

## Institute of Plasma and Fusion Research

### PLASMA PHYSICS: an encyclopedic view

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**Electrical Engineering Department** 

PPG- 1531

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#### **ABSTRACT**

The field of Plasma Physics concerns the behavior of matter in its fourth state, that of an ionized, electrified gas, usually occurring at temperatures higher than those at which the solid, liquid, and gas phases exist. The plasma state is complicated, not only because the particles have electrical charges and are greatly affected by both electric and magnetic fields, but also because the different constituents of the plasma are not usually in thermal equilibrium with one another, and because the medium behaves sometimes like a continuous fluid and sometimes like a large collection of individual particles. Plasmas occur naturally in stellar interiors, in interstellar and interplanetary space, and in the ionospheres of planets. Laboratory plasmas are made for reproducing the nuclear fusion reactions that power the sun and stars, for improving materials and manufacturing semiconductor circuits, and for studying the behavior of matter under conditions of extreme energy bombardment. This survey of the various aspects and applications of Plasma Physics includes an extensive list of references and a glossary for the terminology of this field.

#### PLASMA PHYSICS

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#### INTRODUCTION

The most common type of plasma is a gas of such high temperature that it is ionized; that is, an appreciable number of atoms have been stripped of at least one electron and have become positive ions (Figs. 1, 2). Because of its free electrical charges, plasmas differ from ordinary gases because it is subject to electric and magnetic forces. Indeed, the science of plasma physics mostly concerns those plasmas in which the usual molecular forces are negligibly small compared with the electromagnetic forces. Plasma has been called the "fourth state of matter," after solid, liquid, and gas. All matter goes into this electrified state above temperatures of 10,000 K or so, at which point the fastest particles in the thermal distribution undergo collisions energetic enough to ionize an atom.

Plasmas can also exist at room temperature in metals and semiconductors in which there are free charge carriers. At the other extreme, in the vacuum of outer space, hydrogen atoms are ionized by photoionization by starlight. Once ionized, the ions and electrons do not recombine easily because, at a density of about 1 ion per cc, collisions are rare. Consequently, perhaps 99% of all the matter in the universe is in the plasma state. On the earth, however, the temperatures and atmospheric densities which are consistent with life cannot support plasmas, and these can be studied only in laboratory vacuum chambers.

Within the framework of physics and engineering, plasma physics is an interdisciplinary science that impacts many other fields. It is an extension of electromagnetics that can treat conducting gases that exist in stellar interiors and in nuclear fusion devices constructed to reproduce the conditions there. It describes the motion of charge carriers in semiconductor devices and in the machines used to make them. It is used in the design of gas lasers and microwave generators. Plasma physics is essential to the understanding of the behavior of matter under conditions of high energy density, such as in quasars and pulsars, and in targets bombarded by intense laser or particle beams. It has applications even to chemistry through the use of electrons to form new compounds and polymers. The development of femtosecond lasers to probe chemical and surface reactions in real time will be aided by the knowledge of how intense radiation interacts with matter in the plasma state.

Plasmas are more complicated than ordinary fluids for several reasons. First, it is subject to long range electromagnetic forces. Second, plasmas have a particulate nature, unlike fluids, which are continuous media. At high temperatures, collisions are so rare that plasmas can be treated as collisionless fluids; and in the absence of randomizing collisions, the individual particles of the fluid must be taken into account. Third, since the charged particles motions are greatly affected by magnetic fields, the plasma is a highly anisotropic dielectric and support a rich variety of possible wave motions.

Though plasmas have been known for a much longer time, plasma physics as a science did not begin to advance rapidly until the late 1950s, when the search for a infinite energy source through controlled hydrogen fusion began. The principles of plasma science are now considered to be indispensable in many other disciplines besides controlled fusion: in astrophysics and space physics, in particle accelerators and radiation generators, in solid-state devices and lasers, in the understanding of turbulence and chaos in nonlinear dynamics, and, most recently, in manufacturing sciences.

#### 1. History

Plasma physics had its beginnings in two disparate disciplines: gaseous electronics and cosmic electrodynamics. In the 1920s, Irving Langmuir and his colleagues, in their studies of gas discharges used in electron tubes, laid out the principles of plasma measurements with the electrostatic probes which bear his name. In the 1930s, Chapman and Cowling (1939) worked out the Theory of Non-uniform Gases, and Mott and Massey (1933) contributed the Theory of Atomic Collisions. During World War II, research on isotope separation using plasma discharges at Los Alamos led David Bohm to invent the infamous "Bohm diffusion" law whose origin is still shrouded in mystery. Gaseous electronics took a leap after the War as electrical engineers at M.I.T. and Stanford competed to understand beam-plasma interactions and develop better klystrons and other microwave generators for radar. The theory of radio wave propagation in the ionosphere was worked out simultaneously by V.L. Ginzburg (1964) and others in Russia. Experiments were still limited to Langmuir probe measurements in weakly ionized discharges. It was also in the late 1950s and early 1960s that the other stem of plasma physics began its growth. H. Alfvén (1950), S. Chandraskehar (1961), and other astrophysicists became aware of the rôle of plasmas in deep space and worked out the principles of magnetohydrodynamics, now known as hydromagnetics, including the discovery of "Alfvén waves." Astronomers tended to view plasmas as conductive fluids like mercury, whereas the gaseous electronics community treated plasmas from the particle point of view. These opposing views of the dual nature of plasmas were not merged until the era of nuclear fusion research.

Following World War II, dense, hot plasmas were produced by the "pinch" effect at the Harwell military laboratories in England; these experiments foreshadowed the scramble for fusion power. Attempts to produce thermonuclear reactions in the laboratory and thus to "tame the Hbomb" were started in England, Russia, and at four sites in the United States: the national laboratories at Livermore, Los Alamos, and Oak Ridge, and at Princeton University. All of these pro-The project at Princeton was code-named Project Matterhorn by its grams were classified. mountain-climbing leader, astrophysicist Lyman Spitzer, Jr. The leader of the Los Alamos Project was James Tuck, and this connection to Friar Tuck of Robin Hood fame led to the code name "Project Sherwood" for the U.S. fusion program. Classification was removed and international cooperation began at the 1958 Atoms for Peace conference in Geneva, where each nation displayed its best results. The U.S. unveiled two mirror-confinement programs (at Livermore and Oak Ridge), a pinch program at Los Alamos, and a "Stellarator" program at Princeton. The U.K. exhibited a large toroidal pinch called "Zeta," which produced copious neutrons, but which was later discredited when the neutrons were found to be of non-thermonuclear origin. The Russians exhibited an unglamorous piece of hardware which they dubbed a "tokamak," a Russian acronym for "toroidal magnetic chamber." This concept not only survived but has become the dominant one by far.

Spurred by the race for controlled fusion, plasma physicists progressed rapidly on both theoretical and experimental fronts. To generate power from fusing heavy hydrogen into helium requires heating a plasma to over 108 K and confining it for longer than 1 sec. The principles of confining plasmas with the magnetic walls needed to withstand such temperatures were worked out on the basis of classical physics concepts for both closed (toroidal) and open (mirror) systems, but the unexpectedly fast leakage out of such magnetic "traps" or "bottles" was a huge problem.

To solve it required going back to basic principles and establishing a new science which barely resembles its precursors. The instabilities that a magnetized plasma can support required new theoretical and experimental methods. An elegant variational-principle formulation of the problem in a paper by Bernstein *et al.* (1958) is generally credited with establishing the respectability of plasma physics as a new and viable field. Also in 1958, the Physics of Fluids was established as the plasma journal, and the Division of Plasma Physics was founded in the American Physical Society to give plasma physics a home.

When the exploration of space started in the early 1960s, a need for plasma diagnostics arose, and Langmuir probes were quickly adapted for the earth's ionosphere and the interplanetary environment. Exploration of the plasmas in the magnetospheres of the planets, as well as of the solar wind and other solar phenomena have greatly expanded our knowledge of the solar system in the past three decades. The advent of radio astronomy has led to observations of plasma phenomena in deep space. Most recently, the importance of dusty plasmas has been realized in the understanding of comet tails and planetary rings.

In the early 1970s, high-power pulsed lasers were invented, and the concept of inertial confinement was born. The idea was to do away with magnetic fields altogether, using laser beams to compress a small pellet of deuterium-tritium fuel to 10<sup>4</sup> times solid density, and hence to speed up the fusion reaction time to the order of picoseconds. In that case, the inertia of the imploding pellet would confine the fuel long enough for fusion to occur. Unfortunately, the unexpected effects of plasma physics could not be avoided. When intense laser light strikes a solid object, the surface material is immediately ionized into a plasma cloud. A new type of instability, called a parametric instability, was driven by the laser beam; and these had the effect of reflecting the laser light before it could reach the solid surface of the pellet. Inertial confinement spawned a new branch of plasma physics which required extensive development of nonlinear and computational techniques. Simultaneously, it was found that intense electron and proton beams of order 1 MA at 1 MV could be produced by storing electrical energy in capacitor banks and discharging it into a diode in a short pulse. Pulsed power became yet another branch of plasma physics, permitting the study of matter subject to terawatts of power, and leading to the invention of new plasma switches and x-ray generators.

Until recently, most plasma physics research fell into one of the four areas described above: magnetic fusion, inertial fusion, space plasma physics, and pulsed power. In the 1990s, the use of partially ionized plasmas in manufacturing began to expand rapidly, along with the prevalent use of semiconductor ULSI (Ultra Large Scale Integrated) circuits and personal computers. Plasma etching and deposition is essential in the fabrication of computer chips with millions of internal features. It is anticipated that the production of flat-panel displays will ultimately also depend on plasma processing. Surface hardening of turbine blades, tools, construction materials, medical prostheses, and so forth is already a large commercial application of plasma physics. Plasma treatment of plastics and fibrous materials can be used, for instance, to make plastic containers more leakproof, to make paint adhere to plastic bags, clothing, currency, and automobile parts; to make fabrics absorb or repel water; and to tailor the refractive indices of optical fibers. In summary, though the original impetus for developing plasma physics was the lofty goal of an inexhaustible energy source for mankind, fusion power plants will not be commercially viable for several more decades. In the near future, however, plasma physics is likely to have a large im-

pact on our daily lives because of its utility in the manufacture of a diverse variety of common materials.

#### 2. Basic Plasma Phenomena

#### 2.1 Methods of Treatment

#### 2.1.1 Fluid Description

At the simplest level, plasmas can be treated as an electrified fluid, without regard for its particulate nature. A common approach is to consider the positive ions and negative electrons of the plasma to be two interpenetrating fluids which interact via the electric and magnetic field which they each generate in their motions (Chen, 1983). The equation of motion for fluid elements of each species *j* can be written

$$m_{j}n_{j}\left(\frac{\partial \mathbf{v}_{j}}{\partial t}+\mathbf{v}_{j}\cdot\nabla\mathbf{v}_{j}\right)=q_{j}n_{j}(\mathbf{E}+\mathbf{v}_{j}\times\mathbf{B})-\nabla p_{j}-m_{j}n_{j}v_{jk}(\mathbf{v}_{j}-\mathbf{v}_{k}),$$
(1)

where  $q_j$  and  $m_j$  are the charge and mass of species j;  $n_j$ ,  $p_j$ , and  $\mathbf{v}_j$  are the fluid density, pressure, and velocity of that species; and  $\mathbf{v}_{jk}$  is its average collision frequency with species k. The pressure  $p_j$  is given by  $p_j = \gamma_j n_j K T_j$ , where K is Boltzmann's constant,  $T_j$  is the temperature of the fluid in ° K, and  $\gamma_j$  is an adiabatic constant equal to 3, 2, or 5/3 for 1-, 2-, and 3-dimensional plasmas, respectively. For constant  $T_j$ ,  $\gamma_j$  is unity. A plasma can have a different temperature and even a different value of  $\gamma$  for each species. In a fully ionized plasma, j takes on the values i and e, while k is the opposite. In a partially ionized plasma with less than about 10% ionization fraction, collisions with the neutral atoms are dominant; then  $v_{jk}$  is  $v_{io}$  or  $v_{eo}$ , the collision rate with the neutral gas. Each species also follows the equation of continuity

$$\frac{\partial n_j}{\partial t} + \nabla \cdot (n_j \mathbf{v}_j) = 0 . {2}$$

A set of Eqs. (1) and (2) can be written for each species of positive or negative ions. The electric and magnetic fields **E** and **B** are governed by the vacuum Maxwell's equations:

$$\boldsymbol{\varepsilon}_0 \nabla \cdot \mathbf{E} = \boldsymbol{\sigma} \tag{3}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \tag{4}$$

$$\nabla \cdot \mathbf{B} = 0 \tag{5}$$

$$\nabla \times \mathbf{B} = \mu_0 \left( \mathbf{J} + \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} \right) \tag{6}$$

Together with the equation of state for  $p_j$  given above, the system of equations is closed by defining the plasma charge and current as

$$\sigma = \sum_{j} n_{j} q_{j}, \quad \mathbf{J} = \sum_{j} n_{j} q_{j} \mathbf{v}_{j} \tag{7}$$

The charged particle motions generate the internal electromagnetic fields, and the fields move the particles; thus, Eqs.(1)-(7) must be solved self-consistently. It is also possible to describe the plasma as a dielectric medium with a susceptibility  $\chi$  and a permittivity  $\varepsilon = \varepsilon_0 (1 + \chi)$  which incorporate the effects of the plasma charges and currents, as given by Eqs. (1) and (2). J and  $\sigma$  are then omitted from Eqs. (3) and (6), and  $\varepsilon_0$  replaced by  $\varepsilon$ . Plasmas in a magnetic field are anisotropic, and  $\varepsilon$  is then a rather complicated dielectric tensor. It is a characteristic of plasmas that  $\chi$  is often negative. The electromagnetic quantities **D** and **H** need not be defined and are generally not used in plasma physics.

An alternative description of a plasma is that of a single conducting fluid. The single-fluid, or MHD (magnetohydrodynamic), equations of motion are linear combinations of Eq. (1) for electrons and Eq. (1) for a single species of ions with charge number Z=1, with a number of terms neglected:

$$\rho \frac{\partial \mathbf{v}}{\partial t} = \mathbf{J} \times \mathbf{B} - \nabla p \tag{8}$$

$$\mathbf{E} + \mathbf{v} \times \mathbf{B} = \eta \mathbf{J} , \qquad (9)$$

where the single-fluid density  $\rho$ , velocity  $\mathbf{v}$ , pressure p, and resistivity  $\eta$  are defined by

$$\rho = \sum_{j=i,e} m_j n_j, \quad \mathbf{v} = \sum_{j=i,e} n_j M_j \mathbf{v}_j / \rho, \quad J = \sum_{j=i,e} n_j q_j \mathbf{v}_j, \quad p = p_i + p_e, \quad \eta = m_e v_{ei} / n_e e^2.$$
 (10)

Eq. (9) is known as the generalized Ohm's Law. Eqs. (2) for ions and electrons are replaced by

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0 , \quad \frac{\partial \sigma}{\partial t} + \nabla \cdot \mathbf{J} = 0 . \tag{11}$$

Note that the quasineutrality condition  $n_i = n_e$  used here is almost always a good approximation because only a small difference  $\sigma = e(n_i - n_e)$  is enough to create a sizable field E. The single-fluid equations are convenient for resistive plasmas, in which the dominant collisions are between ions and electrons, but the equations are less accurate than the two-fluid equations (for instance, the Hall current is neglected in the generalized Ohm's Law), and multiple species are not easily handled.

#### 2.1.2 Kinetic Description

The fluid description is valid when collisions are frequent enough to maintain a Maxwell-Boltzmann velocity distribution for each species, so that a temperature T can be defined. Fluid theory is also a good approximation for motions perpendicular to a strong magnetic field even when collisions are rare. In the general case, deviations from a Maxwellian distribution are pos-

sible, and one has to solve for the distribution function  $f_j(\mathbf{r}, \mathbf{v}, \mathbf{t})$  for each species. A function of seven independent scalar variables,  $f_i(\mathbf{r}, \mathbf{v}, \mathbf{t})$  is governed by the *Boltzmann equation* 

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla f + \frac{q}{m} (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \frac{\partial f}{\partial \mathbf{v}} = \left(\frac{\partial f}{\partial t}\right)_{C},$$
(12)

where  $(\partial f/\partial t)_{\rm C}$  is a collision term. "Collisionless" plasmas are hot enough that this term can be set equal to zero, in which case Eq. (12) is called the *Vlasov equation*. Such "Vlasov plasmas" have been studied intensively by plasma theorists (Nicholson, 1983). Other forms of the collision term are the *Fokker-Planck* term, which describes binary Coulomb collisions, and the *Krook* term, which is used to treat collisions with a neutral species. A simplified form of Eq. (12) in which the Larmor gyrations [cf. Sec. 2.2.3] have been averaged and replaced by the motion of a guiding center is called the *drift-kinetic equation*.

