The floating potential of cylindrical Langmuir probes

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The floating potential of a cylindrical probe is computed numerically, and the results are fitted to analytic functions. They differ significantly from the plane approximation. *© 2001 American Institute of Physics.* [DOI: 10.1063/1.1409346]

The formula normally used to calculate the space potential V_s from the measured floating potential V_f is derived for plane probes and is erroneous when applied to cylindrical probes in the low-density plasmas ($n<10^{12}\,\mathrm{cm}^{-3}$) used in industrial plasma processing. The plasma potential V_s is of interest because it determines the sheath drop which governs the energy of ions bombarding the walls or a silicon wafer, for instance. Since V_f is easily measurable, being the potential at which no net current is drawn, it is often used to estimate V_s. The difference V_s-V_f can be calculated from the theory of plane probes, in which a "sheath edge" artificially separates the plasma from the sheath region. If A_p is the probe area, n_0 the density in the body of the plasma, and the potential V_s there is defined as 0, the electron flux ΓA_p to a floating probe is

$$I_e=A_p n_0 \nu_{\mathrm{th}} \exp(V_f/KT_e), \quad \nu_{\mathrm{th}}\equiv(KT_e/2\pi m)^{1/2}. \tag{1}$$

(Note: $I\equiv$total particle current for plane probes and current per unit length for cylindrical probes; the electrical current $\pm eI$ is not used here.) Following the so-called Bohm criterion, the sheath edge is defined as the plane where $eV=\frac{1}{2}KT_e$ and $n=n_s=n_0\exp(-\frac{1}{2})$, so that the ions, having fallen through this potential, have a velocity c_s. The ion current is therefore

$$I_i=A_p n_s c_s=\alpha_0 A_p n_0 c_s, \quad c_s\equiv(KT_e/M)^{1/2}, \tag{2}$$

M being the ion mass and $\alpha_0=\exp(-\frac{1}{2})\approx 0.61$. A spread in ion energies can bring α_0 closer to the convenient value of 0.5. Setting $I_i=I_e$ yields the usual formula for the floating potential:

$$-\frac{eV_f}{KT_e}=\ln\left[\frac{1}{\alpha_0}\left(\frac{M}{2\pi m}\right)^{1/2}\right]\approx 5.18 \text{ in argon.} \tag{3}$$

The ion collection area for a *cylindrical* probe, however, depends on the radius R_{sh} of the sheath, which is not known *a priori*. In this case, there is no need for the artifice of a sharp sheath edge, since solutions of Poisson's equation can be extended to infinity, but it is no longer possible to solve for V_f analytically; numerical solutions of a differential equation are required. Two collisionless theories are available for calculating $V(r)$: the Bernstein–Rabinowitz[1] (BR) theory, which takes into account the angular momentum of the ions which orbit the probe; and the Allen–Boyd–Reynolds[2] (ABR) theory, which neglects orbiting, so that ions move only radially and axially. Chen[3] has recently shown that the BR theory overestimates the ions' angular momentum in partially ionized plasmas because of collisions in the presheath. Hence, for plasma processing purposes, we shall employ the ABR equation as modified by Chen[4] for cylindrical probes:

$$\frac{\partial}{\partial\xi}\left(\xi\frac{\partial\eta}{\partial\xi}\right)=J\eta^{-1/2}-\xi e^{-\eta}, \tag{4}$$

where

$$\eta\equiv -eV/KT_e, \quad \xi\equiv r/\lambda_{\mathrm{D}}, \quad \lambda_{\mathrm{D}}\equiv(\epsilon_0 KT_e/n_0 e^2)^{1/2}. \tag{5}$$

The normalized ion current J is defined by

$$J\equiv\frac{1}{2\pi\sqrt{2}}\frac{I_i}{n_0}\frac{1}{\lambda_{\mathrm{D}}c_s}. \tag{6}$$

The unknown current J has to be assumed beforehand and Eq. (4) integrated to yield $\eta(J,\xi)$ for all ξ. The $J-\eta$ (or $I-V$) curve is then found by varying J. The constraint that the probe be floating can be expressed as follows. The radius ξ_p of a floating probe that corresponds to the assumed J can be found from the condition $I_i=I_e$ at the probe surface, where

$$I_e=2\pi R_p n_0 \nu_{\mathrm{th}}\exp(-\eta_f). \tag{7}$$

If the proper area is substituted, this Maxwellian formula is valid for all $\eta_f>0$, regardless of the probe shape. Substituting Eq. (7) into Eq. (6) gives

$$J=\frac{1}{\sqrt{2}}\xi_p\left(\frac{M}{2\pi m}\right)^{1/2}e^{-\eta_f}, \tag{8}$$

so that

$$\eta_f=\ln\left[\frac{\xi_p}{J}\left(\frac{M}{4\pi m}\right)^{1/2}\right]. \tag{9}$$

Integration of Eq. (4) is nontrivial, and care must be taken to join smoothly to the quasineutral solution at large radii. This procedure yields the potential distribution

$$\eta=\eta(J,\xi). \tag{10}$$

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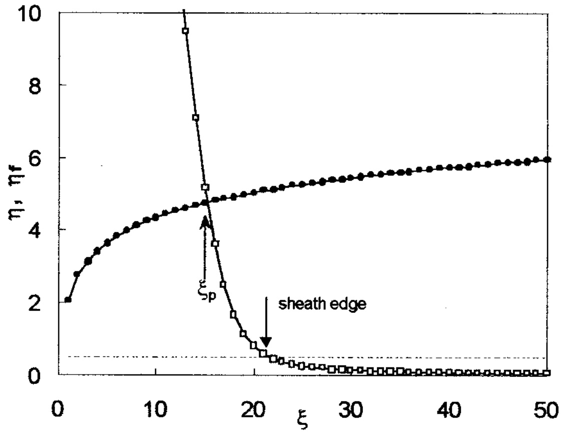

FIG. 1. The radial potential profile $\eta(\xi)$ (□) and the floating potential condition $\eta_f(\xi)$ (●) for the case $J=10$, $\xi_p=15$ in argon. The dashed line indicates the "sheath edge" where $\eta=\frac{1}{2}$, but for cylinders the plasma is not quasineutral there.

