Stability Analysis in a Cognitive Radio System with Cooperative Beamforming

Mohammed Karmoose\textsuperscript{1}  Ahmed Sultan\textsuperscript{1}  Moustafa Youseff\textsuperscript{2}

\textsuperscript{1}Electrical Engineering Dept, Alexandria University
\textsuperscript{2}E-JUST
Agenda

1 Motivation and Related Work

2 System and Data Models

3 Problem Insights and Formulation

4 Queue Service Rates

5 Stability analysis using dominant systems
Motivation and Related Work

- Queuing analysis in CRNs to study the stability of the primary and secondary networks
- Shortcomings of such models: no time-frequency sharing between primary and secondary networks
- Cooperative beamforming to increase the available spectrum opportunities for the secondary networks
- No queuing analysis to characterize the stability regions of such systems
- Our goal: study a CR scenario in which cooperative beamforming is enabled for the secondary network, and obtain the stability region of the system for the primary and secondary network
System and Data Models

**Primary Network:**
- Single link (Tx-Rx pair)
- Primary transmit power $P_p$
- Infinite buffer $Q_p$ - Bernoulli arrival R.P with mean $\lambda$
- Noise at the receivers $\mathcal{CN} \sim (0, 1)$

*Figure: System Model*
**System and Data Models**

**Secondary Network:**

- Single link (Tx-Rx pair)
- Relay-assisted transmission ($K$ relays) operating in a decode-and-forward fashion
- Relays are close to the secondary source
- Power control on total transmitted power by all relays ($P_s \leq P_{\text{max}}$)
- Infinite buffer $Q_s$ - Bernoulli arrival R.P with mean $\lambda_s$
- Noise at the receivers $\mathcal{CN} \sim (0, 1)$

**Figure:** System Model

Mohammed Karmoose, Ahmed Sultan, Moustafa Youseff

Stability Analysis in a Cognitive Radio System with Cooperative Beamforming
Motivation and Related Work

System and Data Models

Problem Insights and Formulation

Queue Service Rates

Stability analysis using dominant systems

System and Data Models

Channel Conditions:

- Small pathloss between secondary source and relays (error free communications)
- Imperfect sensing of primary transmission
- Channel between relays and receivers are $CN \sim (0, 1)$
- Slow fading (channel constant over many time slots)

Figure: System Model
System and Data Models

System Operation:

- Perfect channel estimation at relays based on transmitted ARQs
- Channel estimations are reported to secondary transmitter (negligible time)
- PU packets are sensed by secondary transmitter - imperfect sensing ($p_{md}$ and $p_{fa}$)

Figure: System Model
System and Data Models

- Secondary transmitter computes required beamforming weights
- Secondary data packet is sent to relays which decode perfectly (beamforming vector included in the header)
- Time required for relay reception and decoding is negligible

**Figure:** System Model

Mohammed Karmoose, Ahmed Sultan, Moustafa Youseff
Alex. Univ.

Stability Analysis in a Cognitive Radio System with Cooperative Beamforming
System and Data Models

- If PU is detected
  \[ w_p = \sqrt{P_s} \frac{(I - \Phi)H_s}{\sqrt{H_s^H (I - \Phi) H_s}} \]
- If PU is not detected
  \[ w_a = \sqrt{P_s} \frac{H_s}{\|H_s\|}. \]

Figure: System Model

Mohammed Karmoose, Ahmed Sultan, Moustafa Youseff
Stability Analysis in a Cognitive Radio System with Cooperative Beamforming
If sensing was perfect: PU will not be aware of the secondary transmitter. What would only limit its achievable service rate ($\mu_p$) is the channel conditions between PU Tx-Rx.

If sensing is not perfect: SU can unintentionally interfere with PU transmission, reducing achievable service rate of primary user.