Since the integral of  $f(\mathbf{v})$  over all velocities is just the density, the continuity equation (2) can be recovered by integrating the Vlasov equation. The first moment, found by multiplying Eq. (12) by  $\mathbf{v}$  and then integrating, yields Eq. (1). The second moment gives the heat-flow equation, and so on; but the process is usually truncated after the first moment. Solutions of Eq. (12) with Maxwell's equations reveal two main features not found in Eqs. (1) and (2). First, electric fields can distort  $f(\mathbf{v})$  away from a Maxwellian in the direction parallel to a magnetic field or when there is no magnetic field. Second is the finite-Larmor-radius (FLR) effect, in which the thermal motions of the particles causes them to react to the average field that they experience during a Larmor, or cyclotron, orbit. The fluid equation, Eq. (1), correctly describes the Larmor orbits of cold fluids driven by electric fields, but the pressure term  $\nabla p_j$  does not include all the thermal effects; in particular, the FLR effect is contained in the viscosity tensor, which has been omitted from Eq. (1).

Though plasmas can often be treated as fluids, plasma physics differs from hydrodynamics because non-Maxwellian distributions sometimes occur which require the kinetic treatment described above; it differs also from electromagnetics because the details of the dielectric tensor  $\epsilon$  are treated from the particulate point of view. Indeed, the complex motions of charged particles in electric and magnetic fields support a rich variety of wave phenomena which do not occur in ordinary fluids or dielectrics.

#### 2.1.3 Computer Simulation

Small amplitude waves and instabilities can be analyzed by linearizing the plasma equations given above. That is, each fluctuating quantity-for instance, the fluid density n--is divided into an undisturbed, time-independent part and an oscillating part:  $n = n_0 + n_1$ , where  $n_1$  is assumed to be small relative to  $n_0$ . When higher-order terms containing more than one power of fluctuating quantities are discarded, the resulting set of linear equations can be solved to give the possible wave modes. These linear modes can be superimposed without affecting one another. Large-amplitude waves, and instabilities which have grown large, however, are difficult to treat with these equations. In the early years of plasma physics history, there was a gap between the small-amplitude phenomena accessible by theory and the large-amplitude phenomena observed

experimentally. The advent of high-speed computers provided the Rosetta Stone, particle simulation (Dawson and Lin, 1984), needed to relate experiment to theory.

In computer simulation (Hockney and Eastwood, 1988; Birdsall and Langdon, 1991), a large number of particles—typically 10<sup>4</sup> but as large as 10<sup>6</sup>—are placed in a "box," and the position and velocity of each ion and electron is recorded in the memory. The E and B fields caused by these particles are then found from Maxwell's equations by fast Fourier transform. The particles are then allowed to move in response to these fields during a short time step, and the fields are then recalculated with the new positions and velocities. Thus, the response of the plasma to applied forces or unstable initial conditions can be followed step by step from threshold to turbulence regardless of the amplitudes of the perturbations. The boundaries of the "box" can be made to absorb particles, reflect particles, or to be reentrant or periodic, so that particles leaving one side reappear on the opposite side. To facilitate computation, the box is divided into a grid of cells, and a particle that ends up in any particular cell is placed at the center of that cell for the next time step. Such a scheme is called a Particle-In-Cell, or PIC, code. Since each computer particle represents many real particles, it cannot be given the full electric charge of all those particles, for then the Coulomb collisions between particles would be unduly magnified. These collisional effects have to be properly scaled.

The number of particles, cells, and time steps in the simulation is limited only by the speed and memory of the computer. Programs range from those that can be run on personal computers (PCs) to those that require many hours or days on the most advanced supercomputers. Initially, computer simulation was limited to one-dimensional systems, in which each "particle" was merely a charge sheet. Later, two-dimensional grids could be used, with axes corresponding, for instance, to (x, y) in a Cartesian system or (r, z) in a cylindrical system. A common compromise is a so-called 2½D computation, in which two components of position and all three components of velocity are retained. Fully three-dimensional simulations are now possible. Once a computational run has been finished, it is easy to do diagnostics on the plasma. For instance, one can cull from the data the distribution functions as a function of space or time, the frequency or wavenumber spectrum of the fluctuations, the conservation of energy and momentum, or the direction of the Poynting vector. The results of such computer "experiments" can be used to compare with real measurements, but plasma simulations can be formulated and interpreted only with the insight gained from an intimate knowledge of analytic theory.

#### 2.2 Characteristics of Plasma Behavior

#### 2.2.1 Quasineutrality and Debye Shielding

Plasmas are characterized by an overriding tendency to be neutral. The numbers of positive and negative charges per unit volume cannot be exactly equal, of course; else there would be no electric fields at all. Since the opposing charge densities need differ only by, say, one part per million to create the electric fields normally observed in a plasma, for all intents and purposes these densities can be treated as being equal. This approximation, known as "quasineutrality," is an important concept in plasma physics. The consequences of quasineutrality may not be apparent to physicists from other disciplines. For instance, at frequencies low enough that the heavy

and light particles can move together, **E** cannot be calculated from Poisson's equation, Eq.(3), since  $n_i - n_e$  is not known accurately enough; rather, Eqs. (1) and (2) determine the value of **E** because the ions and electrons must move in such a way as to make their densities track each other. The density  $n_i$  or  $n_e$  is simply known as the plasma density  $n_i$ .

A plasma shields itself from external fields by *Debye shielding*. When a positively charged object is placed in a plasma, the mobile electrons are attracted to it and form an electron cloud covering the charge; similarly, a negative object will repel electrons, leaving an ion cloud for shielding. The thickness of these clouds is of the order of the Debye length  $\lambda_D$ , defined by

$$\lambda_D^2 = \varepsilon_0 K T_e / ne^2 . ag{13}$$

The shielding is not perfect, however, since electrons near the edge of the cloud can escape because of their thermal energies; consequently, electric potentials of order  $\frac{1}{2}KT_e$  can exist in the plasma. [The temperatures of a plasma are specified by the value of KT in electron-volts (eV), where 1 eV corresponds to 11,600 °K. Thus, the average thermal energy of the electrons in a plasma with  $KT_e = 10$  eV is  $\frac{3}{2}KT_e$ , or 15 eV.] Debye shielding also occurs at the walls confining a plasma, where an ion-rich sheath of about  $5\lambda_D$  thickness forms to repel electrons. The Coulomb barrier of the sheath is just high enough to make the escaping electron flux equal to the ion flux,

barrier of the sheath is just high enough to make the escaping electron flux equal to the ion flux, so that no net charge is lost. Unless the plasma is extremely tenuous, these sheaths are much thinner than the dimensions of the plasma, so that the interior is quasineutral.

#### 2.2.2 Characteristic Frequencies and Velocities

When the electrons of a plasma are displaced from their equilibrium positions, the ions left behind draw them back with an electric field. The electrons then overshoot and oscillate about their equilibrium positions with a characteristic frequency  $\omega_p$ ,, called the *plasma frequency*, which depends on the density of electrons as well as their charge and mass:

$$\omega_p = (ne^2 / \varepsilon_0 m)^{1/2} \text{ rad/sec}, \qquad f_p = \omega_p / 2\pi \text{ Hz}. \qquad (14)$$

An ion plasma frequency  $\omega_{pi}$  is similarly defined with the ion charge and mass but is not a natural oscillation frequency because the ions are shielded by Debye shielding.

When a background magnetic field B is present, the ions and electrons gyrate in circular orbits, called Larmor orbits, in a plane perpendicular to the field lines. The frequency of these gyrations is the cyclotron frequency  $\omega_c$ , given for species j by

$$\omega_{cj} = \left| q_j B / m_j \right|, \qquad f_{pj} = \left| q_j B / 2\pi m_j \right|, \tag{15}$$

where  $q_j$  is -e for electrons and Ze for ions. The radius of the orbit--the Larmor radius, is given by

$$r_L = v_{\perp} / \omega_c = m v_{\perp} / |q| B, \qquad (16)$$

where  $v_{\perp}$  is the velocity component perpendicular to **B**. For a Maxwellian distribution, the root-mean-square value of  $v_{\perp}$  is used to evaluate the average  $r_L$ , where

$$\langle v^2 \rangle = 3KT/m, \quad \langle v_\perp^2 \rangle = 2KT/m.$$
 (17)

Charged particles are constrained from moving freely across  $\bf B$  and act as if they were tied to the field lines. When these mass-loaded field lines are displaced, the disturbance propagates along the field with a characteristic velocity known as the *Alfvén speed*  $\bf v_A$ :

$$v_A = B / \sqrt{\mu_0 \rho} \ . \tag{18}$$

This behavior can occur only if the B-field is strong enough to constrain the particles; that is, if the plasma pressure is sufficiently small. The critical quantity is the ratio between plasma pressure and magnetic field pressure, known as "beta":

$$\beta = \frac{p_i + p_e}{B^2 / 2\mu_0} \ . \tag{19}$$

Laboratory plasmas have  $\beta << 1$ , but gravitationally confined plasmas in space may have  $\beta \geq 1$ .

Sound waves in a plasma, called *ion acoustic waves*, travel at a characteristic velocity  $v_{\rm s}$  given by

$$v_s^2 = \left(ZKT_e + \gamma_i KT_i\right) / m_i. \tag{20}$$

Unlike sound waves in air, this velocity does not vanish when  $KT_i = 0$ . The wave motion is sustained not by pressure fluctuations but by the electric fields, of order  $KT_e$ , which extend beyond the Debye clouds shielding the ion density fluctuations.

#### 2.2.3 Particle and Fluid Drifts

In a strong magnetic field  $\mathbf{B}$ , the parallel motions of the particles are unaffected by  $\mathbf{B}$ , while the perpendicular motions can be considered to be circular Larmor gyrations about a moving center, called a *guiding center*. In crossed electric and magnetic fields, the guiding center drifts with a velocity  $\mathbf{v}_{\rm E}$  given by (Kruskal, 1965a):

$$\mathbf{v}_F = \mathbf{E} \times \mathbf{B} / B^2 \,. \tag{21}$$

This velocity is the same for all species, and this is true to the extent that the FLR effect can be neglected. If we replace the force  $q\mathbf{E}$  used in deriving Eq. (21) with a general force  $\mathbf{F}$ , the resulting guiding center drift  $\mathbf{v}_{\mathbf{f}}$ , would be in opposite directions for ions and electrons:

$$\mathbf{v}_f = \mathbf{F} \times \mathbf{B} / qB^2 \,. \tag{22}$$

If the magnitude of B is nonuniform, the guiding center suffers a gradient-B drift v<sub>vB</sub>:

$$\mathbf{v}_{\nabla B} = \pm \mathbf{v}_{\perp} r_L \frac{\mathbf{B} \times \nabla B}{2B^2},\tag{23}$$

where  $\pm$  indicates the sign of the charge. If the direction of **B** is not constant, one can define a local vector radius of curvature  $\mathbf{R}_c$  pointing toward the center of curvature; the guiding center then has a *curvature drift*  $\mathbf{v}_R$  given by:

$$\mathbf{v}_R = \frac{m v_\parallel^2}{q} \frac{\mathbf{R}_c \times \mathbf{B}}{R_c^2 B^2} \ . \tag{24}$$

When the E-field varies in time, the Larmor gyrations are disrupted, and the guiding center suffers a polarization drift  $\mathbf{v}_p$  given by:

$$\mathbf{v}_p = \pm \frac{1}{\omega_c B} \frac{d\mathbf{E}}{dt} \,. \tag{25}$$

In fluid theory, the drifts of the fluid elements are similar to the guiding center drifts, but with some differences. The drifts  $\mathbf{v}_E$ ,  $\mathbf{v}_R$ , and  $\mathbf{v}_p$  are the same; but the gradient-B drift  $\mathbf{v}_{VB}$  cancels out when summed over the particles in a fluid element. The summation process gives rise to a fluid drift, called the *diamagnetic drift*  $\mathbf{v}_D$ , which does not exist for particles (Spitzer, 1967):

$$\mathbf{v}_D = \frac{\mathbf{B} \times \nabla p}{q n B^2} \tag{26}$$

Since  $\mathbf{v}_D$  is in a different direction for ions and electrons, there is a net *diamagnetic current*  $\mathbf{j}_D$  whose  $\mathbf{j} \times \mathbf{B}$  force is just sufficient to keep the  $\nabla p$  force from pushing the plasma across  $\mathbf{B}$ , as seen from Eq. (8).

Because of the electrons' high mobility along  $\mathbf{B}$ , the net parallel force on the electron fluid must be close to zero except at high frequencies. The dominant terms in Eq. (1) are then

$$0 = e n_e \nabla_{\parallel} \phi - K T_e \nabla_{\parallel} n_e , \qquad (27)$$

where we have taken  $E_{\parallel} = -\nabla \phi$  and  $\gamma_e = 1$ . For  $n_e = n_0 + n_1$ , integration of Eq. (27) gives the Boltzmann relation

$$n_{_{1}} = n_{0} \exp(e\phi/KT_{e}), \qquad (28)$$

which has wide applicability and is useful for relating density to potential fluctuations.

#### 2.2.4 Adiabatic Invariants

In addition to energy and momentum, the action of a particle in periodic motion is conserved, permitting a simple way to describe the motion of gyrating particles in slowly varying fields (Northrop, 1963; Kruskal, 1965b). The first such adiabatic invariant is the magnetic moment  $\mu$ , defined by

$$\mu = mv_{\perp}^2 / 2B \,. \tag{29}$$

This quantity, equal to the magnetic flux enclosed in a Larmor orbit, remains constant if B changes in time or if the guiding center moves to a region of different B. The invariance of  $\mu$  is broken if the field seen by the particle changes on the timescale  $1/\omega_c$  or if B varies spatially over a Larmor radius. The second adiabatic invariant J, defined by

$$J = \int v_{\parallel} ds , \qquad (30)$$

concerns the motion of a gyrating particle between two "magnetic mirrors" (Fig. 3). When such a particle moves into a region of stronger B, conservation of  $\mu$  causes  $\nu_{\perp}$  to increase. Since  $\nu_{\parallel}$  must then decrease to conserve energy, the particle is reflected. If the bounce frequency of a particle trapped between two regions of strong field, such as the polar regions of the earth's magnetic field, is larger than the rate of change of B, the integral J evaluated between turning points is conserved. Other adiabatic invariants involve slower periodic motions and are rarely used.

#### 2.2.5 Resistivity and Diffusion

#### 2.2.5.1 Weakly Ionized Plasmas

Driven by a pressure gradient or an electric field, electrons and ions in a weakly ionized discharge have their velocities along **B** limited by collisions with the constant neutral gas background. For the isothermal case  $\gamma_j = 1$ , solution of Eq. (1) with the  $\mathbf{v} \times \mathbf{B}$  term and the left-hand side set equal to zero gives the particle flux (Chen, 1983)

$$n_i \mathbf{v}_{\parallel i} = \mu_i n_i E_{\parallel} - D_{\parallel i} \nabla_{\parallel} n_i , \qquad (31)$$

where 
$$\mu_{\parallel j} = q_j / m_j v_{0j}, \quad D_{\parallel j} = KT_j / m_j v_{0j} \quad (j = i,e),$$
 (32)

 $\mu$  is the mobility and D the diffusion coefficient. In the direction perpendicular to **B**, inclusion of the  $\mathbf{v} \times \mathbf{B}$  term in Eq. (1) results in

$$n_j \mathbf{v}_{\perp j} = \mu_j n_j \mathbf{E}_{\perp} - D_{\perp j} \nabla_{\perp} n_j + \frac{\mathbf{v}_E + \mathbf{v}_{Dj}}{1 + \mathbf{v}_{0j}^2 / \omega_{cj}^2} , \qquad (33)$$

$$\mu_{\perp j} = \frac{\mu_{\parallel j}}{1 + \omega_{cj}^2 \tau_j^2} , \quad D_{\perp j} = \frac{D_{\parallel j}}{1 + \omega_{cj}^2 \tau_j^2} , \quad \tau_j = \frac{1}{\nu_{0j}} , \tag{34}$$

and  $\mathbf{v}_E$  and  $\mathbf{v}_D$  are given by Eqs. (21) and (26). We see that  $\mathbf{v}_E$  and  $\mathbf{v}_D$  are slowed by collisions, whereas cross-field mobility and diffusion exist only by virtue of collisions.

