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Control of Linear Systems with Nonlinear Disturbance Dynamics

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Abstract

Globally asymptotic tracking and disturbance rejection is a desirable performance in many applications. Linear feedback control based on internal model principle achieves asymptotic tracking performance for linear systems with linear exogenous signal dynamics. This paper investigates the case of tracking or rejecting unknown exogenous signals with known nonlinear generating dynamics particularly chaotic signals for linear systems in the continuous time domain. Two different control structures are investigated: nonlinear internal model principle control and predictive internal model control. It is shown that globally asymptotic tracking or disturbance rejection can be achieved when perfect model matching for the linear system is possible. The closed loop robust stability and performance rely on the relative size of the model matching errors to the exogenous signal's local growth rate.

1. Introduction

Tracking or rejection of exogenous signals is of major concern in feedback control design. The exogenous signals can often be modeled as unknown deterministic signals with known signal generating dynamics. For linear systems with linear disturbance dynamics, this problem has been studied by Davison (1976) and Francis and Wonham (1976) etc.. The terminology of "Internal Model Principle" (IMP) was coined by Francis and Wonham (1976), which shows that it is necessary to place the disturbance dynamics in the feedback control loop for achieving asymptotic tracking or disturbance rejection. Integral control and repetitive control are examples of IMP type controllers. Recently more research efforts have been concentrated on developing similar concepts for nonlinear systems. Isidori and Byrnes (1990) discussed the local output regulation of nonlinear systems and gave necessary and sufficient conditions for solvability of the problem. The so called "nonlinear regulator equation" is proposed and discussed. Isidori (1997) further extended the results to the semiglobal output regulation problem, where the initial condition of the disturbance dynamics is confined in a bounded set. This approach is a natural extension of linear internal model principle to nonlinear settings. However the nonlinear regulator equation which is a partial differential equation (PDE) is usually hard to solve and in many cases, the solution of the nonlinear regulator equation may not exist at all. Huang and Lin (1991) and Huang (1995, 1998) proposed a kth-order robust nonlinear servomechanism design, and discussed the existence conditions of the kth-order servomechanism. A k-fold exosystem is built based on the linearized disturbance model. Then a feedback controller including the k-fold exosystem is designed based on the linearized plant model. It is shown that the controlled system output will be bounded by the disturbance up to the kth order. However this is a local result which requires the initial conditions of the plant and disturbance close enough to the origin. Also to reject the disturbance to the k-th order, one need to use a very complex exosystem model in the controller design which will result in a high order controller and may not be able to be implemented for real time applications.

Internal model control (IMC) is another class of control design for disturbance rejection. Garcia and Morari (1982, 1985, 1986) provided a unifying review on internal model control and further extended it to multivariable systems and nonlinear systems. A crucial step for applying internal model control is system inversion. Unstable zero dynamics pose fundamental constraints on system inversion performance. Robustness to modeling errors is always a concern for internal model control. A detailed discussion was provided by Morari and Zafiriou (1989). Tsypkin (1993) proposed a robust internal model control. The so called "absorption principle" was used, which essentially embedded the disturbance signal model in the internal model control structure for asymptotic tracking performance.

Another closely related topic to asymptotically tracking or rejecting exogenous nonlinear signals is chaos synchronization (Kapitaniak, 1996, etc.). Feedback controller is designed to synchronize a nonlinear system with a chaotic system. One important application of chaos synchronization is secure communication. The information signal is coded into a chaotic signal and sent out through the transmitter. The receiver can automatically synchronize the chaotic signal and then decode the information signal. Theoretically chaos synchronization is a special case of the general problem for tracking or rejecting nonlinear exogenous signals. It is assumed the chaotic signal can be directly measured, so plant dynamics are not considered. For the case of secure communication, it means that the transmitted signal can be directly obtained by the receiver without filtering through certain channel dynamics.

This paper investigates the controller synthesis problem of globally asymptotically rejecting exogenous nonlinear input or output disturbances for linear systems in the continuous time domain. Sun and Tsao (1999) presented a discrete time output regulation control design for rejecting nonlinear exogenous disturbances. There are two major difference between the continuous and discrete time disturbance rejection problem. First, in the discrete time domain, future disturbance can be obtained from the disturbance model based on current and previous disturbances. While in the continuous time domain, only high order derivatives can be obtained. Second, in the continuous time domain, relative degree of the control plant will confine the solution of the model matching problem. Two different control schemes will be presented in this paper. The nonlinear internal model principle (NIMP) control scheme is designed by incorporating nonlinear disturbance model in the feedback loop. The predictive internal model control (PIMC) scheme is proposed by predicting the disturbance and its derivatives based on the disturbance model in the internal model control structure. It will be shown that these two schemes are identical in certain cases. Sufficient conditions to achieve asymptotic disturbance rejection are derived. Closed loop robust stability and performance due to unmodeled dynamics and model matching errors are discussed and then demonstrated in the simulations.

