

MAE 270B

Linear Optimal Control

Course Description

This course treats Linear Optimal Control, which usually refers to problems in which the plant is linear and the performance index is a quadratic function of the state and control functions (or sequences).

Prerequisites Students should have completed a course in linear systems, preferably a graduate course. Knowledge of differential equations, complex variables, real analysis, and linear algebra that is required for a graduate-level linear systems course is required for this course.

Although linear operators in Hilbert spaces are used extensively in the course, no prior background in functional analysis is assumed. The course begins with an introduction to Hilbert space and linear operators. This introduction should provide the limited knowledge of functional analysis and operator theory that is required for this course.

Goals The linear operator/Hilbert space framework is used in an attempt to convey a fundamental understanding of the underlying principles of linear optimal control, but an equally important goal of the course is that students learn how to solve a variety of the most important linear optimal control problems.

Class Notes and References The course will be taught from the class notes, which should be sufficient. However, outside reading from suggested references will enhance students' mathematical backgrounds and provide alternative, sometimes more classical, approaches to linear optimal control theory.

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