These relations have a simple physical meaning. Ordinary diffusion is a random walk with a step length  $L=\lambda_{\rm m}$ , the mean free path, and mean time  $\tau=1/\nu$  between steps. For particles of velocity v,  $\lambda_{\rm m}=1/n_{\rm n}\sigma({\rm v})={\rm v}\tau$ , where  $n_{\rm n}$  is the neutral density. The collision frequency  $\nu_0$  averaged over velocities is thus  $\nu_0=n_{\rm n}<\sigma{\rm v}>$ . The diffusion coefficient is then  $D=L^2/\tau={\rm v}^2\tau^2/\tau\approx KT/m\,\nu_0$ . Mobility is like diffusion except that the force  $qn{\rm E}$  takes the place of the pressure force  $KT\nabla n$ , so that the coefficient of E should be q/KT times the coefficient of  $\nabla n/n$ , as in Eq. (32). For perpendicular diffusion across a magnetic field, a collision displaces the guiding center a distance of order  $r_{\rm L}$  or  $\lambda_{\rm m}$ , whichever is smaller. Taking  $L^{-1}=r_{\rm L}^{-1}+\lambda_{\rm m}^{-1}$  gives the reduction factors in Eq. (34) for perpendicular transport.

Since ions and electrons have different transport coefficients, an *ambipolar electric field* will develop to retard the faster species and accelerate the slower species. In the direction along  $\bf B$ , electrons will escape rapidly, leaving behind a positive charge the field of which equalizes the electron and ion escape rates to maintain quasineutrality. The net diffusion rate, found from Eqs. (31) and (32), is described by an *ambipolar diffusion coefficient*  $D_a$ :

$$D_{a} = \frac{\mu_{i} D_{e} + \mu_{e} D_{i}}{\mu_{i} + \mu_{e}} \ . \tag{35}$$

For diffusion across **B**, the same formula obtains if the definitions in Eq. (34) are used. In this case, the ambipolar field is reversed, since ions generally escape faster than electrons across **B**. In the general case, diffusion in a given density gradient is described by the equation  $\nabla \cdot (nv_i) = \nabla \cdot (nv_e)$ , with the parallel and perpendicular fluxes given by Eqs. (31) and (33), respectively. Though this equation is not separable into perpendicular and parallel components, in principle the solution will yield the ambipolar potential  $\phi$ , which is the only unknown in the problem. Unfortunately, the problem is not well posed because of the *short-circuit effect*. In plasmas whose length along **B** is not extremely large relative to the radius, electrons escaping along field lines can short-circuit the radial ambipolar field through the sheaths at the ends. The ambipolar fields in the axial and radial directions then depend on the details of the end conditions.

#### 2.2.5.2 Fully Ionized Plasmas

Because of detailed balance, Coulomb collisions between like particles--for instance, electrons with electrons--cause neither a drag of the fluid motion along **B** nor a net diffusion of the fluid across **B** unless there is a large gradient on the scale of  $r_L$ . Collisions between electrons and ions cause friction between the two fluids, giving rise to a resistivity  $\eta_{\parallel}$  for current flow along **B**. The value of  $\eta_{\parallel}$ , called the *Spitzer resistivity* (Spitzer, 1967), to be used in Eq. (9) for  $E_{\parallel}$ , is approximately

$$\eta_{\parallel} = 5.2 \times 10^{-5} \, Z \ln \Lambda / T_e^{3/2} (\text{eV}) \quad \text{ohm-m}, \qquad \ln \Lambda \approx 31 - \ln(n_e^{1/2} / T_e).$$
(36)

Here Z is the ion charge number,  $n_e$  is in m<sup>-3</sup>, and  $T_e$  is in eV. The quantity  $\ln \Lambda$ , called the *Coulomb logarithm*, is unique to plasma physics (Spitzer, 1967). In averaging the scattering of electrons by ions over all velocities and impact parameters h, one chooses the minimum value of h to be that for a 90° deflection (to avoid the singularity at h = 0), and the maximum value of h to be  $\lambda_D$ , to account for Debye shielding.  $\Lambda$  is the ratio  $h_{\text{max}}/h_{\text{min}}$  and  $\ln \Lambda$  varies so slowly that it can be taken as 10 in most cases. The Spitzer resistivity is nearly independent of density and falls rapidly with temperature. At temperatures of 1 keV and above, the value of  $\eta_{\parallel}$  is so small that, according to Eq. (9),  $E_{\parallel}$  must vanish. An ideal, collisionless plasma is a superconductor along **B** and a superinsulator across **B**.

Particles in a fully ionized plasma can move across a magnetic field only by electron-ion collisions. At each collision, the guiding center can be displaced a distance of order  $r_L \propto v_\perp/B$ . In the random walk, the net displacement is proportional to  $r_L^2$ , or  $\propto KT/B^2$ . Because of momentum conservation at each collision, ions and electrons move across **B** at the same rate; hence, there is a transverse diffusion but no transverse mobility. Similar considerations govern the transport of heat (Braginskii, 1963). The flux of either species is given by

$$nv_{\perp} = -D_{\perp}\nabla n, \quad D_{\perp} = \frac{\eta_{\perp}n}{B^2} \sum_{j} KT_{j} \equiv D_{c},$$
 (37)

where  $\eta_{\perp} \approx 2 \eta_{\parallel}$ , differing from  $\eta_{\parallel}$  only in the way different velocities are weighted. The *classical diffusion coeffcient*  $D_{\rm c}$  depends on density, making the diffusion problem nonlinear, and decreases with temperature because of the  $T_{\rm e}^{-3/2}$  dependence of  $\eta_{\perp}$ . The value of  $D_{\rm c}$  is so small that it is rarely observed; plasmas normally escape from magnetic traps by *anomalous diffusion* caused by instabilities.

#### 2.2.6 Collisionless Damping

Plasmas are the only substances in which dissipation of energy can occur without collisions. This phenomenon, discovered by L. Landau (1946) in the course of a rigorous solution of the Vlasov equation for the case of linear electron plasma waves, is called *Landau damping*. An apparent damping of these waves is found when Eq. (12) is Fourier analyzed and solved by contour integration, the damping appearing as an imaginary part of the complex frequency  $\omega$  resulting from the residue around a pole in its complex plane (Jackson, 1960). The physical mechanism was subsequently explained by J.M. Dawson (1961). Some electrons in the distribution (the *resonant electrons*) have a thermal velocity close to the phase velocity of the wave, and those which are in the accelerating part of the wave electric field can stay in synchronism long enough to gain energy from the wave. There are also electrons in the decelerating phase of the wave, but for a Maxwellian distribution these are fewer in number; and, hence, there is a net gain of particle energy and a loss of wave energy. In the absence of collisions, the electrons accelerated by this wave-particle interaction maintain their energies. In carefully designed echo experiments, this energy gain can be reversed, and an echo of the wave can be seen after the wave has damped away.

The plasma dispersion function  $Z(\zeta)$  is defined by (Fried and Conte, 1961)

$$Z(\zeta) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \frac{\exp(-s^2)}{s - \zeta} ds, \quad \operatorname{Im}(\zeta) > 0$$
 (38)

where the integral is to be taken along the real s axis but passing below all the poles of the integrand. For linear plasma waves of frequency  $\omega$  and wavenumber k, solution of the Vlasov for B=0 gives the dispersion relation

$$2k^2\lambda_D^2 = Z'(\zeta), \quad \zeta = \omega/kv_{th} \quad , \tag{39}$$

where  $Z'(\zeta)$  is the derivative of  $Z(\zeta)$  with respect to its argument, and the electron thermal velocity  $v_{th}$  is  $(2KT_c/m)^{1/2}$ . Eq. (39) gives the Landau damping rate

$$\left[\operatorname{Re}(\omega)\right]^{2} \approx \omega_{p}^{2} + \frac{3}{2}k^{2}v_{th}^{2}, \quad \operatorname{Im}(\omega) \approx -\omega_{p}\sqrt{\frac{\pi}{e^{3}}}\left(\frac{\omega_{p}}{kv_{th}}\right)^{3} \exp\left[-\left(\omega_{p}/kv_{th}\right)^{2}\right], \tag{40}$$

which has been confirmed experimentally.

#### 2.3 Plasma Waves and Instabilities

A rich variety of coherent wave motions are possible in a plasma because of its electromagnetic, anisotropic, and particulate nature. When a source of free energy is available, some of these waves can grow spontaneously and are called *instabilities* (Fig. 4). Most waves can be treated with fluid theory, but *kinetic* waves and instabilities involve distortions of the velocity distribution and must be treated with kinetic equations. The most common waves in a plasma with electrons and a single species of singly charged ions are summarized below. These simple results are modified by multiple species, finite boundaries, and damping (Allis et al, 1963; Swanson, 1989; Stix, 1992). For further details, see the article on Plasma Waves, Oscillations, and Instabilities.

#### 2.3.1 Electrostatic Waves

When there is no magnetic field, two electrostatic waves exist: 1) a high-frequency electron plasma (Bohm-Gross) wave with a frequency given by Eq. (40), reducing for  $T_e \to 0$  to a Langmuir oscillation Eq. (14); and 2) a low-frequency ion acoustic wave with a constant velocity  $\omega/k = v_s$ , given by Eq. (20). These waves can also propagate along a background magnetic field  $\mathbf{B}_0$ ; in addition, there are three new waves propagating across  $\mathbf{B}_0$ . At high frequencies, electrons undergo an upper-hybrid oscillation, with  $\omega = \omega_{\text{th}}$ ; at low frequencies there is an electrostatic ion cyclotron wave at  $\omega_{\text{esic}}$ ; and at intermediate frequencies, there is a lower-hybrid oscillation at  $\omega_{\text{th}}$ :

$$\omega_{uh}^2 = \omega_p^2 + \omega_c^2$$
,  $\omega_{esic}^2 = \omega_{ci}^2 + k^2 c_s^2$ ,  $\omega_{lh}^{-2} = \omega_{pi}^{-2} + (\omega_{ce}\omega_{ci})^{-1}$ . (41)

The hybrid frequencies are commonly occurring combinations of intrinsic frequencies. For propagation in a direction  $\mathbf{k}$  at an angle  $\theta$  relative to  $\mathbf{B}_0$ , the two high frequencies are given by the solutions of

$$\omega^2(\omega^2 - \omega_{uh}^2) + \omega_p^2 \omega_c^2 \cos^2 \theta. \tag{42}$$

When these waves are confined to a conducting cylinder of radius a, the oscillating potentials of the eigenmodes have the form  $\varphi_1 = AJ_m(p_{mn}r/a)e^{i(m\theta+\beta t-\omega t)}$ , and their frequencies are given by the frequently used *Trivelpiece-Gould* formula:

$$\beta a = \pm p_{mn} \left[ \frac{\omega^2 (\omega_{uh}^2 - \omega^2)}{(\omega^2 - \omega_p^2)(\omega^2 - \omega_c^2)} \right]^{1/2}, \tag{43}$$

where  $p_{mn}$  is the *n*th root of the Bessel function  $J_m$ . As the axial wavenumber  $\beta$  is increased, the frequency of one branch increases from 0 to the lower of  $\omega_p$  and  $\omega_c$ , while that of the other branch decreases from  $\omega_{uh}$  to the higher of  $\omega_p$  and  $\omega_c$ .

Examples of waves not describable by fluid equations are the electron and ion *Bernstein waves*, whose frequencies are near the harmonics of  $\omega_{ce}$  and  $\omega_{ci}$ , respectively. Since these depend on the FLR effect, they are found in solutions of the Vlasov equation.

#### 2.3.2 Electromagnetic Waves

Light waves and microwaves propagate in an unmagnetized plasma with a frequency

$$\omega^2 = \omega_n^2 + c^2 k^2, \tag{44}$$

corresponding to a dielectric constant

$$\varepsilon = \frac{c^2 k^2}{\omega^2} = 1 - \frac{\omega_p^2}{\omega^2}.$$
 (45)

Transmission is cut off for  $\omega < \omega_p$ . In the presence of a field  $\mathbf{B}_0$ , light waves propagating across  $\mathbf{B}_0$  polarized with  $\mathbf{E}||\mathbf{B}_0||$  also obey Eq. (44) and are called *ordinary* waves. Those with  $\mathbf{E}\perp\mathbf{B}_0$  have the dispersion relation

$$\varepsilon = 1 - \frac{\omega_p^2}{\omega^2} \frac{\omega^2 - \omega_p^2}{\omega^2 - \omega_{uh}^2}.$$
 (46)

and are called extraordinary waves. These waves have a "resonance," where  $\varepsilon \to \infty$ , at  $\omega = \omega_{\text{uh}}$ , and two "cutoffs," where  $\varepsilon \to 0$ , at the right- and left-hand cutoff frequencies  $\omega_R$ ,  $\omega_L$  given by:

$$\omega_R = \frac{1}{2} \left[ (\omega_c^2 + 4\omega_p^2)^{1/2} + \omega_c \right], \quad \omega_L = \frac{1}{2} \left[ (\omega_c^2 + 4\omega_p^2)^{1/2} - \omega_c \right]. \tag{47}$$

Extraordinary waves are reflected at the cutoffs and have forbidden bands for  $0 < \omega < \omega_L$  and  $\omega_{uh} < \omega < \omega_R$ . The absorption of these waves at the resonance is a method for heating a plasma.

Light waves propagating along  $\mathbf{B}_0$  have normal modes which are circularly polarized, with dielectric constants

$$\varepsilon = 1 - \frac{\omega_p^2 / \omega^2}{1 \pm \omega_c / \omega}. \tag{48}$$

Since these two components have different phase velocities, plane-polarized electromagnetic waves along  $\mathbf{B}_0$  exhibit Faraday rotation. The left-hand circularly polarized mode (upper sign) is cut off below  $\omega_L$ , and the right-hand mode is cut off between  $\omega_c$  and  $\omega_R$ . Below  $\omega_c$ , the right-hand mode has a low-frequency propagation band in which the waves are called whistler waves, named after their gliding tones after passing through the earth's ionosphere.

Low-frequency electromagnetic waves along  $\mathbf{B}_0$  are called *Alfvén waves* and have the velocity  $v_A$ , given by Eq. (18). Such hydromagnetic waves propagating *across*  $\mathbf{B}_0$  are called magnetosonic waves, or simply "fast" waves and have a velocity

$$v_{ms}^2 = c^2 \frac{v_s^2 + v_A^2}{c^2 + v_A^2} \approx v_s^2 + v_A^2. \tag{49}$$

This wave is useful for heating a plasma. Another wave used for this purpose is the *electromagnetic ion cyclotron* wave, with a frequency given by

$$\omega_{emic}^{2} = \omega_{ci}^{2} \left[ 1 = \frac{\omega_{pi}^{2}}{c^{2}} \left( \frac{1}{k_{\parallel}^{2}} + \frac{1}{k^{2}} \right) \right]^{-1}, \qquad (50)$$

where k is the total wavenumber and  $k_{\parallel}$  is its component along  $\mathbf{B}_0$ .

#### 2.3.3 Electrostatic Instabilities

All the waves of Sec. 2.3.1 can be excited by beams of electrons or ions whose velocity component in the direction of the wave is near the phase velocity (Mikhailovsky, 1974). Beamplasma interactions can involve any combination of beams and waves. The growth rate of the instability varies typically as the cube root of the beam density. The Buneman instability occurs when the electron and ion fluids of a plasma are both cold and drift through each other.

Current-driven instabilities, on the other hand, involve drifts which are small relative to the thermal velocity. In the *ion-acoustic instability*, an ion wave is excited by the relative motion of a warm electron fluid with a threshold drift velocity which varies from  $c_s$  when  $T_i << T_e$  to  $v_{the}$  when  $T_i \approx T_e$ . In the presence of a magnetic field, the *electrostatic ion cyclotron instability* [cf. Eq. (41)] has a lower threshold. An intermediate case occurs when a warm beam is added to a thermal electron population, resulting in a *bump-on-tail* distribution. Resonant electrons can then excite plasma oscillations by *inverse Landau damping*. In addition to this particle effect, there is also the basic hydrodynamic effect, of which the Buneman instability is the zero-temperature limit.

In crossed electric and magnetic fields, electrons and ions can have slightly different  $\mathbf{E} \times \mathbf{B}$  drifts due to small effects such as collisions with neutrals [cf. Eq. (33)]. When  $\mathbf{E}$  and  $\nabla n_0$  are in the same direction, this drift causes an  $E \times B$  instability which is important in the laboratory and in space (Simon, 1963). As in ordinary fluids, velocity shear gives rise to Kelvin-Helmholtz instabilities (D'Angelo and von Goeler, 1966; Jassby, 1972); in plasmas, these are modified by the magnetic field. Sources of free energy on which instabilities feed can occur without beams or drifts. In plasmas confined by magnetic mirrors, the velocity distribution can by anisotropic, leading to velocity-space or loss cone instabilities (Hasegawa, 1975). Any confined plasma must deviate from complete thermal equilibrium, since it must have a pressure gradient, at least at the edge. This gradient drives drift wave instabilities with a velocity given by  $\mathbf{v}_{De}$  [Eq. (26)], and in the collisionless case these are called universal instabilities (Kadomtsev, 1965).