The rest of this paper is organized as follows. Section 2 describes the system and disturbance models; Section 3 presents the feedback control design; Section 4 discusses the robust stability and performance of the closed loop system; Section 5 shows the simulation results and Section 6 is the conclusion.

2. Problem Description

Consider the following single input single output continuous linear time invariant causal system:

$$A(s)y(s) = B(s)u(s) + C(s)d_m(s) \tag{1}$$

where $u(s)$ and $y(s)$ are input and output respectively. $d_m(s)$ is the unmeasurable bounded disturbance. $A(s)$ and $B(s)$ are coprime. $C(s)$ is Hurwitz and $\deg C(s) = \deg A(s)$.

Suppose the disturbance satisfies the following nonlinear model:

$$\begin{aligned} \dot{v}(t) &= f(v(t)) + g(v(t))\sigma(d_m(t)) \\ d_m(t) &= h(v(t)) \end{aligned} \tag{2}$$

where $d_m(t) \in R$, $v(t) \in R^w$, $f: R^w \to R^w$, $g: R^w \to R^{w \times w}$, $\sigma: R \to R^w$ and $h: R^w \to R$.

If the original system (1) is not stable, we can first stabilize it by designing a feedback controller. So without losing generality, we assume the system is stable. For notation convenience, we rewrite system (1) into the following form:

$$y(s) = \pi(s)u(s) + d(s) \tag{3}$$

where $\pi(s) = \dfrac{B(s)}{A(s)}$, $d(s) = r(s)d_m(s) = \dfrac{C(s)}{A(s)}d_m(s)$.

Since $A(s)$ and $C(s)$ are both Hurwitz, $r(s)$ is proper Hurwitz and inversely Hurwitz. Therefore the disturbance $d(t)$ satisfies following nonlinear model:

$$\begin{aligned} \dot{v}(t) &= f(v(t)) + g(v(t))\sigma r^{-1}(d(t)) \\ d(t) &= rh(v(t)) \end{aligned} \tag{4}$$

The control law we are considering is output feedback. Details will be presented in the following section. The control objective is to achieve $\lim_{t\to\infty} y(t) = 0$ for any initial conditions of the plant and disturbance.

The above control system and objective differ our problem from previous output regulation results in two aspects. One is that we will present global asymptotic output regulation results for linear systems with nonlinear disturbance dynamics. There is no constraints that the plant and disturbance initial conditions need to stay in a small neighborhood of the origin. The other is that the disturbance model doesn't need to be neutrally stable. Instead, we allow the Jacobian matrix of the disturbance model at the equilibria contains unstable eigenvalues, which actual represents the case of chaotic disturbances.

3. Feedback Control Design

Two different output feedback control schemes will be presented in this section. The control structures are first outlined. Conditions to achieve asymptotic disturbance rejection are then provided. Robust stability and performance of the closed loop system will be addressed in the following section.

- *Nonlinear Internal Model Principle Control (NIMP)*

As shown in Fig. 1, the nonlinear internal model principle control law is as follows:

$$u(t) = \theta(z(t)),\ z(t) = \phi(-\psi(z(t)) + y(t)) \tag{5}$$

where $\theta(\cdot)$ and $\psi(\cdot)$ are time domain linear stable operators and their Laplace domain representations are $\theta(s)$ and $\psi(s)$ respectively. $\phi(\cdot)$ is a time domain nonlinear operator.

- *Predictive Internal Model Control (PIMC)*

The block diagram of PIMC is shown in Fig. 2 and the control law is as follows:

$$u(t) = \theta(z(t)),\ z(t) = \phi(-\hat{\pi}\theta(z(t)) + y(t)) \tag{6}$$

where $\hat{\pi}(\cdot)$ is the nominal model of the control plant. $\theta(\cdot)$ and $\phi(\cdot)$ are defined as above.

Following are the two conditions $\theta(\cdot)$, $\psi(\cdot)$ and $\phi(\cdot)$ need to satisfy to achieve asymptotic disturbance rejection:

$$\psi(s) = \pi(s)\theta(s) \tag{7}$$

$$\lim_{t\to\infty} \psi\phi(d(t)) = -d(t) \tag{8}$$

Condition (7) is to find the stabilizing feedback controller by solving the model matching problem. If $\pi(s)$ is a minimum phase system, we can design $\theta(s)=\frac{\psi(s)}{\pi(s)}$ by direct inversion. If $\pi(s)$ is a non-minimum phase system, we can obtain an optimal approximate solution by solving

$$\theta(s)=\arg\inf_{\tilde{\theta}(s)\in l_1}\left\|\psi(s)-\pi(s)\tilde{\theta}(s)\right\|_\infty \quad (9)$$

