrogen cross section curve, there is a sharp threshold for electron runaway. In Fig. 6, we
show the distance that a 30-eV electron travels before it slowly comes to a halt while driven by an electric field \( E \) in 2.5 torr of atomic hydrogen. There is evidence of some oscillatory behavior at low fields, but above about 300 V/cm, the distance increases abruptly, and for higher fields the electron continuously gains energy. This runaway threshold, for a 30-eV electron, is shown as a function of pressure in Fig. 7. Though Eq.(4) does not predict that \( E/p \) should be constant, in this case the \( E/p \) curve is indistinguishable from a straight line. Finally, in Fig. 8, we show the runaway threshold field vs. pressure for various initial electron energies. The case of \( W_0 = 100 \) eV has only one point, because the electron is almost collisionless from the beginning, except at the highest pressure. Such results should not be taken too seriously, since we had to extrapolate the cross section data all the way from 12 eV.

III. MOLECULAR HYDROGEN

The cross section data for \( \text{H}_2 \) from three graphs in Brown (4) are shown in Fig. 9. It is clear that there is a lack of agreement below 2 eV. To get a smooth curve, we have rejected some of the points and averaged the others to obtain the composite data shown in Fig. 10. Again we give the fitting procedure in detail, since it is crucial to the credibility of the results. In this case, the data fortunately extend to 100 eV, and the high-energy part can be fitted nicely to the function \( A \exp(-BW)/W^3 \). This fit is shown in Fig. 11. The low-energy part was fitted to a polynomial minus a function of the form \( 1/W \) to account for the steep drop below .2 eV. There was also a prescription that \( \sigma(W) \) should not go negative near 0. The fit with the low-energy composite data is shown in Figs. 12 and 13. The high- and low-energy parts are joined at 10 eV, and the total fit is shown logarithmically in Fig. 14.

The mean free path in \( \text{H}_2 \) vs. electron energy is shown for various pressures in Fig. 15; the low-energy part is shown expanded in Fig. 16. Fig. 17 shows these data with the lines for energy gained per mean free path. Molecular hydrogen differs from atomic hydrogen in two main respects. First, the cross sections are about a factor 2 higher, which is to be expected from a diatomic molecule. Second, the cross section falls at low energies, since the molecule has no dipole moment. This causes the lines on Fig. 17 for moderate E-fields to lie at an angle to the mfp curves, and generally to the left of them. This means that electron runaway is not possible except at very high fields.

Electrons with energies below the cross section maximum tend to be trapped near 1 eV. If they gain energy from the E-field, they see a larger cross section and drop back down in energy at the next collision. An electron starting at 30 eV will usually lose energy until it falls below the cross section peak, and then it can oscillate in energy before achieving a constant equilibrium drift velocity or "stopping". (It does not really stop, but it has made such a large collision that it has lost its identity and starts over again as a low-energy electron.) If the combination of energy, E-field, and neutral pressure is just right, some electrons can travel farther than the average. This is demonstrated in Fig. 18, showing the stopping distance as a
function of E. At a narrow window near 540 V/cm and for E > 550 V/cm, a constant drift is attained, and the electron keeps going, but it does not run away.

Since there is no real runaway, we must show the results in a different way. Fig. 19 gives the field needed to drive a 30-eV electron to the end of the computation, at 1 cm. There are discontinuous jumps as the pressure is increased, and even E/p jumps at these points. Even if the electron escapes from the cross section trap, its energy increases only gradually. In such a case we can define a "runaway" field to be that required to increase the initial energy by a factor of 3 at the end of 1 cm. This is shown as a function of pressure and initial energy in Fig. 20.

In Figs. 21 and 22 we track the energy of a 30-eV electron at the end of 1 cm of travel in 2.5 torr of H₂ as the E-field is increased. For fields below 600 V/cm, the electron is "stopped" before going 1 cm. Then there is a window in which the electron can delay its demise and stay at 6 eV for some distance. Beyond 700 V/cm, the electron begins to gain energy, but only slowly. In Fig. 22 the graph is continued to very large E-fields. Only at fields beyond 3000 V/cm does the electron attain energies much larger than W₂, but even then there is no sharp critical field.

IV. HELIUM

The cross section data on Figs. 23 and 24 were taken from two curves in Brown (4). A composite curve was made by averaging the data where they overlapped and rejecting the two low-energy points of the high-energy data set. The high-energy part of the composite data can be fit well with a curve of the form A/W⁶ (Fig. 25), and the low-energy part with a polynomial (Fig. 26). Joining these at 10 eV gave the reasonable curve of Fig. 27. However, the succeeding graphs revealed that there was too much of a "lump", and we had to improve the junction by making the part between 6 and 11 eV a straight line. This fit, still not perfect, is shown in Fig. 28.