#### 2.3.4 Hydromagnetic Instabilities

The principal hydromagnetic instabilities (Bateman, 1978; Freidberg, 1987), in which the magnetic field fluctuates, are the Rayleigh-Taylor instability and the kink instability. These are the primary ones to be considered in designing a fusion confinement device. As in hydrodynamics, a Rayleigh-Taylor instability occurs when a heavy fluid is supported by a light fluid. In plasma physics, the heavy fluid is the plasma, and the light fluid is the massless magnetic field. The role of gravity is played by the centrifugal force on particles following curved magnetic field lines in their thermal motions. This instability is also called the gravitational or interchange instability. The ripples which grow have a velocity of the order of  $v_R$  [Eq.(24)] and allow the plasma to escape across **B**. At low densities, this instability is electrostatic in nature. The kink instability is driven by currents along **B**<sub>0</sub>. These currents themselves generate a magnetic field **B**<sub>1</sub> which is perpendicular to **B**<sub>0</sub>, causing the total field to twist into a helix, which then grows. The sausage instability is a special case of the kink instability which has azimuthal symmetry; the field lines are pinched into constrictions resembling the links of a sausage.

#### 2.4 Radiation in Plasmas

Most plasmas in the laboratory and in space have densities below  $10^{21}$  m<sup>-3</sup>, or less than  $10^{-4}$  of atmospheric density. At these densities, the optical depth for most radiation is much larger than the plasma. Consequently, though the particles of a plasma can be in thermal equilibrium, radiation in a plasma is rarely in equilibrium with the plasma and will pass through it freely unless the frequency strikes a particular resonance with a characteristic frequency of the plasma or with an atomic or molecular transition.

#### 2.4.1 Bremsstrahlung

The acceleration of electrons when they collide with ions generates *bremsstrahlung* photons, which have energies of the order of thermal electron energies (Bekefi, 1966). In plasmas with  $KT_e$  in the 100-10,000 eV range, bremsstrahlung is in the soft x-ray regime and passes readily through the plasma. X-ray pinhole cameras are often used for imaging the internal features of the plasma. Further information on this appears under Fusion, Magnetic Confinement.

#### 2.4.2 Synchrotron Radiation

In a magnetic field, gyrating electrons emit synchrotron radiation at  $\omega_c$  and its harmonics, in the submillimeter wave regime. Since the fundamental frequency  $\omega_c$  is resonant with the gyrof-requency of other electrons of any energy, this radiation is easily reabsorbed before it leaves the plasma. The higher harmonics, however, depend on distortions of the circular orbits due to relativistic mass increase or field inhomogeneities, and they can escape reabsorption. Thus the plasma radiates like a black body at  $\omega_c$  and its first few harmonics but is transparent to the high harmonics. The transition harmonic number depends on the details (Dawson, 1981).

#### 2.4.3 Scattering of Radiation

Thomson scattering of laser light occurs when the electric field of the light beam accelerates the electrons of a plasma, and the electrons reradiate in other directions and at different frequencies (Bekefi, 1966). The ratio of scattered power to incident power is proportional to  $r_0^2 n_e$ , where  $r_0 = e^2/mc^2$  is the classical electron radius and  $n_e$  is the electron density. The frequency spectrum of the scattered light depends on the parameter  $\alpha = 1/\Delta k \lambda_D \propto 1/k_i \sin(\theta/2)$ , where  $\Delta k = k_s - k_i$  is the difference between the scattered and incident wave vectors, and  $\theta$  is the angle between them. Incoherent scattering from individual electrons occurs when  $\alpha <<<1$ , since the scattering wavelength is smaller than the Debye length  $\lambda_D$  and does not average over a Debye cloud. The scattered spectrum is then Doppler broadened and yields the electron temperature. Coherent scattering occurs at small angles  $\theta$  when  $\alpha \geq 1$ . Then the scattering wavelength extends over plasma structures, and the spectrum yields the frequencies of plasma waves. The high-frequency part will have a peak at the plasma frequency, and the low-frequency part will give the spectrum of ion acoustic waves.

#### 2.4.4 Diagnostics with Radiation

In spite of the small Thomson cross section of  $6.65 \times 10^{-29}$  m<sup>2</sup>, the large intensities available with Nd-glass or CO<sub>2</sub> lasers makes Thomson scattering a commonly used, non-perturbative diagnostic for temperatures and densities inside a plasma (Lochte-Holtgreven, 1968; Griem and Lovberg, 1970; Sheffield, 1975). In this technique, the frequency spectrum of the scattered light (propagation vector  $\mathbf{k}_s$ ) at an angle  $\theta$  to the incident beam ( $\mathbf{k}_0$ ) yields information depending on the value of the parameter  $\alpha = 1/k\lambda_D$ , where  $k = |\mathbf{k}_0 - \mathbf{k}_s| \approx 2k_0\sin(\theta/2)$  is the wavenumber sampled in the plasma. For  $\alpha << 1$ , the spectrum yields the velocity distribution of indvidual electrons. For  $\alpha >> 1$ , the light probes the cooperative motions of the plasma, including electron plasma

waves, whose frequency yields the density, and ion acoustic waves, whose frequency and damping yield the electron and ion temperatures.

Microwave interferometry (Huddlestone and Leonard, 1965) provides a reliable way to measure plasma density; it is based on the change of the index of refraction given by Eq. (45). Since the waves cannot propagate for  $\omega_p > \omega$ , densities above  $10^{19}$  m<sup>-3</sup> cannot be measured by centimeter waves; and, in fact, millimeter waves must be used for such densities because of refraction. To measure higher densities, far-infrared lasers are used for interferometry with submillimeter waves (Fig. 4). The reflection of far-infrared radiation at the cutoff frequency  $\omega_p$  or  $\omega_R$  [Eq. (47)] can be used in reflectometers for density profiles and fluctuations. Emission of electron cyclotron harmonic radiation gives information on electron temperature. Detection of optical emission gives information based on the intensity ratios of spectral lines and on Stark and Doppler broadening (Lochte-Holtgreven, 1968; Griem and Lovberg, 1970; Selwyn, 1993). For the intense plasmas of laser fusion, X-ray spectrometry is used. Finally, the sophisticated method of laser-induced fluorescence permits measurements of ion velocities and of plasma motions by following tagged metastable ions.

#### 2.5 Nonlinear Phenomena

Plasma methods devised for studying nonlinear phenomena been found useful in other field of applied physics also. A few examples of interesting results are given below.

#### 2.5.1 Nonlinear Waves and Wave-Particle Interactions

When ion acoustic waves reach large amplitude, their sinusoidal waveforms steepen into sawteeth, as predicted by the *Korteweg-deVries* equation (Davidson, 1972). Ion acoustic shock waves have a steep front followed by a wave train and have velocities larger than  $c_s$  (Fig. 5). Single pulses called *solitons* are also possible and have been studied experimentally. Plasmas can support collisionless shock waves of both ion acoustic and Alfvén wave nature. Nonlinear electron plasma waves tend to form *envelope solitons*, or wave packets, described by the nonlinear Schrödinger equation (Hasegawa, 1975). The envelopes can shrink in length and grow in amplitude due to a *modulational instability*. Waves and particles can affect each other through nonlinear interactions (Sagdeev and Galeev, 1969), resulting in, for instance, the generation of harmonics.

#### 2.5.2 Sheaths and Double Layers

The ion-rich sheath at the wall of a discharge has an electron density that falls exponentially, so that the description of such *Debye sheaths* is intrinsically a nonlinear problem. A negatively biased electrode has, in addition, a *Child-Langmuir sheath*, which is well known to be nonlinear. In space plasma physics, it was found that free-standing sheaths, called *double layers*, can be formed away from all boundaries if at least one of the species has a streaming velocity. Such double layers have been reproduced in the laboratory.

The oldest and simplest plasma diagnostic is the electrostatic probe, whose current-voltage characteric gives information on density and temperature, and sometimes even the non-thermal electron components. Based on sheath theory, the theory of probes was first worked out by I. Langmuir. Though Langmuir probes cannot be used in plasmas so hot and dense that they would destroy anything inserted into them, probes are especially valuable for localized, rather than integrated, measurements (Griem and Lovberg, 1970).

#### 2.5.3 Solitons, Cavitons, and Collapse

The study of ion acoustic solitons has led to light-wave solitons in optical fibers, useful for long-range communications. A *caviton* is a density cavity filled with radiation that is trapped because the plasma around it is overdense; that is,  $\omega_p > \omega$ . The radiation pressure, which is called *ponderomotive force* when modified by a plasma, creates the density cavity. A caviton can also be filled with electron plasma waves, which also exert a ponderomotive force. Turbulent Langmuir waves are predicted to precipitate into a number of such cavitons by a process called *collapse* (Zakharov, 1984; Hasegawa, 1975).

#### 2.5.4 Turbulence and Chaos

The growth of unstable waves into nonlinear waves and finally into turbulence was studied first analytically (e.g., Galeev and Sagdeev, 1983), and more recently by plasma simulation on high-speed computers (e.g. White, 1983). Plasma turbulence is quite different from turbulence in fluid dynamics, in which the "inertial range" spectrum postulated by Kolmogoroff to vary as  $k^{-5/3}$  has been observed, k being the reciprocal of the eddy size. Plasma turbulence in strong magnetic fields tends to be two-dimensional, in which case small eddies tend to coalesce into larger ones, instead of vice versa. If an inertial range exists for magnetized plasmas, the energy spectrum can be shown to vary as  $k^{-5}$  (Chen, 1965), but many other spectra have also been predicted depending on the assumed origin of the turbulence (e.g., Zakharov, 1984; Sagdeev et al., 1991). For treating drift wave turbulence, many theoretical techniques have been devised because of its perceived importance in magnetic fusion.

The approach to a state of disorder is not always gradual. Mechanisms such as period doubling or mode coupling can cause oscillations to jump suddenly into and out of a chaotic state with a small change in parameters. Such discontinuous changes are mathematically predicted in the field of *chaos* (Ott, 1993), but it is not clear whether or not these processes are the same as those observed in real plasmas. Abrupt changes from order to disorder were first found in computational studies of the behavior of particle orbits and magnetic field lines in fusion devices (Lichtenberg and Lieberman, 1983) (Fig. 6). This modern field of nonlinear dynamics was initiated in large part by plasma physicists.

#### 2.6 Non-neutral and Strongly Coupled Plasmas

#### 2.6.1 Single Species Plasmas

Pure electron plasmas can be trapped for days in a uniform magnetic field with electrostatic confinement at the ends (a *Penning trap*), there being no unlike-particle collisions to cause diffusion. Since the electron charge is not neutralized by ions, the resulting electric field causes the plasma to rotate, but a stable equilibrium is possible (Davidson, 1974). Many of the features of normal plasmas, such as Debye shielding, are retained in *single-species*, or *one-component*, plasmas. Because of their simplicity, they have been successfully used to study basic phenomena such as conservation laws and the details of Coulomb collisions. Single-species plasmas can also be made with ions or positrons. Ions can be confined also by rf in *Paul traps*. Of particular interest are plasmas of extremely low temperature. Electrons can be cooled by synchrotron radiation, but ions can be cooled much more efficiently with lasers. In laser cooling, a laser is tuned to the Doppler-shifted wavelength of ions which are moving rapidly toward the laser. The radiation pressure of the beam is then preferentially applied to slowing down these fast ions.

#### 2.6.2 Liquid and Solid Plasmas

When a plasma is cooled sufficiently, the number of particles in a Debye sphere,  $N = n\lambda_D^3$ , falls to the order of unity or below, and Debye shielding can no longer occur. This is the regime of *strongly coupled plasmas* (Ichimaru, 1994). The controlling parameter  $\Gamma$ , proportional to the ratio of Coulomb energy to thermal energy, is defined by

$$\Gamma = q^2 / a_s KT$$
,  $(4/3)\pi a_s^3 n = 1$ , (51)

where  $a_s$  is the average interparticle distance. When  $\Gamma \geq 2$ , the thermal energy of the particles is too weak to overcome interparticle Coulomb forces, and the plasma exhibits the short-range order of a liquid. When  $\Gamma$  exceeds 100 or so, the ions fall into a solid lattice. This has been seen in a pure ion plasma. An easier method to form a solid plasma is to use negatively charged particulates or powders, with a typical grain size of 1  $\mu$ m. Since these particles can have as many as  $10^4$  electron charges each, the condition  $n\lambda_D^3 \leq 1$  can easily be achieved. Such plasma lattices have been produced in the laboratory. Since  $n\lambda_D^3$  varies as  $n^{-1/2}$ , strongly coupled plasmas also occur at very high densities, such as in white dwarf and neutron stars and in compressed inertial fusion targets (q.v.). The equation of state of matter under such conditions is an active field of study.

#### 3. Physics of Magnetic Confinement

#### 3.1 Effects of Magnetic Fields on Plasmas

Magnetic confinement of plasmas in space and in small laboratory experiments is usually only partially effective: diffusion across field lines is reduced to the rates given by Eq. (34) for partially ionized plasmas and Eq. (37) for fully ionized plasmas. The physics involved with reduced motion across straight magnetic fields is relatively straightforward and contains few surprises. To trap a plasma so that it cannot escape *along* the field lines, however, is considerably more difficult. This requires specially shaped magnetic fields and elegant methods to suppress the instabilities that such fields engender. Total confinement in complex geometries entails a large number of new concepts. For this reason, the extensive body of knowledge on magnetic confine-

ment is connected primarily with controlled fusion, for which plasma containment in all directions is required.

Two types of magnetic containment geometries are possible: open and closed. Open systems consist of two or more regions of strong magnetic field separated by a weak-field regions. Charged particles in the weak-field region are reflected from the strong-field regions by the magnetic mirror effect (Fig. 3). Particles with small magnetic moment [Eq. (29)], however, can escape through the mirrors at too fast a rate for efficient fusion, and methods for stoppering the ends had to be devised. Closed, or toroidal, geometries have magnetic field lines which do not reach the walls, so that plasma cannot escape along field lines. However, the curvature of the field induces a number of instabilities which destroy the confinement. The problem in closed systems is to devise methods for suppressing these instabilities. (Many books on fusion have been written; several are listed under Further Reading.)

#### 3.2 Magnetic Confinement Fusion

Controlled fusion energy is generated when deuterium and tritium ions in a plasma combine to form a helium nucleus and a neutron. Because of Coulomb repulsion, directed beams of ions will scatter elastically and undergo a fusion reaction only in rare head-on collisions. In a plasma with a Maxwellian distribution of ion energies, however, the mean energy will not be changed by elastic collisions, and fusion events will eventually occur. These are called *thermo-muclear* reactions. An ion temperature of about 10,000 eV is usually required. The electron fluid may be cooler; but since the temperatures will equilibrate in about the same time as needed for the fusion reactions, the electrons will also be hot enough to maintain the high conductivity of the plasma.

The helium "ash" of the D-T fusion reaction consists of 3.5-MeV  $\alpha$ -particles which are confined by the magnetic field. *Ignition* is achieved when the transfer of energy from the  $\alpha$ 's sustains the plasma temperature against losses via radiation and particle escape. Electrical energy is derived from the other reaction product, the 14-MeV neutrons, which are captured in a *blanket* and thermalized to produce heat for a conventional power plant. For net energy gain, the *Lawson criterion*  $n\tau_E \ge 10^{20}$  sec/m³ must be satisfied, where n is the plasma density and  $\tau_E$  is the energy confinement time given by the plasma energy content divided by the power input. For toroidal systems, n is typically  $10^{20}$  m³ and  $\tau_E$  is typically 1 sec at this *breakeven* condition. Since high ion temperature is also required, the triple product  $T_i n\tau_E$  should exceed  $10^{21}$  keV-sec/m³. References and details on this subject can be found under Fusion, Magnetic Confinement and Fusion Technologies.

