



Preface

Meshfree methods have undergone substantial development and have received much attention in the last decade. The most significant advantage of meshfree methods is the flexibility in customizing approximation functions for desired accuracy or for capturing essential physics and features of the particular problems of interest. Adaptivity formulation and multiple-scale solution strategies also can be implemented with relative ease. It has become clear that the meshfree methods provide considerable advantages over the conventional finite element methods in solving problems involving moving discontinuities, evolving material interfaces, multiple-scale phenomena, and large material distortion and structural deformation.

While challenges to the computational efficiency of meshfree methods remain, considerable advances in the method have been achieved. This includes mathematical analysis of meshfree methods, enhanced treatment of boundary conditions, hierarchical meshfree approximation, stability analysis of meshfree method, coupling of finite element and meshfree methods, generalization of finite element and meshfree methods, fast and stable domain integration techniques, and application to problems that are difficult or impossible to solve by the conventional finite element methods. This special issue contains 18 papers addressing various subjects of meshfree methods, including theoretical advancement of meshfree method, meshfree techniques for new and emerging research activities in computational materials science, biomechanics, and multiphysics problems, and recent progress of meshfree methods.

W.K. Liu, W. Han, H. Lu, S. Li, and J. Cao introduce a new method that combines the advantages of finite element and meshfree methods, called the reproducing kernel element method (RKEM). This method offers two distinguish features: (1) arbitrary order of smoothness and (2) the interpolation property in the approximation. Thus RKEM can be used to enrich finite element approximation in achieving arbitrary smoothness as well as to enhance efficiency of meshfree method in imposing essential boundary conditions. A rigorous error analysis and a convergence study are also presented.

In part II of this work authored by S. Li, H. Lu, W. Han, W.K. Liu, and D.C. Simkins, RKEM is extended to achieve arbitrary order of continuity and interpolation property in a multi-dimensional finite element setting with minimum degrees of freedom. This method also eliminates a major weakness of meshfree interpolants that often leads to loss of the optimal convergence rate in multiple dimensions when the polynomial reproducing order is larger than one. The numerical results show satisfactory performance of RKEM in solving higher order differential equations.

In part III of RKEM-related work, H. Lu, S. Li, D.C. Simkins, W.K. Liu, and J. Cao use the property of RKEM to introduce a generalized enrichment of the global partition polynomials. The enrichment is accomplished either by multiplying the enrichment function with the global partition polynomials, or by increasing the order of global partition polynomials in the same mesh. Using generalized enrichment, two optimal elements in two dimensional space have been constructed: T10P3I $4/3$ triangular element, satisfying the third order consistency condition with only ten degrees of freedom, and Q15P4I $4/3$ quadrilateral element, satisfying the fourth order consistency condition with 15 degrees of freedom.

RKEM part IV, co-authored by D.C. Simkins, S. Li, H. Lu, and W.K. Liu, introduces triangular element hierarchy based on RKEM interpolation. Several optimum triangular elements were constructed with both minimum degrees of freedom and higher order continuity. The construction of these optimum triangular elements is demonstrated by a direct approach using global polynomials and a parametric approach by mapping global partition polynomials to a parent triangle.

T. Rabczuk, T. Belytschko, and S.P. Xiao present a stable meshfree particle method for large deformation problems that employs a Lagrangian kernel with correction of derivatives. In this approach, the trial functions are corrected derivatives of the kernel functions and the test functions are Shepard functions. A Lagrangian kernel function is introduced to eliminate numerical tension instability. The use of nodal integration and selection of stress points are also studied. Various forms of smoothed particle hydrodynamics (SPH) are compared in numerical examples, and their numerical characteristics are summarized.

D. Wang and J.S. Chen propose a meshfree plate formulation to resolve shear locking in Mindlin–Reissner plate problem. They first introduce Kirchhoff mode reproducing conditions in the construction of shape functions so that the pure bending modes are included in the approximation space. The integration constraints that fulfill bending exactness in the Galerkin meshfree discretization of a Mindlin–Reissner plate are then derived. A nodal integration with curvature smoothing stabilization that fulfills bending exactness is formulated for a Mindlin–Reissner plate. The resulting meshfree formulation is efficient, stable, and free of shear locking in shear deformable plates.

J.S. Chen and D. Wang introduce an extended meshfree method for solving boundary value problems. This approach expresses the total solution by a combination of particular and homogeneous solutions. The particular solution is an analytical expression satisfying the governing differential equation but not necessarily the boundary conditions. The homogeneous solution is solved numerically with auxiliary boundary conditions determined by the particular solution. Numerical results demonstrate a significant improvement of the solution accuracy with commensurately less computational effort using the new approach compared to the conventional meshfree formulation.

A. Huerta, S. Fernandez-Mendez, and W.K. Liu compare bridging scale method and continuous blending method for coupling finite element and meshfree methods. Both methods preserve the desired consistency on the computational domain. The results show that in addition to the computational efficiency, the continuous blending method allows the placement of meshfree discretization only in the locations where they are needed. This property leads to a simple way of imposing Dirichlet boundary conditions. Furthermore, the continuous blending method preserves the optimal order of convergence of meshfree and finite element methods. This is not the case of the bridging scale method.