The mean free path in He is shown in Fig. 29, and the comparison with the energy gained per mfp lines is given in Fig. 30. As with H₂, the lines are at an angle to the curves and generally lie to the left of them, so that runaway would not be expected except at the lowest pressures. Since He has a dip in the cross section at low energies, electrons can be trapped there and come to a constant drift velocity after some oscillations. An example of this is shown in Fig. 31, which shows the energy of a 30-eV electron vs. distance as it is driven by a particularly interesting electric field.

The E-field required to drive a 10-eV electron as far as a centimeter before "stopping" is shown in Fig. 32 vs. pressure. As with H₂, there are peculiar oscillations during which even E/p is not constant. Since there is no true runaway, we show in Fig. 33 the energy of a 30-eV electron at the end of 1 cm, as it is driven by various large E-fields in 5 torr of He. The curve looks smooth, but an examination of the low-energy portion (Fig. 34) shows that there is a threshold at which the energy suddenly becomes finite at 1 cm. Fig. 35 shows the field required to
accelerate electrons of various initial energies to three times that energy in 1 cm, as a function of pressure. The low-pressure part of this graph is shown expanded in Fig. 36. The linearity of these curves shows that there is no runaway phenomenon in He even for extremely high E-fields.

V. SUMMARY AND ACKNOWLEDGMENTS

These simple calculations only show the possibility of electron runaway in weakly ionized rf discharges and the possibility of a fundamental difference between atomic hydrogen and other gases in this regard. The energy loss of an electron during a collision is a statistical process, and even electrons with a given starting energy will have a spread in their final energies. A better calculation would account for the distribution of scattering angles and the variation in differential cross section. To explain experiments, one would have to integrate over the distribution of starting energies and add the effect of local dc electric fields created by the space charge, add the effect of inelastic collisions, and account for the time and space variation of the rf electric field. Such elementary exercises are best left to the student.

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REFERENCES

5. K. Kadota, private communication.
Electron-hydrogen cross section data

![Graph showing electron-hydrogen cross section data with new and old markers.]

Fig. 1

Electron-hydrogen cross section

![Graph showing electron-hydrogen cross section with data and part fit markers.]

Fig. 2
Electron-hydrogen cross section

Fig. 3

Mean free path in atomic hydrogen

Fig. 4
Fig. 5

Energy gained per mean free path

Electron energy (eV)

mean free path (nm)

E (V/cm)

Electrons in atomic hydrogen

Stopping length (mm)

Wo = 30 eV

p = 2.5 Torr

E (V/cm)

Fig. 6
Runaway field vs. pressure

Fig. 7

Runaway field vs. pressure

Fig. 8
Electron cross section with H2

Fig. 11

Electron cross section with H2

Fit of low energy points

Sig 
Sigma - exp(-v^2/v)
Sigma (E-16)

Fig. 12
Fig. 15

Mean free path in molecular hydrogen

Fig. 16

Mean free path in molecular hydrogen
Energy gained per mean free path

Molecular H₂

Electron energy (eV)

mean free path (mm)

E (V/cm)

p (Torr)

1
3
10
30
100
200
300
500

Fig. 17

Stopping length in H₂

p = 2.25 torr, W₀ = 30 eV

Distance (mm)

E (V/cm)

Fig. 18
Runaway field vs. pressure in H2
Field for electron to go 1 cm

Fig. 19

Runaway field vs. pressure in H2
Field to get to 3Wo at end of 1 cm

Fig. 20
Runaway in molecular hydrogen

Fig. 21

Electron runaway in H₂

Energy at end of 1 cm

Fig. 22
Electron-helium cross section
High energy part

Fig. 23

Electron-helium cross section
Low energy part

Fig. 24
Electron-helium cross section

Fig. 25

Electron-helium cross section

Fig. 26
Electron-helium cross section

Fig. 27

Sigma fit in joining region

Fig. 28
Mean free path in helium

Fig. 29

Energy gained per mfp in helium

Fig. 30
Approach to equilibrium energy
Helium, 5 torr, Wo = 30 eV, E = 162.2 V/cm

E-field for electron to go 1 cm
Helium, Wo = 10 eV

Fig. 31

Fig. 32
Energy at end of 1 cm in He

$p = 5\text{ torr}, \ Wo = 30\text{ eV}$

Fig. 33

Energy at end of 1 cm in He

$p = 5\text{ torr}, \ Wo = 30\text{ eV}$

Fig. 34
Fig. 35

Field to give 3Wo at end of 1 cm

Helium

$E_{-field} \text{ (V/cm)}$

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Wo (eV)

- ■ 10
- □ 20
- ● 30
- ○ 50
- ▽ 100

Fig. 36

Field to give 3Wo at end of 1 cm

Helium

$E_{-field} \text{ (V/cm)}$

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Wo (eV)

- ■ 10
- □ 20
- ● 30