#### 3.3 Particle and Energy Transport

In principle, plasma can escape from a closed system only at the classical diffusion rate given by Eq. (37). Under fusion conditions the particle confinement time  $\tau_p$  would then be of the order of hours. In practice, confinement times as long as 1 sec are difficult to achieve because of anomalous diffusion caused by instabilities and field errors. Kinetic energy is also transported

across **B** in Coulomb collisions. Classically, the heat flow  $q_e$  of electrons, for instance, should be proportional to  $n_e \nabla T_e$ :

$$q_e = \chi_e n_e \nabla T_e \,, \tag{52}$$

where the heat diffusivity  $\chi_e$  has the same  $n_i T_e^{-1/2} B^{-2}$  dependence as does  $D_{\perp}$  [Eq. (37)]. In tokamaks, the measured value of  $\chi_l$  agrees approximately with this prediction when it is modified by the neoclassical effects described below, but  $\chi_e$  is highly anomalous. Not only is it large, but it can depend on  $\nabla T_e$  itself, and its dependences on  $T_e$  and B can be inverted. In view of this rapid transport, the loss of energy by radiation can be neglected. The energy confinement time  $\tau_E$ , which should then be proportional to  $1/\chi_e \propto T_e^{1/2} B^2/n_i$ , is actually found to increase with n and decrease with n Much of magnetic fusion physics concerns the explanation of such anomalies through the study of instability and turbulence.

#### 3.4 The Energy Principle

To enumerate the possible instabilities in the complicated magnetic geometries required for confinement would be an impossible task. Fortunately, the discovery of an energy principle for hydromagnetic stability (Bernstein *et al.*, 1958) reduces the distinction between stability and instability to evaluating the sign of a quantity  $\delta W$ , which represents the change of the potential energy of the magnetic field-plasma system when the fluid is given a displacement  $\xi(\mathbf{r})$ . If the field **B** and pressure p are known everywhere in the volume V,  $\delta W$  in its simplest form is given by the integral

$$\delta W = \frac{1}{2} \left[ \left[ \mathbf{Q}^2 + (\nabla \times \mathbf{B}) \cdot (\boldsymbol{\xi} \times \mathbf{Q}) + (\nabla \cdot \boldsymbol{\xi}) (\boldsymbol{\xi} \cdot \nabla p) + \gamma p (\nabla \cdot \boldsymbol{\xi})^2 \right] d^3 V , \tag{53} \right]$$

where  $\mathbf{Q} = \nabla \times (\mathbf{\xi} \times \mathbf{B})$ . Stability requires  $\delta W > 0$  for all trial functions  $\xi(\mathbf{r})$ .

#### 3.5 Toroidal Confinement

Bending straight magnetic field lines into circles, thus forming a torus (pictured here as a horizontal doughnut), would prevent plasma from escaping along **B**. Unfortunately, the field would then be nonuniform, and the gradient-B drifts [Eq. (23)] would separate the ions and electrons, causing a vertical electric field which drifts the plasma outwards by the drift  $v_E$  [Eq. (21)]. Toroidal devices must be designed to have twisted, or helical, B-lines so that the particles, in their thermal motion along **B**, experience an oscillating gradient-B drift with zero average. There are three general types of toroidal devices. Tokamaks employ a plasma current to add a poloidal B-field (around the minor axis) to the basic toroidal field generated by external coils. Helical devices employ currents in windings outside the plasma to generate the poloidal component. Stellarators, torsatrons, heliacs, and spherators are examples of helical devices. If a closed configuration has no hole in the center and all the fields are generated by currents in the plasma, it is a compact torus.

#### 3.5.1 Rotational Transform

Let  $\theta$  be the azimuthal angle at which a given helical field line crosses a given cross section of the torus. After going once around the torus, this field line will have an azimuth angle  $\theta + \Delta \theta$ . The rotational transform  $\iota$ , defined as the average value of  $\Delta \theta$  over many traverses, is a measure of the amount of twisting. For tokamaks, the reciprocal quantity  $q = 2\pi/\iota$  is known as the *quality factor*. Kink instabilities will occur if the plasma current is large enough to give  $\iota > 2\pi$ , or q < 1, a condition known as the *Kruskal-Shafranov limit*. The value of q generally varies with minor radius r. At most radii, q is an irrational number, and the field lines cover an inner torus, called a *magnetic surface*, without ending. In steady state, a perfectly conducting plasma will have  $\nabla_{\parallel}p = E_{\parallel} = 0$ , according to Eqs. (8) and (9), so that such magnetic surfaces are surfaces of constant pressure and potential. Particle drifts [Eqs. (21) and (23)] cannot cross these nested surfaces. However, at radii such that q(r) is a rational number, the field lines close upon themselves and form *magnetic islands* (Fig. 6). Overall hydromagnetic stability of toroidal configurations depends on the existence and positions of these *rational surfaces*, and hence on the profile of q(r).

#### 3.5.2 Instabilities

In a perfectly conducting plasma, in which  $E_{\parallel}=0$ , the gravitational and kink instabilities discussed in Sec. 2.3.4 can be controlled by shaping the magnetic field. Unfortunately, a small resistivity  $\eta$  suffices to invalidate the  $E_{\parallel}=0$  assumption and allow growth of waves localized within a magnetic surface. The primary resistive instability is the *tearing mode*. Modes localized to the outside regions of a torus where the effective "gravity" is strong are called *ballooning* modes. The drift-wave instability discussed in Sec. 2.3.3 is believed to be the cause of electrostatic turbulence, leading to anomalous diffusion. Drift waves can be made unstable not only by finite resistivity but also by the trapping of particles in banana orbits (Sec. 3.5.4); these are called *trapped particle* instabilities. The  $\eta_i$  instability is a drift wave driven by ion temperature gradients. The  $\alpha$ -particle product of fusion reactions is predicted to drive Alfvén waves unstable. The number of known instabilities is large (Wesson, 1978), but fortunately only a few in practice are deleterious to plasma confinement.

#### 3.5.3 Stabilization Methods

The two primary methods for suppression of instabilities are *shear* and *minimum-B*. Magnetic shear occurs when q(r) varies with radius, so that the B-lines have different pitch angles on each magnetic surface. This has the effect of scrambling unstable displacements and suppressing them. A minimum-B configuration is one in which the magnitude |B| is larger on the outside of the plasma than on the inside, forming a potential hill that unstable motions must overcome. These field lines bulge inwards toward the plasma rather than outwards. A similar benefit accrues if only the average of |B| along a field line has an interior minimum, such configurations have minimum-average-B, or minimum- $\overline{B}$ .

#### 3.5.4 Banana Orbits

Let a magnetic surface intersect a cross section of the torus in a circle. As a particle's guiding center moves along a field line on this magnetic surface, the points at which it passes a given cross section should eventually cover the entire circle. However, particles with large magnetic moments will be reflected from regions of strong field by the magnetic mirror effect [cf. Sec. 2.2.4] and will not be able to reach the inside part of the cross section, near the major axis, where the field is strong. The guiding center will then trace out not a circle but a banana-shaped path localized to the outside of the cross section. The particle is then a trapped particle. We define the aspect ratio of a torus as A = R/a, where R is the major radius and a the minor radius, and the inverse aspect ratio as  $\varepsilon = a/R$ . The fraction of trapped particles is proportional to  $\varepsilon^{1/2}$ . The width of the banana orbits is  $(B_t/B_p)r_L$ , where  $B_t$  and  $B_p$  are respectively the toroidal and poloidal field components, or approximately the Larmor radius evaluated with the poloidal field. The steplength for diffusion is therefore increased by the factor  $\varepsilon^{1/2}(B_t/B_p)$  (Kadomtsev and Pogutse, 1971).

#### 3.5.5 Neoclassical Diffusion

Because of the toroidal effects described above, transport by classical Coulomb collisions is increased even without instabilities. The resulting "neoclassical" diffusion coefficient has three ranges as the collision frequency  $\nu$  is increased (Hinton and Hazeltine, 1976). In the banana diffusion regime (small  $\nu$ ),  $D_{\perp}$  is given by

$$D_{\perp} = D_{c} q^{2} / \varepsilon^{3/2} = q^{2} r_{L}^{2} v / \varepsilon^{3/2}.$$
 (54)

In the large-v limit, diffusion is governed by the Pfirsch-Schlüter coefficient

$$D_{PS} = (1 + 2q^2)D_c \cong q^2 r_L^2 v. \tag{55}$$

Here the enhancement over  $D_c$  [Eq. (37)] is caused by the added electron-ion friction as electrons flow along **B** to cancel their gradient-B drifts, as described above. At intermediate collisionalities,  $D_{\perp}$  is independent of v; in this *plateau* regime,  $D_{\perp}$  is given by

$$D_{\perp} = q r_L^2 v_{th} / R \,. \tag{56}$$

#### 3.5.6 Bohm and Gyro-Bohm Diffusion

Observed losses are much faster than neoclassical. In the presence of electrostatic turbulence, the guiding centers of ions and electrons suffer random  $\mathbf{E} \times \mathbf{B}$  drifts, and the resulting diffusion can be shown to scale as  $KT_e/B$ . The infamous Bohm diffusion coefficient contains an empirical factor of 1/16:

$$D_B = \frac{1}{16} \frac{KT_e}{eB} \,. \tag{57}$$

This scaling holds for both random diffusion and steady convection as long as the eddy sizes are scaled to the plasma radius. Improvements in stabilization have reduced  $D_{\perp}$  to below this value, at least for electrons. If the correlation lengths are scaled instead to the ion Larmor radius, as is

predicted for certain instabilities such as the *ion temperature gradient instability*, the diffusion coefficient would have a smaller value, called the *gyro-Bohm* coefficient:

$$D_{GB} = \frac{r_{Li}}{a} \frac{KT_e}{eB} \,. \tag{58}$$

Electron transport has been observed to follow this scaling law, which recovers the  $1/B^2$  dependence of classical diffusion but is still much larger in magnitude.

#### 3.6 Tokamak Physics

#### 3.6.1 Fields and Currents

A tokamak (Furth, 1981) has at least three sets of coils: TF (toroidal field) coils, which encircle the doughnut and produce the primary toroidal field; VF (vertical field) coils, which provide a  $\mathbf{j} \times \mathbf{B}$  force to prevent expansion of the torus; and OH (ohmic heating) coils (or a transformer) for inducing the toroidal current. In addition, there are often field shaping coils. The all-important q-profile is determined by the radial distribution of the toroidal current j(r). The ohmic dissipation of this current heats the plasma in the absence of auxiliary heating. Since the resistivity is sensitive to electron temperature [Eq. (36)], q(r), j(r), and  $T_e(r)$  are related to one another. The value of q typically varies from below 1 on the minor axis to above 3 at the periphery. The current tends to be concentrated on reentrant field lines, for instance on the q = 2 or 3 surfaces, generating localized poloidal fields. These form magnetic islands which degrade the confinement.

#### 3.6.2 Sawteeth and Disruptions

Sawtooth-shaped fluctuations in n and  $T_{\rm e}$  are invariably seen near the q=1 surface. Inside this surface, the plasma is unstable and turbulent because the Kruskal-Shafranov limit is not satisfied. Heat and density are redistributed by the sawtooth oscillations; this is a mechanism for the plasma to readjust j(r) to regain stability. This self-regulation mechanism is not available to stellarators, where  $\iota(r)$  is primarily fixed by external currents. In contrast to sawteeth, disruptive instabilities occur unpredictably and are believed to be caused by the growth and overlapping of magnetic island chains. A major disruption stops the plasma current completely and brings the plasma to the wall.

#### 3.6.3 Auxiliary Heating

Ohmic heating can be supplemented in four main ways. In ECRF (electron cyclotron range of frequencies) heating, microwaves from powerful gyrotrons are used to generate the extraordinary wave [Eq. (46)], which is absorbed at the upper hybrid resonance. In ICRF (ion cyclotron range of frequencies), radiofrequency is applied from antennas to generate the fast wave [Eq. (49)]. In lower hybrid heating (LHH), an intermediate frequency is applied by phase grid arrays to excite the lower hybrid resonance [Eq. (41)]. The most successful method has been neutral beam injection (NBI) heating, in which 100-200 kV beams of neutral deuterium atoms are injected across the magnetic field. The atoms are then ionized by the plasma and trapped. The

resulting fast ions transfer their energy first to electrons, and then to the ions. In fusion reactors, these fast ions enhance the fusion rate, since they pass through the region of maximum fusion cross section as they slow down. The current of these fast ions significantly modifies the plasma equilibrium, depending on *co-injection* (in the OH current direction) or *counter-injection* (opposite) and can cause new instabilities.

#### 3.6.4 H-Mode Confinement

With neutral beam injection, it was found that the plasma sometimes makes a spontaneous transition to a new mode of confinement in which the density and potential gradients at the edge of the discharge become extremely steep. When this happens, the oscillation level drops near the edge, presumably because instabilities are suppressed by the large, sheared electric field there. There is then a transport barrier at the edge which increases the overall confinement time by about a factor of two. The details of the edge physics are still under study.

#### 3.6.5 Current Drive

Since the toroidal current in a tokamak must be induced by a transformer, with or without an iron core, the length of a current pulse is limited by the number of volt-seconds available. Helical systems do not require an internal current and can be operated cw. Currents can be driven without transformers by using the principle of Landau damping [Sec. 2.2.6], by which electrons are accelerated by radiofrequency waves excited in the plasma. In toruses in which plasma is escaping with a radial velocity  $v_r$ , there is an automatic current drive due to the  $-ev_r \times B_p$  force on the electrons. Since this is always in the direction of the current creating  $B_p$ , it is called the *bootstrap current*. Steady-state operation of tokamaks may be possible using the combination of bootstrap current and rf current drive.

#### 3.7 Magnetic Mirrors

#### 3.7.1 Loss Cone

A simple magnetic mirror field can be produced by two separated circular coils with currents in the same direction. The B-field along the common axis is strongest at the coils (B =  $B_{max}$ ) and weak at the midplane between them (B =  $B_{min}$ ). The mirror ratio  $R_m$  and the loss cone angle  $\theta_m$  are defined by Eq. (59):

$$R_m = B_{\text{max}} / B_{\text{min}} , \quad \sin \theta_m = 1 / R_m$$
 (59)

Particles with  $v_{\perp}/v_{\parallel} < \tan \theta_{m}$  at the midplane cannot be reflected at the mirror throats and are lost. Particles diffuse in velocity space into the loss cone by collisions. Since electrons have a large collision frequency, they are lost rapidly, creating an ambipolar potential which accelerates ions through mirror. Thus, the ions are driven out with a finite drift velocity. In the limit of large  $R_{\rm m}$ , the confinement time is proportional to  $\ln R_{\rm m}$ . The loss rate from a simple mirror is too rapid to satisfy the Lawson criterion [Sec. 3.1].

#### 3.7.2 Instabilities

Since the field lines of a simple mirror curve outwards instead of inwards, the configuration is subject to the Rayleigh-Taylor gravitational instability [Sec. 2.3.4]. Early experiments were stabilized by *line-tying*, the short-circuiting of electrostatic potentials by conducting plates on which the field lines end. Mirrors can easily be stabilized by the addition of a quadrupole field applied by *loffe bars*, which are four equally spaced current-carrying conductors parallel to the axis, placed outside the plasma. Above a critical current, these bars convert the mirror into a minimum-B configuration, with a dramatic increase in stability (Post, 1981). It is common to combine the Ioffe bars with the mirror coils into a single conductor, called a *baseball coil*, which has the shape of the seam on a baseball. For ease of assembly, the baseball coil can be split into two halves, which together are called a *yin-yang* coil. In addition to hydromagnetic instabilities, magnetic mirrors are subject to velocity-space instabilities due to the absence of particles in the loss cone. The most dangerous of these is the drift cyclotron loss cone (DCLC) instability, which can be stabilized by adding a cool but isotropic plasma to the hot plasma (Baldwin, 1977).

#### 3.7.3 Tandem Mirrors

Even after instabilities have been brought under control, magnetic mirrors still suffer from large end losses. To minimize these losses, it is necessary to build a tandem mirror (Fowler, 1981). This has a large central cell with a weak, uniform B-field connected to a mirror with strong B-field at each end. These "plugs" have yin-yang coils to provide MHD stability and must be filled with hot, dense plasma. Their volumes, however, are relatively small. In addition, the injection of ion beams and ECRF power into the plugs produces a specially tailored potential distribution which, in one section, electrostatically confines the ions and, in another section, forms a thermal barrier for the electrons. These devices for plugging the end losses have been successfully demonstrated, but they detract from the basic simplicity of the mirror concept.