A. Huerta, Y. Vidal, and P. Villon propose a novel meshfree formulation for incompressible flow. In this approach, a pseudo-divergence-free interpolation is introduced so that the approximation functions satisfy the divergence-free condition in a diffuse derivative sense. It is shown that the diffuse derivative approaches the full derivative as the discretization is refined, and thus the pseudo-divergence-free interpolation approaches a divergence-free space in the discretization refinement. It is also noted that such an interpolation can be obtained under the standard moving least-squares implementation.

D. Sulsky and A. Kaul present an implicit material-point method (MPM) for large deformation analysis. The discrete equations of MPM are solved using conjugate gradients (CG) for the linear case, Newton–CG for the nonlinear case, and Newton’s method combined with the generalized minimum residual (GMRES) method, also for the nonlinear case. These Newton–Krylov solvers are implemented in a matrix-free fashion for numerical efficiency. In all test problems, if the time step is chosen appropriately, the implicit solution technique is more efficient than an explicit method without loss of desired features in the solutions.

X. Jin, G. Li, and N.R. Aluru provide a new viewpoint for analysis of meshfree collocation methods, namely through the concept of positivity conditions. In this paper, the positivity conditions commonly used

in the finite difference literatures are employed for constructing approximation functions and their derivatives. The significance of positivity conditions is pointed out by an error analysis of the finite cloud method, which is a collocation-based meshfree method. Procedures for constructing meshfree approximation that satisfy positivity conditions are also discussed. Numerical results demonstrate the effectiveness of the proposed approach in collocation methods.

P. Breitkopf, A. Rassineux, J.-M. Savignat, and P. Villon examine whether the diffuse element method can pass a linear patch test. First, the integration constraint for passing linear patch test is presented. It is shown that the use of complete derivatives is not a sufficient condition to pass the patch test. A Gauss integration with custom weights computed from the patch test conditions on numerical integration on a tile-by-tile basis is then proposed. The method also is extended to a beam problem with Hermite shape functions. Numerical tests show the success of this numerical integration for both complete and diffuse derivatives.

Z. Wu develops a quasi-interpolation method for radial basis functions applied to the solution of one-dimensional time dependent partial differential equations with adaptive repositioning of the sampling points. In this approach, a moving knots scheme for a nonlinear propagations equation is proposed. The sampling data points (knots) are moved relatively denser near the locations where the solution is relatively more oscillatory. The variation-diminishing property that guarantees the stability of the scheme is discussed. Burger's equation is solved to demonstrate the effectiveness of the proposed method.

G.E. Fasshauer introduces an approximate moving least squares method by combining moving least squares approximation method and the theory of approximate approximations proposed earlier by Maz'ya and Schmidt. This approach offers the simplicity in Shepherd's method as well as accuracy of higher order moving least squares approximations. The numerical results show the method for irregularly spaced points does not compare very favorably with the one for regularly spaced data.

J. Bonet, S. Kulasegaram, M.X. Rodriguez-Paz, and M. Profit present a variational formulation of smooth particle hydrodynamics. The discrete equations are derived directly from the particle form of Hamiltonian and thus the method preserves linear and angular momentum. The constitutive equation of the material is represented via an internal energy term. This proposed formulation works well for fluids as well as for solids with moderate plastic deformation.

Fernandez-Mendez and H. Huerta present a general overview on the existing techniques to enforce essential boundary conditions in Galerkin-based meshfree methods. Special attention is paid to methods that couple meshfree and finite element methods for the imposition of prescribed values and to methods based on a modification of the Galerkin weak form. In particular, a Nitsche's method is introduced to impose essential boundary conditions as an alternative of the penalty method. It is shown that the Nitsche's method maintains consistency and coercivity of the bilinear form, and the method also avoids ill-conditioning in the penalty method.

J.S. Chen, V. Kotta, H. Lu, D. Wang, D. Moldovan, and D. Wolf propose a variational formulation and a double-grid method for modeling stressed grain growth in polycrystalline materials. Modeling of stressed grain growth requires discretization of grain interiors as well as grain boundaries with the consideration of topological changes in grain structures. In the proposed double-grid method, the material domain is discretized by the meshfree shape functions with strain discontinuity enrichment across the grain boundaries. The grain boundaries, on the other hand, are discretized by the standard finite elements. This approach allows modeling of arbitrary evolution of grain boundaries without remeshing.

X. Wang and W.K. Liu develop an extended immersed boundary method (EIBM) for fluid–structure interaction. In EIBM, the submerged elastic solid, which occupies a finite volume within the fluid domain, is considered. Equivalent nodal forces calculated in the context of finite element formulations replace the kinematic and dynamic matching of the fluid–solid interface. Further, the discretized delta function commonly used in the IB method is replaced with the kernel functions to allow the use of required

smoothness and non-uniform discretization in the fluid domain. This approach is suitable for application in biological fluid involving molecular, cellular, and flexible vessel–flow interactions.

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