The floating potential of cylindrical Langmuir probes

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The floating potential of a cylindrical probe is computed numerically, and the results are fitted to analytic functions. They differ significantly from the plane approximation. © 2001 American Institute of Physics.

\begin{abstract}
The formula normally used to calculate the space potential \( V_s \) from the measured floating potential \( V_f \) is derived for plane probes and is erroneous when applied to cylindrical probes in the low-density plasmas \((n<10^{12} \text{ cm}^{-3})\) used in industrial plasma processing. The plasma potential \( V_s \) is of interest because it determines the sheath drop which governs the energy of ions bombarding the walls or a silicon wafer, for instance. Since \( V_f \) is easily measurable, being the potential at which no net current is drawn, it is often used to estimate \( V_s \). The difference \( V_s - V_f \) can be calculated from the theory of plane probes, in which a “sheath edge” artificially separates the plasma from the sheath region. If \( A_p \) is the probe area, \( n_0 \) the density in the body of the plasma, and the potential \( V_f \) there is defined as 0, the electron flux \( \Gamma A_p \) to a floating probe is

\[ I_e = A_p n_0 \nu_{th} \exp(V_f/KT_e), \quad \nu_{th} = (KT_e/2\pi m)^{1/2}. \]  

(Note: \( I = \) total particle current for plane probes and current per unit length for cylindrical probes; the electrical current \( \geq eI \) is not used here.) Following the so-called Bohm criterion, the sheath edge is defined as the plane where \( eV = \frac{1}{2}KT_e \) and \( n = n_j = n_0 \exp(-\frac{V}{2}) \), so that the ions, having fallen through this potential, have a velocity \( c_s \). The ion current is therefore

\[ I_i = A_p n_j c_s = \alpha_0 A_p n_0 c_s, \quad c_s = (KT_e/M)^{1/2}, \]  

\( M \) being the ion mass and \( \alpha_0 = \exp(-\frac{1}{2}) \approx 0.61 \). A spread in ion energies can bring \( \alpha_0 \) closer to the convenient value of 0.5. Setting \( I_i = I_e \) yields the usual formula for the floating potential:

\[ -\frac{eV_f}{KT_e} = \ln \left( \frac{1}{\alpha_0} \left( \frac{M}{2\pi m} \right)^{1/2} \right) \approx 5.18 \text{ in argon.} \]  

The ion collection area for a cylindrical probe, however, depends on the radius \( R_{th} \) of the sheath, which is not known \textit{a priori}. In this case, there is no need for the artifice of a sharp sheath edge, since solutions of Poisson’s equation can be extended to infinity, but it is no longer possible to solve for \( V_f \) analytically; numerical solutions of a differential equation are required. Two collisionless theories are available for calculating \( V(r) \): the Bernstein–Rabinowitz\textsuperscript{(1)} (BR) theory, which takes into account the angular momentum of the ions which orbit the probe; and the Allen–Boyd–Reynolds\textsuperscript{2} (ABR) theory, which neglects orbiting, so that ions move only radially and axially. Chen\textsuperscript{3} has recently shown that the BR theory overestimates the ions’ angular momentum in partially ionized plasmas because of collisions in the presheath. Hence, for plasma processing purposes, we shall employ the ABR equation as modified by Chen\textsuperscript{4} for cylindrical probes:

\[ \frac{\partial}{\partial \xi} \left( \frac{\xi \partial \eta}{\partial \xi} \right) = J \eta^{1/2} - \xi e^{-\eta}, \]  

where

\[ \eta = -eV/KT_e, \quad \xi = r/\lambda_D, \quad \lambda_D = (\epsilon_0 KT_e/n_0 e^2)^{1/2}. \]  

The normalized ion current \( J \) is defined by

\[ J = \frac{1}{2\pi \sqrt{2}} \frac{I_f}{n_0 \lambda_D c_s}. \]  

The unknown current \( J \) has to be assumed beforehand and Eq. (4) integrated to yield \( \eta(J, \xi) \) for all \( \xi \). The \( J - \eta \) (or \( I - V \)) curve is then found by varying \( J \). The constraint that the probe be floating can be expressed as follows. The radius \( \xi_p \) of a floating probe that corresponds to the assumed \( J \) can be found from the condition \( I_i = I_e \) at the probe surface, where

\[ I_e = 2\pi R_p n_0 \nu_{th} \exp(-\eta_j). \]  