#### 4. Physics of Inertial Confinement

#### 4.1 Laser Fusion

In laser fusion, a small pellet of frozen deuterium-tritium fuel is suddenly heated by laser or particle beams impinging symmetrically on the surface, turning it into a hot plasma. As the plasma expands, its momentum is transferred to the solid part of the pellet, compressing it in a violent implosion to a density between  $10^3$  and  $10^4$  times solid density (Duderstadt and Moses, 1982). The fusion rate at these densities is high enough to give a net energy gain in the short time, of the order of picoseconds, during which this compressed state is maintained by inertia. The Lawson criterion of Sec. 3.1 can be expressed here as  $\rho R > 3$  g/cm<sup>2</sup>, where  $\rho$  is the density and R the radius in the compressed state. Here, the range of the  $\alpha$ -particles produced is so small that their energy is efficiently used in keeping the plasma at fusion temperature. In practice, the pellets are hollow shells covered with various layers of absorbers, a small piece of fuel at the center is ignited first, and the  $\alpha$ -particles from this are used to ignite the rest of the fuel in a propagating burn. In laser fusion, there are two approaches. The laser energy can be directed at the fuel pellet (direct drive), or it can be directed through the ends into a tiny high-Z canister, in the cen-

ter of which the fuel pellet is mounted (*indirect drive*) (Yamanaka, 1991). The laser light is converted into X-rays, and the latter are used for heating the pellet symmetrically. *Inertial confinement* is a generalization of the compression-ignition process to include drivers other than lasers (see Sec. 4.4). A more detailed treatment of this subject can be found under Fusion, Inertial Confinement.

#### 4.2 Parametric Instabilities

The hope that inertial fusion would be free from the problems of plasma instabilities that plagued magnetic confinement was not realized: laser fusion had instabilities of its own, called parametric instabilities (Nishikawa, Liu, Kaw, and Kruer, 1976). Ideally, laser light of frequency  $\omega_0$  should penetrate the plasma cloud up to the critical density  $n_c = \varepsilon_0 m \omega_p^2/e^2$  where  $\omega_0 = \omega_p$ [Eq.(14)]. At that point the laser energy would be absorbed by collisional damping (inverse bremsstrahlung) and by the generation of plasma waves which are subsequently Landau damping (resonance absorption). Unfortunately, reflective instabilities occur in the underdense ( $n < n_c$ ) region, preventing the absorption of the light. Furthermore, these instabilities generate fast electrons which can more forward into the solid fuel and preheat it, making it harder to compress. In stimulated Brillouin scattering (SBS), the laser light excites an ion acoustic wave, which then acts as a grating to reflect the laser beam at a red-shifted frequency. Stimulated Raman scattering (SRS) is similar except that an electron plasma wave is generated, and the redshift is of order  $\omega_{\rm p}$ . If the plasma wave has  $k\lambda_D \leq 1$  so that it is strongly Landau damped, the induced scattering from individual electrons is called *stimulated Compton scattering*. The laser light can also decay into two plasma waves in a two-plasmon decay instability occurring at the quarter-critical layer, where  $n = n_c/4$ . The *filamentation* instability can divide the laser beam into smaller beamlets, thus reducing the uniformity of power deposition. Fast electrons accelerated out from the pellet have such large current densities that they can generate multi-megagauss magnetic fields, which degrade the smoothing effect of heat conduction. These parametric effects can be minimized by increasing  $\omega_0$ , since the critical layer is then moved to higher densities, where classical collisions are more dominant. Nd-glass lasers are used for the 100-TW power levels required, the 1.06-µm (red) fundamental being frequency doubled to 0.53 µm (green) for diagnostics and tripled to 0.35 um (blue) for compression. Though the conversion efficiency in crystals can be of order 75%, frequency-quadrupled light at 0.25 µm (UV) would require unusual optical materials. In indirect drive, the X-rays have such high frequency that parametric instabilities are not a problem, but the light still has to pass through a low-density plasma before reaching the metal canister.

#### 4.3 Rayleigh-Taylor Instability

After parametric instabilities, the most important remaining instability is the Rayleigh-Taylor instability (Bodner, 1991). As the pressure of the ablating plasma pushes on the solid wall of the pellet, small irregularities in the surface or the driving force create growing ripples which prevent symmetric compression. Large aspect pellets which have an outer shell that is thin compared with its radius are more easily imploded, but they are also more susceptible to R-T instability. A beneficial effect not available in magnetic fusion is *vorticity shedding*, in which the escaping plasma carries away some of the instability energy. In direct drive, the laser energy must be divided into many beams to spread out the driving force uniformly over a sphere. In indirect drive, the X-ray *hohlraum* is supposed to provide a uniform pressure, but the laser illumination of

the canister and the blockage of X-rays escaping from the ends of the cylinder have to be adjusted carefully.

#### 4.4 Driver technology

Glass lasers are exceptionally inefficient and cannot be pulsed rapidly because of heat dissipation in the glass. For fusion power production, therefore, other drivers are necessary. The KrF laser, with 0.3 µm wavelength, is a possible alternative, being a gas laser that can run cw, but its development is in a relatively early stage. Beams of light ions, such as Li, can be produced by pulsed power [cf. Sec. 6.2 below] and directed toward a larger target than in laser fusion, heating the surface in the same way. The main problem is the focusing and transport of the ion beam from the diode structure in which it is produced to the target through an evacuated space which must also accommodate the neutron blanket. In light-ion fusion, the pulse length, energy per pulse, target size, and fusion energy per shot are all scale upward from the laser case. The difficulties with transport are overcome in heavy-ion drivers. In heavy-ion fusion, beams of ions such as uranium are accelerated to MeV energies by particle accelerator techniques. As in high-energy physics, the beams can be transported and focused to small radii. The energy deposition in the target is sharply concentrated at the end of the ions' range. New types of linear accelerators are being developed for these drivers.

#### 5. Plasmas in Space

#### 5.1 Plasmas in the Solar System

That plasmas exist naturally in outer space has long been apparent from displays of the Aurora Borealis and from disruptions of shortwave radio communications, including the mysterious glide tones now called whistler waves (Helliwell, 1965). The exploration of the environments of the earth and other planets with spacecraft has brought not only an understanding of these phenomena, but also the realization that the sun and planets form an electromagnetic system controlled by the laws of plasma physics. The dipole magnetic field of the earth traps ions and electrons coming from the sun by the magnetic mirror effect (Sec. 3.1.5), forming the Van Allen Radiation Belts. The plasma density in this ionosphere rises from zero in the atmosphere, where the neutral gas cools the plasma so that it recombines readily, to a peak of the order of 10<sup>5</sup> cm<sup>-3</sup>, and then falls to the order of 1-10 cm<sup>-3</sup> in interplanetary space. Occasionally, electromagnetic disturbances drive energetic electrons into the atmosphere, exciting the oxygen atoms to emit the light of the aurora. A thunderstorm in the southern hemisphere can generate electromagnetic plasma waves which are channeled along the earth's magnetic field to give whistling sounds detected by radio near the north pole, since the higher frequencies travel faster. A stream of charged particles from the sun, called the solar wind (Brandt, 1970), populates the ionosphere and controls its behavior. A solar storm can cause the ionosphere to fluctuate and disrupt its normal reflection of communications signals. The ionospheric plasma can even be manipulated by man, using powerful radiofrequency generators at the plasma frequency.

Outside the ionosphere, the plasma is nearly collisionless and is therefore frozen to the magnetic field. Since this is a high-beta plasma [beta is defined in Eq. (19)], the magnetic field

cannot hold the plasma; rather, the field lines are shaped by the plasma motion. In particular, the plasma of the solar wind pushes the dipole field of the earth in to a long tail on the night side, called the *magnetotail*. The field lines of the earth's field are joined to those of the interplanetary magnetic field discontinuously at a boundary called the *magnetopause*, which encloses the *magnetosphere*. The discontinuity is possible because of sheet currents in the plasma. As the earth orbits the sun, it can be viewed as plowing through the interplanetary magnetic field, dragging its own plasma and magnetic field with it, the field lines adjusting themselves at every instant by the process of *reconnection*. Since the earth's velocity is supersonic in the interplanetary plasma, a collisionless shock front, called the bow shock, forms ahead of the earth.

For further information and references, see the article under Ionosphere and Magnetosphere.

Each planet has a different plasma environment depending on its magnetic field and ionosphere. The magnetosphere of Jupiter has been explored extensively and is particularly interesting because of its major satellites, which have their own magnetic fields and can even inject particles into Jupiter's magnetosphere through volcanic action. The rings around Saturn have features that may depend on plasma effects. For instance, it has been proposed that the radial spokes in these rings may be caused by charged dust that orbits over the poles of the planet. Comets can often be seen to have two tails, one blown by the sun's photons and the other by the solar wind. The dust cloud that is ejected from a comet as it nears the sun can be ionized if it collides with an existing plasma at greater than the *critical ionization velocity*. Dusty plasmas are of importance not only in comets and planetary rings, but also in plasma processing (see Sec. 7.2).

#### 5.2 Plasmas in the Sun

The sun is a fusion reactor with gravitational plasma confinement. The heat which is generated at the core is convected out to the surface by turbulent motions. In a manner not yet understood, these motions generate complicated magnetic fields, which control the appearance of the sun and its interaction with the earth via the solar wind. Some magnetic lines leave the sun's surface and loop back, forming the north and south poles of a sunspot pair. Solar prominences reveal the presence of magnetic loops further out. Some field lines do not loop back but reach out into space through holes in the corona, carrying the solar wind with them. The corona itself is extremely hot (Fig. 7). The mechanism of coronal heating and the dynamo which creates the magnetic fields are unsolved problems. For further information and references, see the article on Solar Radiation.

## 5.3 Magnetic Field Reconnection

Since the particles in a collisionless plasma gyrate around magnetic field lines and are effectively "frozen" to them, it is not possible to change the magnetic configuration without a mechanism for particles to jump from one field line to another. This problem of magnetic reconnection or field annihilation arises both in space plasma physics (Fig. 8) and in magnetic confinement fusion (Sec. 3.1.4.2). If two oppositely directed field lines come together to cross each other at an X-point, entraining plasma with them, eventually the plasma is squeezed into a thin layer in which the density is high enough for collisions to occur. Collisions permit particles to cross field lines, and so the field lines can lose their identity momentarily and reconnect. In this process, some of the magnetic energy is converted into kinetic energy, and acceleration of particles can occur. Reconnection is independent of collisionality, because the thickness of the inter-

face layer can adjust itself to guarantee a sufficiently high density. A peculiarity of plasmas is that reconnection can occur even if the collision rate is identically zero, because collisionless processes such as Landau damping or instabilities can provide an effective resistivity.

## 5.4 Plasma Astrophysics

Hydrogen atoms are photoionized in interstellar space by starlight, and the plasma does not readily recombine when the density is of order 1 per cm<sup>3</sup>. The Debye length is nonetheless much shorter than the dimensions of the system, so that interstellar and intergalactic space is filled with true plasmas. The motions of magnetic field lines are subject to the same constraints as in the solar system. The presence of magnetic fields can be seen in the Crab Nebula, in which the organization of gas into filaments can only be due to magnetic fields (Fig. 9). The directions of these fields can be measured from the polarization of the synchrotron radiation emitted by the magnetized electrons.

Quasi-stellar objects such as neutron stars, quasars, pulsars, and black holes involve plasmas of extremely high density and magnetic fields as high as  $10^{12}$  G. Radioastronomy has revealed the presence of extended radio sources and astrophysical jets, which also emit plasma radiation. When an object with a large magnetic field spins, possibly with plasma streaming from the poles, there will be a radius at which the field lines will be rotating at the speed of light. Relativistic astrophysics is the name of the science of such phenomena, but the concepts of plasma physics are deeply imbedded in this science. The acceleration of cosmic rays can be attributed to these objects, but also to other phenomena such as field annihilation and magnetic mirroring.

On a larger scale, the formation of galaxies (Fig. 10) has been treated as a plasma problem, with the particles of a plasma replaced by stars and the electromagnetic force replaced by gravity. Spiral arms can thus be considered to grow as a result of an instability. The formation of stars from gas and dust is a problem in plasma hydrodynamics involving dusty plasmas, which also occur in the solar system and in industrial plasmas (Secs. 5.1 and 7.2). For further information and references, see the article on Astrophysics.

## 6. Light and Particle Beams

#### 6.1 Coherent Radiation Generators

#### 6.1.1 Microwave Sources

Originally developed for radar, microwave generators are now used by everyone in personal communications, television, and microwave ovens. For the scientist, microwave sources are useful in particle accelerators, controlled fusion, and spectroscopy of organic substances. Coherent radiation is generated by electron beams, which are basically single-species plasmas, and their motions, space charge effects, and instabilities are described by plasma theoretical techniques. In a *klystron*, an electron beam is shot through two resonant cavities. The electric field in the first cavity modulates the electrons' velocities so that they become bunched. The bunched beam then passes through the second cavity and excites the microwaves in it. A portion of this energy is fed

back to drive the first cavity, and the rest is the output energy. In a traveling wave amplifier (TWT), a slow-wave structure such as a coil surrounds the beam and propagates electromagnetic waves with approximately the electrons' velocity. A beam-plasma instability occurs, with the plasma replaced by the slow-wave structure, and the wave on it is amplified. In both cases, radiation is generated at the expense of electron energy. In a magnetron, a magnetic field causes the electrons to drift at the  $E \times B$  velocity. Magnetrons at 2.45 GHz are mass produced economically and are used in microwave ovens. In a gyrotron, the electron beam travels both perpendicular and parallel to a magnetic field, and they are bunched in their gyro-motions. The radiation is near the cyclotron frequency. To produce higher frequencies at a reasonable magnetic field, harmonic gyrotrons operate at high multiples of  $\omega_c$ . Gyrotrons can produce prodigious power in the MW range for use in heating fusion plasmas. The technology of gyrotrons has also been used in isotope separation and materials processing. A recent variant is the cyclotron autoresonant maser (CARM), in which the electrons' parallel velocity is larger than their perpendicular velocity, providing an energy source for maintaining the proper relativistic mass to stay in cyclotron resonance.

#### 6.1.2 Free Electron Lasers

The microwave sources mentioned above are generally not tunable over a large range of frequencies. Tunability is the distinguishing feature of the free electron laser (FEL) (Roberson and Sprangle, 1989). In an FEL, a relativistic electron beam and a beam of electromagnetic radiation are injected together into a channel in which there is a spatially alternating magnetic field (a wiggler or undulator). The combined fields cause the electron beam to oscillate and generate radiation at higher frequency and higher power. Tunability is achieved by varying the wiggler wavelength or the electron energy, since the frequency upshift is proportional to  $\gamma^2$ , where  $\gamma$  is the relativistic Lorentz factor. Thus, FELs are useful not only for applications where tunability is essential, such as spectroscopy, but also for generating short wavelength radiation. Powerful FELs have been proposed for defense purposes. When the density of electrons is so large that  $k\lambda_D \leq 1$  in their rest frame, microwave devices are said to operate in the Raman regime, but no such devices have yet been made practical.

#### 6.2 Pulsed Power

### 6.2.1 Relativistic Electron Beams

Extremely intense beams of electrons can be produced by charging a room-sized array of capacitors immersed in oil (a *Marx bank*) and switching the voltage to a pulse-forming transmission line (a *Blumlein*) which ends in a diode (Miller, 1982). By charging the capacitors in parallel and connecting them in series via spark gaps before discharging, voltages of 0.1-10 MV can be achieved. The Blumlein is usually insulated by pure water, which can maintain a high dielectric constant ( $\approx 89$ ) for short pulses, and can deliver  $\approx 80$  nsec pulses of order 1-20 MA, depending on whether the device is designed for high or low impedance. The relativistic electron beams (REBs) emitted by the diode can carry powers up to the order  $10^{13}$  W. The beam density is large enough to prevent propagation in a vacuum; a gas fill is normally used to provide the ions needed for space-charge neutralization. The beam current also has to be neutralized, since it is beyond the *Alfvén limit*, at which the Larmor radius of the electrons in their self-generated magnetic field

is smaller than the beam radius. A return current of dense but slow electrons flows backwards in the beam to cancel the magnetic field. REBs can be used to study the behavior of matter under extreme energy conditions, such as in nuclear weapons tests, and to generate multi-megagauss magnetic fields (Turchi, 1980).

## 6.2.2 Light Ion Beams

By reversing the polarity of the diode in a pulsed power machine and introducing a source of light ions such as plastic fibers, it is possible to produce intense ion beams up to the order of 1 MA. Magnetic insulation can be used to prevent the electrons from short-circuiting the diode. Light ion beams are studied as alternate drivers for inertial confinement fusion (q.v.). For this purpose, circular arrays of tens of pulsed power generators have been built to concentrate the power onto a fusion target (cf. the article on Fusion, Inertial Confinement).

### 6.2.3 Pinches, Switches, and Exploding Wires

The advent of pulsed power opened up research on different types of single-shot devices. A Z-pinch is a linear, high-current discharge whose self-magnetic field is sufficient to confine, or even pinch, the plasma radially. Such a plasma is known to be unstable to kink or sausage instabilities, but useful radiation, for instance for x-ray lithography, can be created before breakup. A Marx bank discharged through a solid deuterium fiber or an imploding metal cylinder has been used to study the possibility of a simple fusion device that requires no external coils. Pulsed discharges through solid wires or wire arrays are used to generate short pulses of light or pressure. Fast-opening switches that can transfer large currents have a number of uses including the charging of an inductor for inductive energy storage (Früngel, 1976). A dense plasma generated by pulsed power can be used for this purpose. (See also the article on Sparks, Arcs, and Other Electric Discharges.)