This curve is insensitive to the presence of a probe at $r=R_p$. The curve simply terminates at R_p, whatever its value, and is unaltered for all $r>R_p$. This is because no ions return from the probe, and the ion distribution depends only on $V(r>R_p)$, while the electron distribution is assumed Maxwellian and depends only on $V(r)$. For each J, Eqs. (9) and (10) give two curves whose intersection yields a pair of values (η_f, ξ_p), as illustrated in Fig. 1, where ξ_p is the normalized radius of a floating probe collecting the assumed current J.

Varying J generates the function $\eta_f(\xi_p)$, shown in Fig. 2 for argon, which approaches the plane limit of 5.18. If we now define

$$\alpha \equiv \sqrt{2} J/\xi_p, \tag{11}$$

Eq. (9) takes the same form as Eq. (3), with α in place of α_0. Thus, from Eq. (2), αA_p is the effective collection area of a floating probe, and the ratio α/α_0 expresses the expansion of this area as ξ_p is decreased. The functions α and α/α_0 are also shown in Fig. 2. There is no need to define a "sheath edge," but if one is defined at the radius R_{sh} where $\eta=\frac{1}{2}$, as in Fig. 1, conservation of current requires $I_i=2\pi R_{sh} n_s c_s$. However, n_s is not $0.61 n_0$ as in the plane case, since quasineutrality has not been assumed at R_{sh}, and $n_i \neq n_e$ there. Using Eqs. (11) and (6), we can conveniently express the ion current to a floating probe in terms of the function $\alpha(\xi_p)$:

$$I_i = 2\pi R_p \alpha n_0 c_s, \tag{12}$$

with α acting as an effective Bohm coefficient.

The following analytic fits to the computed curves may be useful for probe analysis:

$$\frac{1}{(\eta_f)^6} = \frac{1}{(A \ln \xi_p + B)^6} + \frac{1}{(C \ln \xi_p + D)^6}, \tag{13}$$

where $A=0.583$, $B=3.732$, $C=-0.027$, and $D=5.431$; and

$$\frac{\alpha}{\alpha_0} \approx \frac{R_{sh}}{R_p} = 1 + E \exp(-F \xi_p^G), \tag{14}$$

where $E=4000$, $F=7.01$, and $G=0.096$. In the plane probe limit $\xi_p \to \infty$, η_f approaches the value of 5.18 for argon, and α and α/α_0 approach 0.61 and 1, respectively. In the range $\xi_p = 1-10$ commonly encountered in rf discharges, η_f is of order 3.7–4.6 for argon, significantly less than the usual value of 5.2. The reason is that the sheath thickness at V_f causes a cylindrical probe of given area to collect more ion current than a plane probe, and thus the sheath drop has to be lowered to permit more electron flow.

The difference in V_s calculated from the plane and cylindrical formulas is therefore of the order of KT_e, or 2–5 V in most rf discharges. This is not a large effect, but the popular misconception on the use of Eq. (3) should be corrected. More important is the effect of inadequate rf compensation, which could increase the apparent value of KT_e and of $|V_f|$. Non-Maxwellian electron tails would also increase $|V_f|$. Electron collisions do not affect our results as long as the electrons are Maxwellian, but ion collisions in the sheath could. However, that effect would not be noticeable below about 1 Torr. Finally, the use of the ABR theory needs to be justified. For low-pressure plasmas with a few collisions, it has been shown[3] that the BR theory overestimates the effect of ion orbiting around the probe, while the ABR theory underestimates it. The ABR theory is more accurate than the plane approximation, though at the lowest densities orbiting does occur and does affect the results. Evidence for this is deferred to a full-length paper.

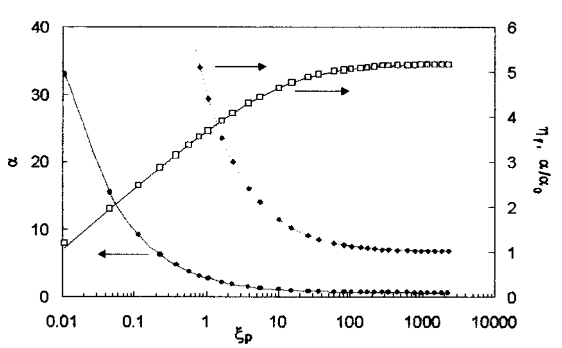

FIG. 2. Decrease of floating potential η_f (□) with decreasing $\xi_p = R_p/\lambda_D$ due to the increase in sheath area as measured by α (●) and α/α_0 (♦). The line through the η_f points is an analytic fit.

[1] I. B. Bernstein and I. N. Rabinowitz, Phys. Fluids **2**, 112 (1959).
[2] J. E. Allen, R. L. F. Boyd, and P. Reynolds, Proc. Phys. Soc. London, Sect. B **70**, 297 (1957).
[3] F. F. Chen, Phys. Plasmas **8**, 3029 (2001).
[4] F. F. Chen, J. Nucl. Energy, Part C **7**, 47 (1965).