For the SU: Two factors govern its achievable service rate ($\mu_p$):

1. Channel conditions between relays and secondary destination
2. The rate at which PU uses the channel (i.e., $\lambda_p$ and $\mu_p$)
Problem Insights and Formulation

Problem Formulation

- Stability for a queue is achieved iff $\lambda < \mu$
- We try to find the stability region of the system (the regions of $\lambda_p$ and $\lambda_s$ for which the two queues are stable), i.e.:

$$
\lambda_p < \mu_p \\
\lambda_s < \mu_s
$$
**Queue Service Rates**

**Primary Queue:** If secondary queue is empty OR secondary queue is not empty and PU is detected:

\[ p_{\text{out},p} = \Pr\{P_p|H_p|^2 < \beta_p\} = 1 - \exp\left(\frac{-\beta_p}{P_p}\right) \]

If secondary queue is not empty and PU is misdetected:

\[ p_{\text{out},p}^{\text{md}} = \Pr\left\{ \frac{P_p|H_p|^2}{|H_{sp}^H w_a|^2 + 1} < \beta_p \right\} \]

Service rate:

\[ \mu_p = (1 - p_{\text{out},p}) \left( \Pr\{Q_s = 0\} + (1 - p_{\text{md}})\Pr\{Q_s \neq 0\} \right) \]

\[ + (1 - p_{\text{out},p}^{\text{md}}) p_{\text{md}} \Pr\{Q_s \neq 0\} \]
Queue Service Rates

Secondary Queue:

PU queue is empty
AND not detected:

\[ p_{\text{out},s} = \Pr \{ P_s \| H_s \|^2 < \beta_s \} \]

PU queue is empty
AND detected (false alarm):

\[ p_{\text{out},s}^{fa} = \Pr \{ |H_s^H w_p|^2 < \beta_s \} \]

PU queue is not empty
AND detected:

\[ p_{\text{out},s}^{(d)} = \Pr \{ \frac{|H_s^H w_p|^2}{P_p |H_{ps}|^2 + 1} < \beta_s \} \]

PU queue is not empty
AND misdetected:

\[ p_{\text{out},s}^{md} = \Pr \{ \frac{P_s \| H_s \|^2}{P_p |H_{ps}|^2 + 1} < \beta_s \} \]

\[ \mu_s = \left( p_{\text{out},s}^{(d)} (1 - p_{fa}) + p_{\text{out},s}^{fa} p_{fa} \right) \Pr \{ Q_p = 0 \} + \left( p_{\text{out},s}^{md} (1 - p_{md}) + p_{\text{out},s}^{md} p_{md} \right) \Pr \{ Q_p \neq 0 \} \]
Stability analysis using dominant systems

- Continuing analysis is hard to interacting queues
- We use the concept of “dominance”: We assume two auxiliary systems
  1. Primary queue is dominant (sends dummy packets when queue is empty)
  2. Secondary queue is dominant (sends dummy packets when queue is empty)
- It is proven that the union of the stability regions of both systems is exactly the stability region of the original system
Stability analysis using dominant systems

Primary dominant queue:

- \( \Pr\{Q_p = 0\} = 0 \)
- \( \mu_s^{pd} = \left( p^{(d)}_{out,s} (1 - p_{md}) + p^{md}_{out,s} p_{md} \right) \)
- Secondary queue does not depend on the state of primary queue (channel stationarity is guaranteed). We apply Little’s theorem \( \Pr\{Q_s = 0\} = 1 - \frac{\lambda_s}{\mu_s^{pd}} \)
- \( \mu_p^{pd} = (1 - p_{out,p}) - \frac{\lambda_s}{\mu_s^{pd}} p_{md} \left( p^{md}_{out,p} - p_{out,p} \right) \)
Stability analysis using dominant systems

**Effect of changing** $P_s$:

- **On secondary rate:** Increasing $P_s$ increases $\mu_{s}^{pd}$
- **On primary rate:**
  - Increasing $P_s$ increases interference on the primary receiver when misdetection occurs
  - Increasing $P_s$ helps SU empty its queue faster and evacuates the channel