#### 6.3 Plasma Accelerators

New ideas from plasma physics have been applied to the technology of high-energy accelerators used for fundamental particle physics research and for synchrotron light sources. For instance, the *modified betatron* adds a toroidal magnetic field to the betatron accelerator, and the *inverse free-electron laser* (IFEL) accelerates a beam of electrons using a laser beam and an undulator. Conventional accelerators driven by microwave sources are limited to acceleration rates of the order of 100 MeV/m by voltage breakdown. Since a plasma is already ionized, plasma-based schemes do not suffer from this limit and in principle can achieve gradients of order 100 GeV/m (Chen, 1991a).

#### 6.3.1 Beat-wave Accelerator

In this concept, two infrared laser beams of different frequencies are sent into a plasma whose plasma frequency matches the lasers' difference frequency. The beat between the laser beams then resonantly excites an electrostatic plasma wave with a velocity equal to the group velocity of the light (Fig. 11). Electrons injected into this wave at the right phase will be continuously accelerated by the electrostatic field. The maximum electron energy in eV is roughly equal

to  $n^{1/2}$ , where n is the density per cm<sup>-3</sup>, so that  $n = 10^{18}$  cm<sup>-3</sup> would give 1GeV/cm. Such linear accelerators would be a factor of 1000 shorter than conventional ones and make the *in situ* production of medical isotopes much easier. Preliminary tests have shown acceleration to 30 MeV, in agreement with theory.

### 6.3.2 Wake-field Accelerator

Another method to generate a fast plasma wave is to inject a short pulse of electrons into a plasma. A "wake" oscillating at  $\omega_p$  then follows the pulse like the wake of a boat. If another pulse of electrons is injected into this wake, it can be accelerated in the same way as in the beatwave accelerator. The wake-field accelerator is effectively a transformer, turning a large bunch of electrons of low energy--say, 100 MeV--into a smaller bunch of electrons at a higher energy--say, 1 GeV. A related concept employs a short laser pulse to generate the wake. In any case, these methods all require a dense, uniform plasma source and either a large laser or a large accelerator.

### 6.3.3 Plasma Lenses

To maximize the rate of high-energy reactions, particle beams must be focused to sub-micron diameters. Plasmas can provide a focusing force orders of magnitude larger than conventional magnets. A charged relativistic beam can propagate without spreading because its electrostatic repulsion is balanced by the inward force of the self-magnetic field. When sent through a plasma, the beam electrons replace the thermal electrons, and the ions cancel the electrostatic field. The pinch force is then able to focus the beam.

#### 6.4 X-ray and Short-Pulsed Lasers

## 6.4.1 X-ray lasers

The dream of generating coherent light in the x-ray regime has been realized by the use of the powerful pulsed lasers developed at the Lawrence Livermore Laboratory for laser fusion. Inner shell transitions in multiply ionized atoms can be induced by irradiating a plasma several centimeters in length with intense laser pulses (Kauffman, 1991). The medium for x-ray lasers is necessarily in the plasma state. X-rays have also been produced by laser heating of carbon fibers and with z-pinches. In each case, special methods are used to depopulate the lower level of the transition. So far, no efficient x-ray reflectors have been developed, and all x-ray lasers are superradiant.

#### 6.4.2 Femtosecond Lasers

Pulse compression techniques on glass lasers have been developed to the extent that tens of terawatts can be delivered in 100 femtoseconds (1 fsec = 10<sup>-15</sup> sec), with peak intensities exceeding 10<sup>18</sup> W/cm<sup>2</sup>. At these intensities, the electric field of the light wave exceeds that which holds the orbital electrons in an atom, and the ponderomotive force is measured in Mbars. Under such conditions of high energy density, almost any material would be turned into a plasma. The behavior of matter in this plasma state would be highly nonlinear; for instance, the electrons

moving in the light wave would change their mass relativistically in each half cycle. Production of even shorter pulses will permit the observation of ultrafast biological and chemical phenomena, such as the generation of nerve impulses or the formation of a molecule.

## 7. Industrial Applications

Plasmas are extensively used in manufacturing and in the production and improvement of materials. These applications call for low-temperature, partially ionized plasmas, which have properties different from the hot, collisionless plasmas used in fusion and present in space. Industrial plasma are likely to have many species of atoms and molecules, including negative ions, and are usually collisional, involving simultaneously many different scattering, excitation, and ionization processes (Lieberman and Lichtenberg, 1994).

#### 7.1 Plasma Sources

In some manufacturing procedures, the type of plasma is not crucial; only the existence of electrons is needed. In the "barrel etcher," for instance, a simple discharge between two electrodes is used to create a low degree of ionization. For the fabrication of large-scale integrated semiconductor circuits, however, the plasma performs several functions and must be carefully controlled. By far the most common and the best developed plasma generator for this purpose is the parallel-plate capacitor discharge, called the RIE (Reactive Ion Etching) source (Fig. 12). The RIE plasma is made by applying an rf voltage at 13.56 MHz between two parallel electrodes about 5-10 cm apart. The gas is a mixture containing Cl or F compounds, which are dissociated by the plasma electrons; it is the neutral Cl or F atoms which chemically attack the target in etching. A sheath forms on both electrodes to confine the electrons, that on the negative electrode being thicker. During the rf cycle, these sheaths oscillate, and the plasma "sloshes" back and forth between electrodes. On average, the ions experience a sheath potential which accelerates them toward the substrate, or object to be treated, which is mounted on one electrode. That electrode can be grounded and a separate rf power supply connected to it to adjust the rectified sheath voltage. Since the sheath drop, rf oscillation, and plasma density all depend on rf power, the degree of control over the plasma is restricted.

New plasma sources have been proposed to be more flexible than the RIE source and to produce higher plasma densities (Lieberman and Gottscho, 1994). (See also under Plasma Devices.) The leading contenders are the electron cyclotron resonance (ECR) source; inductively coupled plasmas (ICP); the radiofrequency inductive (RFI) or transformer coupled plasma (TCP) source; and the helicon source. The ECR source, driven by 2.45 GHz microwaves, is a spinoff from ECRF technology used for plasma heating (cf. Sec. 3.1.4.3). This source can produce high densities at low pressures but requires a magnetic field of 875 G at the resonance layer. The other sources operate at the industrial frequency of 13.56 MHz or its harmonics. ICPs utilize a helical winding around a cylindrical tube for applying the rf power, usually with an electrostatic shield to eliminate direct electrostatic coupling. RFI and TCP sources are similar, each with a pancake-shaped rf antenna lying on a quartz plate, which separates it from the plasma chamber. These rf sources do not require a magnetic field and do not resonate with a natural frequency of the plasma. In the helicon source, an external antenna launches a low-frequency whistler wave, called

a helicon wave, which ionizes the plasma as it is damped. A magnetic field of 0.1-1 kG is required. When the helicon resonance is struck, ionization is particularly efficient. In all these sources, the plasma is often drifted into a chamber with magnetic confinement at the walls provided by arrays of permanent magnets, called a *magnetic bucket*, to smooth out density variations before the plasma arrives at the substrate.

# 7.2 Plasma Etching and Deposition

Plasma processing is essential in the production of semiconductor integrated circuits, in which the individual features can be smaller than 0.5 µm. The manufacture of flat-panel displays, such as the active matrix liquid crystal displays (AMLCDs) used in portable computers, also benefits from plasma processing, though it is not essential, since the elements here are on a larger scale. Integrated circuits are made, hundreds at a time, on a polycrystalline silicon wafer, typically eight inches or larger in diameter. The various components of a chip—transistors, capacitors, conductors, etc.—are made in a series of steps involving deposition, masking, stripping, and etching (Manos and Flamm, 1989). Different processes are used for handling semiconductor material, oxide or nitride insulators, conductors, and photoresist. The patterns are of such fine scale that photolithography with short wavelength x-rays must be used; the x-ray sources are often plasma devices. Since the gases, pressures, and even plasma sources are generally different for each step, the wafers are shuttled under vacuum from one "reactor" to the next in a transfer mechanism known as a cluster tool.

The role of plasma in the etching process is two-fold. First, the electrons produce the active species, and ion bombardment prepares the surface so that chemical etching occurs more readily. The combined effect is to increase the etch rate by over an order of magnitude over chemical etching without plasma or plasma sputtering without chemicals. Second, the sheath electric field accelerates ions so that they strike the substrate a right angles, thus guiding the etching process to proceed in a straight line, so that sharply defined features can be chiseled. Low pressure operation is desirable for reducing ion scattering in the sheath. The plasmas used for etching are designed for the following attributes: 1) uniformity of density over an entire wafer; 2) directionality, or anisotropy of ion orbits; 3) high density, and hence high etch rate; 4) high selectivity in etching the desired material; and 5) compactness, simplicity, and low cost. Examples of problems which occur in plasma etching and deposition are contamination by particulates and damage to oxide insulators. Micron-size dust particles are formed from the ambient gases, and, being charged negatively by electron bombardment, they are trapped in local maxima of electric potential. When the plasma is turned off, the particles are driven into the wafer and cause defects in some of the chips. Device damage occurs when energetic electrons or ions imbed themselves into an insulating layer and cannot be removed. To prevent damage to delicate circuits, one can use a downstream reactor, in which the plasma produces the active species but is swept away before any charged particles can reach the wafer. More detailed information on this subject may be found in the article on Plasma Etching.

### 7.3 Gaseous Electronics

Traditionally, gaseous electronics is the study of plasmas in gas filled electronic tubes and switches and of atomic processes such as collisions, ionization, and radiation. Subjects such as

positive columns, glow discharges, arcs, coronas, field emission, breakdown, and cathode spots have now been replaced by the microwave and radiofrequency discharges used for plasma processing. The most common efficient light source, the fluorescent light, retains its eminent position in gaseous electronics; but new light sources with higher brightens and efficiency and with no internal electrodes are being developed using microwave and rf technology. For more information on this subject, see the article on Sparks, Arcs, and Other Electric Discharges.

# 7.4 Implantation and Polymerization

Ion implantation with energetic ion beams is used to improve the surface hardness, for instance, of metals or to create special semiconductor materials (Conrad and Sridharan, 1994). In this process, a three-dimensional object has to be rotated to expose its surfaces to the ion beam. In plasma source ion implantation (PSII), the object is immersed in a plasma and is given a negative pulse of tens of kilovolts. If the Debye length is sufficiently small, a sheath is formed on all surfaces, and the sheath voltage drives ions at near-normal incidence into all surfaces regardless of their irregular shapes. This method is used for implanting nitrogen into cutting tools and other metallic objects to increase hardness and reduce corrosion. Medical prostheses can be hardened and smoothed by PSII. Eventually, PSII might be applied to the manufacture of magnetic and optical disks.

Strong plastic coatings can be created with plasma discharges in organic compounds such as methyl methacrylate (d'Agostino, 1990). When polymerized in the presence of electrons, a plastic material becomes highly cross-linked, resulting in a strong and resistant layer. Organic plasmas can be used for a number of practical purposes: for barrier coatings in gasoline tanks, soft drink bottles, and medical capsules; for making textile fibers more absorbent to dyes; for increasing the water resistance of papers and wood; for cleaning and sterilization of biomedical containers; for producing optical fibers with graded indices of refraction; and eventually for producing integrated optics chips for optical computing.

#### 7.5 Thermal Plasmas

Thermal plasmas are plasma at near-atmospheric pressure which are so collisional that they are in kinetic, though not radiative, equilibrium (Boulos et al., 1994). Plasma spray treatment of aircraft and automobile parts such as turbine blades is a widespread industrial application of plasma physics. Thermal plasma jets are capable of growing diamond coatings at a much faster rate than with low-pressure plasmas. Development of this process can lead to diamond coatings on all cutting tools, and even to diamond substrates for semiconductors. Thermal plasmas can also be used to develop new types of ultrahard materials. Further information can be found in the article on Thermal Plasma Devices.

## 7.6 Isotope Separation

The production of  $U^{235}$  by gas diffusion methods is notoriously slow, and many ideas for faster processes have been proposed. The use of plasmas for isotope separation started during World War II with the ill-fated Calutron project, named after the Univ. of California, in which instabilities were so strong that the plasma was lost at what is now called the Bohm diffusion rate.

Though new types of plasma centrifuges are still being researched, the major avenues for advance isotope separation use lasers or radiofrequency generators.

# 7.6.1 Laser Isotope Separation

With narrow linewidth, lasers can selectively excite an isotopic species either as an atom or in a molecule. In atomic vapor laser isotope separation (AVLIS), the naturally occurring isotopic mixture of an element is vaporized and introduced into a long chamber. A laser beam tuned to a particular line is directed into the gas and selectively ionizes the desired isotope. A high voltage applied to the walls of the chamber then causes the ions to be collected at the cathode (Vitello et al., 1992). This method is limited by the rate at which the ions can drift across the chamber, and to those isotopes for which a suitable laser line can be found.

# 7.6.2 Ion Cyclotron Isotope Separation

In this scheme, the raw material is vaporized and ionized by microwaves from a gyrotron, and is confined radially by a uniform magnetic field (Chen, 1991b). A radiofrequency signal near the cyclotron frequency of the desired isotope is then applied to the plasma with a helical antenna. Those atoms which are in cyclotron resonance are selectively accelerated to large perpendicular energies and, because of their large Larmor radii, can be "scraped off" onto suitable collectors. Since both the frequency and the magnetic field can be tuned, this method will obviously work for any isotope; for this reason, it is particularly useful for producing medical isotopes.

### Glossary

adiabatic invariant: a physical quantity that stays constant under slowly changing conditions.

Alfvén speed: the propagation speed of displacements of magnetic field lines imbedded in a plasma.

Alfvén wave: a low-frequency wave in a magnetized plasma traveling at the Alfvén speed.

ambipolar diffusion: collisional diffusion of ions and electrons to the wall at rates which are made equal by an electrostatic field.

**ballooning mode:** a plasma instability localized to a region in which the magnetic field has unfavorable curvature (see **magnetic curvature**).

banana orbit: the curve, in a cross-sectional plane of a toroidal confinement device such as a tokamak, traced by an ion or electron as it moves along the magnetic field lines. These orbits are caused by the magnetic mirror effect, which reflects some particles before they can reach the strong-field region on the inside of the torus.

Bernstein wave: electrostatic waves near the ion and electron cyclotron frequencies and their harmonics.

beta: ratio of plasma pressure to magnetic field pressure.

**blanket:** a meter-thick region surrounding the plasma in a fusion reactor, containing compounds of lithium and used to absorb the product neutrons and convert their energy into heat.

**Blumlein:** a high-voltage transmission line, filled with water or oil, used for pulse-shaping in a pulsed power device.

**Bohm diffusion:** instability or convection driven transport of plasma across a magnetic field, which not only is much faster than collisional diffusion but also scales more weakly with field strength.

Bohm-Gross wave: an electron plasma wave.

Boltzmann distribution: the Gaussian velocity distribution of a gas in thermal equilibrium.

**Boltzmann relation:** relation between the local electric potential and the density of a charged species with a Boltzmann velocity distribution.

**bootstrap current:** in a closed fusion device, a self-induced toroidal current driven by the momentum loss of radially escaping plasma.

**breakeven:** in fusion reactors, the condition for the nuclear energy release to exceed the energy used to produce the plasma.

**bremsstrahlung:** x-radiation emitted by the electrons of a plasma by the accelerated charge effect when they collide with ions.

bremsstrahlung, inverse: classical collisional absorption of laser light.

**CARM:** Cyclotron AutoResonant Maser: one of several types of microwave generator based on the cyclotron motion of electrons in a magnetic field.

caviton: a density cavity in a plasma caused by the radiation pressure of electromagnetic radiation trapped in it.

**chaos:** the mathematically random behavior of a variable subject to a small perturbation from a well organized, periodic state.

**Child-Langmuir sheath:** the layer surrounding a charged electrode which contains only one charge species and in which the potential varies according the theory of space-charge-limited diodes.

collapse: the ultimate fate of Langmuir turbulence, according to theory: Langmuir waves become trapped in density cavities which they themselves create, and these cavities become smaller and deeper until the wave energy is dissipated by the acceleration of fast electrons.

**confinement time:** particle confinement time is the average time an ion-electron pair resides in a plasma before it is lost to the walls; energy confinement time is the ratio of the heat content of a plasma to the power input in steady state.