If the proper area is substituted, this Maxwellian formula is valid for all \( \eta_j > 0 \), regardless of the probe shape. Substituting Eq. (7) into Eq. (6) gives

\[ J = \frac{1}{\sqrt{2}} \frac{\xi_p}{J} \left( \frac{M}{2\pi m} \right)^{1/2} e^{-\eta_j}, \]  

so that

\[ \eta_j = \ln \left( \frac{\xi_p}{J} \left( \frac{M}{4\pi m} \right)^{1/2} \right). \]  

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Thus, from Eq. (5) current for argon, which approaches the plane limit of 5.18. If we return from the probe, and the ion distribution depends only on $V_s$, whatever its value, and is unaltered for all $r>R_p$. This is because no ions return from the probe, and the ion distribution depends only on $V_s$. For each $J$, Eqs. (9) and (10) give two curves whose intersection yields a pair of values ($\eta_f, \xi_p$), as illustrated in Fig. 1, where $\xi_p$ is the normalized radius of a floating probe collecting the assumed current $J$.

Varying $J$ generates the function $\eta_f(\xi_p)$, shown in Fig. 2 for argon, which approaches the plane limit of 5.18. If we now define

$$\alpha = \sqrt{2J}/\xi_p,$$

Eq. (9) takes the same form as Eq. (3), with $\alpha$ in place of $\alpha_0$. Thus, from Eq. (2), $\alpha A_p$ is the effective collection area of a floating probe, and the ratio $\alpha/\alpha_0$ expresses the expansion of this area as $\xi_p$ is decreased. The functions $\alpha$ and $\alpha/\alpha_0$ are also shown in Fig. 2. There is no need to define a “sheath edge,” but if one is defined at the radius $R_{sh}$ where $\eta = \frac{1}{2}$, as in Fig. 1, conservation of current requires $I = 2\pi R_{sh} n_c$. However, $n_r$ is not 0.61$n_0$ as in the plane case, since quasineutrality has not been assumed at $R_{sh}$, and $n_r \neq n_s$ there. Using Eqs. (11) and (6), we can conveniently express the ion current to a floating probe in terms of the function $\alpha(\xi_p)$:

$$I = 2\pi R_p n_0 c_s,$$

with $\alpha$ acting as an effective Bohm coefficient.

The following analytic fits to the computed curves may be useful for probe analysis:

$$\frac{1}{(\eta_f)^\alpha} = \frac{1}{(A \ln \xi_p + B)^\alpha} + \frac{1}{(C \ln \xi_p + D)^\alpha},$$

where $A = 0.583$, $B = 3.732$, $C = -0.027$, and $D = 5.431$; and

$$\frac{\alpha}{\alpha_0} = R_{sh}/R_p = 1 + E \exp(-F x_p^G),$$

where $E = 4000$, $F = 7.01$, and $G = 0.096$. In the plane probe limit $\xi_p \to \infty$, $\eta_f$ approaches the value of 5.18 for argon, and $\alpha$ and $\alpha/\alpha_0$ approach 0.61 and 1, respectively. In the range $\xi_p = 1-10$ commonly encountered in rf discharges, $\eta_f$ is of order 3.7–4.6 for argon, significantly less than the usual value of 5.2. The reason is that the sheath thickness at $V_f$ causes a cylindrical probe of given area to collect more ion current than a plane probe, and thus the sheath drop has to be lowered to permit more electron flow.

The difference in $V_f$ calculated from the plane and cylindrical formulas is therefore of the order of $KT_s$, or 2–5 V in most rf discharges. This is not a large effect, but the popular misconception on the use of Eq. (3) should be corrected. More important is the effect of inadequate rf compensation, which could increase the apparent value of $KT_s$ and of $|V_f|$. Non-Maxwellian electron tails would also increase $|V_f|$. Electron collisions do not affect our results as long as the electrons are Maxwellian, but ion collisions in the sheath could. However, that effect would not be noticeable below about 1 Torr. Finally, the use of the ABR theory needs to be justified. For low-pressure plasmas with a few collisions, it has been shown that the BR theory overestimates the effect of ion orbiting around the probe, while the ABR theory underestimates it. The ABR theory is more accurate than the plane approximation, though at the lowest densities orbiting does occur and does affect the results. Evidence for this is deferred to a full-length paper.