- The impact of both effects depend on $\lambda_s$ and $\mu_{p}^{pd}$

- Optimization problem over $P_s$ to maximize $\mu_{p}^{pd}$

$$\max_{P_s} \lambda_p = \mu_{p}^{pd} \quad \text{s.t.} \quad \lambda_s < \mu_{s}^{pd}, \quad P_s \leq P_{\text{max}}$$
Stability analysis using dominant systems

Primary dominant queue:

Figure: $P_p = 1, K = 4, p_{md} = 0.1, p_{fa} = 0.01, \beta_p = \beta_s = 1$ and $P_{max} = 2$. 
Stability analysis using dominant systems

Primary dominant queue:

Figure: $P_p = 1$, $K = 4$, $p_{md} = 0.1$, $p_{fa} = 0.01$, $\beta_p = \beta_s = 1$ and $P_{\text{max}} = 2$. 
Stability analysis using dominant systems

Secondary dominant queue:

- \( \Pr\{Q_s = 0\} = 0 \)
- \( \mu_{sp}^{sd} = (1 - p_{out,p})(1 - p_{md}) + (1 - p_{out,p}^{md})p_{md} \)
- Primary queue does not depend on the state of secondary queue (channel stationarity is guaranteed). We apply Little's theorem \( \left( \Pr\{Q_p = 0\} = 1 - \frac{\lambda_p}{\mu_{sp}^{sd}} \right) \)

\[
\mu_{sp}^{sd} = \frac{\lambda_p}{\mu_{sp}^{sd}} \left( p_{out,s}^{(d)}(1 - p_{md}) + p_{out,s}^{md}p_{md} - p_{out,s}(1 - p_{fa}) \right.
- p_{out,s}^{fa}p_{fa} \) + \left( p_{out,s}^{fa}(1 - p_{fa}) + p_{out,s}^{fa}p_{fa} \right)
\]
Stability analysis using dominant systems

Effect of changing $P_s$:

- **On primary rate**: Increasing $P_s$ decreases $\mu_p^{sd}$
- **On secondary rate**:
  - Increasing $P_s$ enhances SINR of secondary transmission and increase $\mu_s^{sd}$
  - Increasing $P_s$ interferes with primary transmission - PU will occupy the channel for longer times which decreases $\mu_s^{sd}$

- The impact of both effects depend on $\lambda_p$ and $\mu_s^{sd}$
- Optimization problem over $P_s$ to maximize $\mu_s^{sd}$

$$\max_{P_s} \lambda_s = \mu_s^{pd} \quad \text{s.t.} \quad \lambda_p < \mu_p^{pd}, \quad P_s \leq P_{\text{max}}$$
Stability analysis using dominant systems

Secondary dominant queue:

Figure: $P_p = 1$, $K = 4$, $p_{md} = 0.1$, $p_{fa} = 0.01$, $\beta_p = \beta_s = 1$ and $P_{max} = 2$. 

Mohammed Karmoose, Ahmed Sultan, Moustafa Youseff

Stability Analysis in a Cognitive Radio System with Cooperative Beamforming
Stability analysis using dominant systems

Secondary dominant queue:

\[ \text{2nd Dominant System} \]

\[ \lambda_p = 0, \lambda_p = 0.1, \lambda_p = 0.35, \lambda_p = 0.36 \]

**Figure:** \( P_p = 1, K = 4, p_{md} = 0.1, p_{fa} = 0.01, \beta_p = \beta_s = 1 \) and \( P_{\text{max}} = 2 \).
Stability analysis using dominant systems

Stability region of the original system:

Figure: $K = 4$, $p_{md} = 0.1$, $p_{fa} = 0.01$, $\beta_p = \beta_s = 1$ and $P_{max} = 2$. 
Stability analysis using dominant systems

Stability region of the original system:

Figure: \( P_p = 1, 10, K = 4, p_{md} = 0.1, p_{fa} = 0.01, \beta_p = \beta_s = 1 \) and \( P_{\text{max}} = 2 \).
Thank you

For any questions, feel free to contact the author at m_h_karmoose@alexu.edu.eg