**Coulomb collision:** deflection of an ion or electron by the electric field of another charged particle.

Coulomb logarithm (In  $\Lambda$ ): a numerical factor of order 10 which accounts for the cumulative effect of many small-angle deflections; here  $\Lambda$  is the ratio of the maximum to minimum impact parameters used in the calculation.

current drive: the induction of a steady toroidal current in closed fusion device by the entrainment of electrons in a toroidally propagating wave.

cutoff frequency: a characteristic frequency of a wave at which its phase velocity becomes infinite and beyond which it cannot propagate

**cyclotron frequency:** the frequency at which an ion or electron executes circular orbits in a magnetic field.

**cyclotron radiation:** radiation at the cyclotron frequency and its harmonics emitted by electrons gyrating in a magnetic field; also called **synchrotron radiation**. The origin of signals received from space, as well as a cause of energy loss from fusion plasmas.

**Debye length:** a characteristic length below which a plasma behaves like individual particles without long-range cooperative motions.

**Debye sheath:** a thin layer, of the order of several Debye lengths, separating a plasma from walls and electrodes. The sheath forms a self-adjusting potential barrier to equalize the escape rates of the positive and negative charge carriers.

**Debye shielding:** the formation of sheaths around externally imposed electrodes to prevent strong electric fields from appearing in a highly conductive plasma.

**Debye sphere:** a sphere with radius equal to a Debye length.

diamagnetism: the tendency for a material to generate magnetic fields opposite to those imposed on it; plasmas are diamagnetic because of the field generated by gyrating particles.

**diffusion, anomalous:** transport across magnetic fields at a rate exceeding that expected from classical Coulomb collisions, and caused by such effects as instabilities, convection in dc electric fields, and magnetic errors.

diffusion, Bohm: see Bohm diffusion.

diffusion, classical: transport of particles or heat via ordinary Coulomb collisions.

**diffusion, gyro-Bohm:** an anomalous diffusion rate smaller than the Bohm rate by the ratio of Larmor radius to plasma size.

diffusion, neoclassical: classical diffusion in a torus, embodying no anomalous processes other than predictable toroidal effects such as the trapping of particles in banana orbits.

**direct drive:** in laser fusion, the use of laser light to compress the fuel pellet without first converting the radiation to x-rays.

**disruption:** a sudden termination, unpredictable and sometimes violent, of the discharge in a to-kamak, resulting in deposition of the stored energy into the walls.

**double layer:** a region inside a plasma which contains alternating layers of excess positive and negative charge and which is sustained by the presence of non-Maxwellian velocity distributions.

**drift wave:** a low-frequency wave, with a phase velocity near the diamagnetic drift velocity, which exists only in nonuniform plasmas.

drift instability: unstable growth of drift waves driven by a pressure gradient; also called a universal instability in the collisionless limit.

**ECRH.** Electron Cyclotron Resonance Heating, a method for heating plasmas with microwave power at the electron cyclotron frequency. Also called ECRF: Electron Cyclotron Range of Frequencies.

**electron plasma wave:** a Langmuir oscillation which propagates by virtue of finite electron temperature; also called **Langmuir wave** or **Bohm-Gross wave**.

energy principle: a formulation, based on the calculus of variations, of the criterion for stability or instability of a plasma in a magnetic field of arbitrary shape.

extraordinary wave: an electromagnetic wave propagating in a plasma in a direction normal to the background magnetic field.

**Faraday rotation:** rotation of the plane of polarization of a light wave traveling through an anisotropic medium, such as a magnetized plasma.

favorable curvature: concavity of magnetic field lines which bulge toward the center of the plasma rather than in the normal, convex direction; this sign of the curvature stabilizes the gravitational instability.

**FEL:** Free Electron Laser: a generator of coherent radiation based on the excitation of waves of upshifted frequency when an electromagnetic wave is passed through a periodically varying magnetic field in the presence of an electron beam.

field annihilation: see reconnection.

**filamentation:** in laser-plasma interactions, the tendency for an intense laser beam to break up into smaller beamlets when passing through a plasma; in magnetic fusion, the tendency for a toroidal current to do likewise, thus creating **magnetic islands**.

fusion: release of nuclear energy by the transmutation of light elements into heavier ones; the conversion of heavy hydrogen into helium requires a hot plasma in near-thermal equilibrium and is then called **thermonuclear fusion** or **controlled fusion**.

**gravitational instability:** a hydrodynamic instability of a plasma confined by a magnetic field; the centrifugal force of particles following curved field lines acts like a gravitational force in pushing small ripples in the plasma surface into large bulges.

## guiding center

gyrotron: one of several types of microwave generator based on the cyclotron motion of electrons in a magnetic field.

**H-mode:** a regime of operation of tokamaks characterized by <u>High</u> confinement, small internal gradients, and an edge layer with highly sheared electric fields which forms a transport barrier.

ICRH: Ion Cyclotron Resonance Heating: a method for heating plasmas with radiofrequency power at the ion cyclotron frequency. Also called ICRF: Ion Cyclotron Range of Frequencies.

**ignition:** maintenance of fusion conditions in a thermonuclear plasma by the retained heat of the charged particle products of the reactions.

indirect drive: in laser fusion, compression of the fuel pellet by a uniform bath of x-rays, which are generated when laser beams strike the interior of a metal canister surrounding the pellet.

inertial confinement: attainment of fusion conditions by using radiation or particle beams to compress the fuel to more than 1000 times solid density; at such densities, the fusion reactions take place during the instant when the fuel is held together by its own inertia.

interchange instability: same as gravitational instability. The plasma and magnetic field regions are imagined to be interchanged, with a decrease in potential energy.

inverse bremsstrahlung: classical collisional absorption of laser light.

ion wave: plasma analogue of an acoustic, or sound, wave in air. Only one of the species in the plasma needs to have a finite temperature for the wave to exist.

ionosphere: the region surrounding the earth or other planet which contains a plasma.

kink instability: the bending or coiling of a current channel in a magnetized plasma.

Landau damping: reversible damping of a wave in a collisionless plasma, caused by the loss of wave energy to particles which surf on the wave.

Langmuir oscillation: the high-frequency oscillation of electrons in a plasma when displaced from their normal positions relative to the relatively immobile ion background. These oscillations do not propagate in an infinite, cold plasma, but they become propagating electron plasma waves in a warm plasma and propagating Trivelpiece-Gould modes in a bounded plasma.

Langmuir probe: a small electrode which, when inserted in a plasma, gives information on local temperatures, densities, and potentials via its current-voltage characteristic.

Langmuir waves: same as electron plasma waves.

Langmuir turbulence: the disorganized state of a plasma subject to large amplitude Langmuir waves.

**Larmor orbit:** the circular path of a charged particle in a magnetic field.

**Lawson criterion:** the value of the product of plasma density and confinement time above which energy breakeven is achieved in a fusion reactor.

**line-tying:** in an open confinement device, the stabilizing effect of currents in conductors on which magnetic field lines terminate.

loss cone: the range of pitch angles for which particles are not reflected by a magnetic mirror.

magnetic mirror: a region of increasing magnetic field which reflects a fraction of the impinging plasma particles; also refers to a fusion confinement system consisting of a set of two magnetic mirrors.

magnetosphere: the region surrounding a planet in which the planet's magnetic field is dominant over the interplanetary magnetic field.

Marx bank: a capacitor bank that is charged in parallel but discharged in series, in order to obtain high voltages.

magnetic bottle: popular term for fusion confinement device.

magnetic curvature: technical term for the convex (unfavorable) or concave (favorable) curvature of magnetic field lines, with reference to the stability properties of the plasma in a fusion confinement device; see favorable curvature.

magnetic island: one of a chain of azimuthally distributed regions in the cross section of a torus which contain an isolated group of magnetic field lines that do not reach other parts of the cross section.

magnetic well: a magnetic configuration containing a point from which the field strength increases in all directions.

magnetic trap: same as magnetic well.

MHD: Magneto Hydro Dynamics: a discipline in which the plasma is treated as a conducting fluid without regard for individual particle motions or deviations from thermal equilibrium.

**minimum-B:** characterizing an absolute **magnetic well**; a minimum-B configuration is an average magnetic well, in which the average **magnetic curvature** experienced by a particle is in the favorable direction.

mirror ratio: the ratio between maximum and minimum field strengths in a magnetic mirror.

**modulational instability:** in nonlinear plasma theory, an instability in which the unstable waves develop a growing ripple in the envelope of their amplitudes.

**NBI:** Neutral Beam Injection: the leading method for heating the plasma in controlled fusion experiments.

non-neutral plasma: a plasma containing only positive or only negative charges.

**nonlinear:** beyond the scope of validity of small-amplitude, linear theory, in which the behavior of a system can be described by a simple sum over Fourier components.

parametric instability: a phenomenon occurring in laser-plasma interactions in which one wave is excited by another (usually the laser beam), but only in the presence of a third wave. The name comes from parametric amplifiers in electronics.

particulate (adj.): possessing the characteristics of individual particles, rather than of a continuous fluid.

particulate (n.): a charged "dust" grain, of the order of 1µm to 1mm in size, usually negative in industrial plasmas and positive in space plasmas.

PECVD: Plasma Enhanced Chemical Vapor Deposition.

**pinch:** an electrical discharge in which the plasma is squeezed by the magnetic field pressure generated by currents within the plasma.

plasma dispersion function: a complex function of a complex argument, found by contour integration, arising in the theory of collisionless plasmas.

plasma etch: a step in the manufacture of semiconductors in which material is removed chemically with the aid of a plasma.

plasma frequency: the characteristic electron oscillation frequency in a plasma; see Langmuir oscillation.

plasma instability: a growing perturbation or wave in a plasma.

plasma processing: the use of plasmas in manufacturing.

plasma torch: an atmospheric pressure discharge used for treatment of materials or wastes.

plasma wave: any wave in a plasma, but usually referring to an electron plasma wave.

**ponderomotive force:** the force exerted on a plasma by a wave with a gradient in its amplitude; radiation pressure modified by the presence of plasma.

**PSII:** Plasma Source Ion Implantation.

pulsed power: a discipline involving the generation of terawatt bursts of energy by means of Marx banks and Blumleins, usually resulting in an intense electron or ion beam.

**q:** a quality factor of tokamaks, inversely proportional to the degree of twist of the magnetic field lines.

quarter-critical layer: in laser-plasma interactions, the density region in which the plasma frequency is 1/4 of the laser frequency.

quasineutrality: a characteristic trait of plasmas in which the positive and negative charges move so as to maintain almost equal numbers everywhere, but with small deviations to permit the existence of electric fields.

Rayleigh-Taylor instability: the hydrodynamic instability of a heavy fluid supported by a light fluid, which, in plasma physics, is the magnetic field or laser beam; see gravitational instability.

**REB:** Relativistic Electron Beam generated by pulsed power.

reconnection: the joining of one group of magnetic field lines to another in the presence of plasma. Since plasma particles are entrained by magnetic fields, reconnection can occur only if the particles can cross field lines, and reconnection can occur only in a region, no matter how thin, in which dissipation, including Landau damping, is important.

resonance absorption: absorption of laser radiation by the generation and subsequent damping of plasma waves.

**RIE:** Reactive Ion Etching, a common process in the fabrication of semiconductors; also a type of plasma discharge used for this.

resistive instability: a hydromagnetic instability which occurs only if the plasma resistivity is nonzero.

rotational transform: in a torus, the number of turns a magnetic field line encircles the minor axis for each time around the major axis; the reciprocal of q.

sawtooth: a temperature or density fluctuation with triangular waveform observed in tokamaks, thought to be caused by small kink instabilities and to be the mechanism by which a tokamak plasma adjusts its internal current distribution.

SBS: Stimulated Brillouin Scattering: a parametric instability in which a laser beam excites an ion acoustic wave and a reflected light wave.

shear: the change of pitch angle of magnetic field lines on different surfaces; the radial variation of rotational transform or q.

**simulation, computer:** numerical calculations, stepwise in time, following the development of plasma motions.

**simulation, particle:** a computer simulation in which the particle motions, rather than fluid elements or distribution functions are followed.

solar wind: a stream of ions and electrons ejected from the sun.

soliton: a wave with only one maximum; a solitary wave.

**Spitzer resistivity:** resistivity of a fully ionized plasma accounting only for binary Coulomb collisions of electrons with ions.

SRS: Stimulated Raman Scattering: a parametric instability in which a laser beam excites an electron plasma wave and a reflected light wave.

**strongly coupled plasma:** a plasma of such high density and low temperature that the number of particles in a Debye sphere is of order unity or below.

synchrotron radiation: same as cyclotron radiation.

tandem mirror: a magnetic mirror fusion device plugged at both ends with short mirror sections containing a hot, dense plasma.

tearing mode: a type of resistive instability driven by current and involving reconnection.

thermonuclear: pertaining to nuclear reactions in a Maxwellian plasma.

theta pinch: a pinch in which the current is in the azimuthal direction.

**Thomson scattering:** a plasma diagnostic technique in which laser light is scattered by plasma particles or density fluctuations.

**tokamak:** a toroidal fusion device in which the magnetic configuration is determined by the distribution of toroidal current within the plasma.

toroidal confinement: trapping of plasma in a closed magnetic field configuration.

**trapped particle:** in tokamaks, ions or electrons which are prevented from circulating around the minor axis by the magnetic mirror effect of the strong fields near the major axis.

turbulence: a statistically steady state of random fluctuations outside the realm of linear theory. universal instability: a drift wave instability occurring in collisionless plasmas.

Van Allen belts: plasma trapped in the earth's magnetic field by the magnetic mirror effect.

**Vlasov equation:** an equation describing the evolution of velocity distribution functions in the absence of collisions.

whistler wave: an electromagnetic wave in a plasma propagating along magnetic field lines; first detected in radio reception as whistling sounds from the ionosphere.

**Z-pinch:** a **pinch** in which the current is in the axial direction.

## **Figure Captions**

- Fig. 1. The Aurora Borealis is an example of a plasma naturally occurring on earth. The glow is excited by electrons precipitating from the van Allen radiation belts.
- Fig. 2. Electrical discharges in the form of lightning produce plasmas at atmospheric pressure.
- Fig. 3. Charged particles gyrate around magnetic field lines and are trapped between strong-field regions by the magnetic mirror effect, which results from the adiabatic invariant  $\mu$ . The second adiabatic invariant, J, controls the frequency of the longitudinally bounce motion between mirrors (Fowler and Post, 1966).
- Fig. 4. A plasma instability is revealed by laser interferometry of an initially circular cylinder of dense plasma (Fowler and Post, 1966).

- Fig. 5. Ion acoustic shock waves, as produced and detected in the laboratory. Many such nonlinear plasma phenomena are well described by theory (Chen, 1983).
- Fig. 6. Numerical mapping of a perturbed Hamiltonian reproduces the magnetic island structure of the field lines in a toroidal tokamak fusion machine. In this cross section, the magnetic surfaces break up into "islands" at those radii where the field lines return to the same position after a rational number of turns around the torus. Between these islands, the field lines lie in KAM (Kolmogoroff-Arnold-Moser) surfaces and cover the space ergodically (Lichtenberg and Lieberman, 1983.)
- Fig. 7. The sun's corona, prominences, and sunspots are examples of magnetized plasmas. The solar wind carries some of this plasma to the earth and controls its plasma environment. Fusion reactions in the interior of the sun generates its energy, as in all stars.
- Fig. 8. The magnetic environment of the earth, showing the neutral sheet in which field lines of opposite polarity coalesce and magnetic reconnection occurs. The transition between the interplanetary magnetic field and the earth's field is controlled by the presence of plasma from the solar wind which is entrained by the magnetic fields.
- Fig. 9. The filamentary structure of the Crab Nebula reveals the presence of plasma constrained by magnetic fields.
- Fig. 10. Ionized gases, dust, and magnetic fields are the substance of galaxies as well as of plasmas used in industry. The spiral arms are thought to be caused by an instability of the "plasma", the particles of which, in this case, are stars.
- Fig. 11. A two-dimensional (r, z) particle simulation of the beat-wave accelerator mechanism. The laser beams enter from the right (along z), and the ponderomotive force at their beat frequency bunches the electrons into the density fluctuations of a plasma wave. The electric potential of the wave is plotted vs. z and radius r. The pattern travels along z at the laser beams' group velocity. Beam electrons injected into the wave can be trapped in the potential wells and accelerated to the wave's phase velocity (Mori, 1990).
- Fig. 12. Diagram of an RIE (Reactive Ion Etching) plasma source commonly used for etching and deposition of silicon wafers in semiconductor fabrication. A 13.56 MHz radiofrequency source applied between the parallel plate electrodes creates the plasma, which impinges on the wafer mounted on the bottom plate.

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