Comment on "Comment on 'Magnetic field effects on gas discharge plasmas' " [Phys. Plasmas 14, 024701 (2007)]

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In a curious exchange between Allen[1] and Sternberg *et al.*,[2] they expressed general agreement but disagreed on the reason for that agreement. There are more interesting questions regarding the Bohm criterion than those they raised. The potential in a gas discharge varies continuously from plasma to wall. Because of the disparity in scale lengths, it is customary for computational convenience to divide space into a quasi-neutral plasma region and a sheath region. The plasma region contains effects such as collisions and ionization, while the sheath region, at least in low-pressure discharges, is collisionless. These two regions are joined by a transition region, the presheath, where ions gain a unidirectional flow to the walls. What happens there was a bone of contention in 2002, and I defer to the analyses by Riemann.[3,4] The place called the sheath edge is a personal choice and not a physically significant location. I choose to define it as the place near the wall where the average velocity of the dominant ion species is the acoustic speed c_s. Others may find another definition more useful, but there is nothing to argue about.

The Bohm sheath criterion is based on the necessity for the electric field to point toward the wall, in order to repel electrons. This requirement does not depend on what happens in the plasma but only on the rigidity of ion motion in the collisionless region. The quasi-neutral solution, by contrast, depends on collisions and ionization; and the place where it breaks down (becomes double-valued) should depend on these effects also. That the Bohm criterion should be nearly satisfied at this point is not surprising, since it is exactly the lack of the correct charge separation to form an ion sheath that makes the quasi-neutral solution invalid.

It is well known that the presence of a dc magnetic field does not alter a Maxwellian distribution because the Lorentz force is perpendicular to the velocity and therefore cannot impart energy to any electron. If a magnetic field is slowly applied to an initially field-free discharge with Maxwellian electrons, the discharge dynamics will be changed, and so will the plasma solution. However, the electrons will remain Maxwellian at the same temperature. In the Allen-Magistrelli experiment,[5] an azimuthal B-field was applied with a central wire, causing changes in the I-V characteristic of the discharge. The axial E-field caused the electrons to drift in the azimuthal direction, but the electrons should have been Maxwellian in the rotating frame, since the drift is much slower than their thermal velocities. The sheath solution is affected by the magnetic field since the ions can impinge at an angle, but the electrons remain Maxwellian and follow the Boltzmann relation. At the wall, a few fast electrons overcome the Coulomb barrier, leading to a small error-function depletion of the tail. Their temperature is not cooled because these fast electrons are slowed to the bulk temperature by the potential hill. Thus, in deriving the Bohm criterion, use of the Boltzmann relation for electrons is entirely justified, especially in view of the uncertainly in the Bohm coefficient, which depends on the ion distribution.

However, there are two effects that have been inadequately treated. If inelastic collisions cause holes in the electron distribution[6] or if there are enhanced tails, these will carry over into the sheath. Fortunately, the Bohm criterion depends on the shape of the electron distribution at its peak, and these effects at higher energies should be unimportant. Secondly, there is the problem of instabilities. It would be impractical to solve the strong B-field case classically, since almost all such discharges are drift-wave unstable. Even with no B-field, the Bohm criterion for monoenergetic ions is exactly the threshold for an ion acoustic instability. This was pointed out by the author[7] in 1962, and a more detailed analysis was then made using the ion distribution given by the classical theory of a collisionless plane discharge. However, the plasma solution did not extend into the sheath, and it was found that stability of the ion stream depended on more subtle effects such as ionization in the sheath. Since this was done before modern computers were available to treat the presheath, this problem would be an interesting one to reexamine in the context of the Bohm criterion.

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