Condition (8) is to build a disturbance observer based on the disturbance model (4). Due to its asymptotic observer performance, we name condition (8) as the generalized disturbance model. For general nonlinear disturbance dynamics, to build the generalized disturbance model is not trivial. Consider a special case of disturbance model (4):

$$\begin{aligned}\dot{v}(t)&=Fv(t)+\sigma r^{-1}(d(t))\\ d(t)&=r(Hv(t))\end{aligned} \quad (10)$$

One example for building the generalized disturbance model for (10) is given as follows:

$$\psi(s)=-1$$

$$\phi(d(t)):\begin{cases}\dot{\hat{v}}(t)=F\hat{v}(t)+\sigma r^{-1}d(t)+Kr^{-1}(\hat{d}(t)-d(t))\\ \hat{d}(t)=r(H\hat{v}(t))\end{cases} \quad (11)$$

From (10) and (11), we have

$\dot{e}(t)=(F+KH)e(t)$, where $e(t)=v(t)-\hat{v}(t)$

If $F+KH$ is Hurwitz, $\hat{v}(t)\to v(t)$ and $\hat{d}(t)\to d(t)$, so condition (8) is satisfied.

Remark: The generalized disturbance model is incorporated in the feedback loop of NIMP control to achieve asymptotic performance. Also note the proposed predictive internal model control (PIMC) is different from ordinary internal model control because the disturbance model is inserted into the closed loop to predict the disturbance. Actually it is a integration of internal model control and internal model principle control.

Remark: Condition (7) and (8) are two independent conditions. As we mentioned above, condition (7) is to find the stabilizing feedback controller, while condition (8) is to build the asymptotic disturbance observer. We can design them separately, which satisfies the separation principle. $\psi(\cdot)$ is used to connect these two conditions.

Remark: Disturbance model (10) represent a large class of nonlinear signal dynamics, including chaotic signals, such as Chua's circuit, Duffing's equation and van der Pol's equation etc..

4. Robust Stability and Performance Analysis for NIMPC and PIMC

Denote

Δ_1 : Unmodeled dynamics of plant, i.e. $\pi(s)=\hat{\pi}(s)(1+\Delta_1)$

Δ_2 : Model matching error, i.e. $\hat{\pi}(s)\theta(s)=\psi(s)(1+\Delta_2)$

As shown in Fig.1 and 2, in both control schemes, only $\phi(\cdot)$ contains nonlinear dynamics. We can always divide the closed loop system into two parts: one is the linear part, the other part contains only static nonlinear functions. Without losing generality, in the following theorem, we assume $\phi(\cdot)$ is a static nonlinear function, otherwise the linear dynamics in $\phi(\cdot)$ can always be absorbed into $\psi(\cdot)$ and $\theta(\cdot)$. Also for simplicity, we assume the input to $\phi(\cdot)$ is a scalar in the following theorem, while similar result can be obtained for the vector input case using the same approach.

Theorem 1: Suppose $\phi(0)=0$ and $\frac{|\phi(\sigma)|}{|\sigma|}\le M$ for some finite positive number M.

(a): The NIMP control is globally asymptotically stable if

$$M\,\|\psi(s)(\Delta_1+\Delta_2+\Delta_1\Delta_2)\|_\infty<1 \quad (12)$$

(b): The PIMC is globally asymptotically stable if

$$M\,\|\psi(s)(\Delta_1+\Delta_1\Delta_2)\|_\infty<1 \quad (13)$$

(c): Asymptotic disturbance rejection can be achieved with both schemes if conditions (7) and (8) are satisfied.

(d): If the disturbance model (4) is linear, the NIMP control will achieve asymptotic disturbance rejection even when $\Delta_1,\Delta_2\neq 0$, while the PIMC won't.

Proof:

(a): As shown in Fig.1, we can separate the system into a feedback loop with a linear block $\psi(s)-\pi(s)\theta(s)$ and a memoryless nonlinear block. By the Circle Criterion (Khalil, 1996), the feedback system is globally asymptotically stable if

$$\|\psi(s)-\pi(s)\theta(s)\|_\infty<\frac{1}{M}$$

Since $\psi(s)-\pi(s)\theta(s)=\psi(s)(\Delta_1+\Delta_2+\Delta_1\Delta_2)$, (12) follows.

(b): Similarly, in view of Figure 2, $\|\pi(s)\theta(s)-\hat{\pi}(s)\theta(s)\|_\infty<\frac{1}{M}$, which implies (13).

(c): For the nonlinear internal model principle control (NIMP), the steady state output is:

$$y(t)=d(t)+\pi\theta\phi(-\psi(z(t))+y(t))$$
$$y(t)=d(t)+\pi\theta\phi(-\psi(z(t))+\pi\theta(z(t))+d(t))$$

If condition (7) and (8) are satisfied, we have

$$y(t)=d(t)+\psi\phi(d(t))=0$$

For predictive internal model control (PIMC), the steady state output is:

$$y(t)=d(t)+\pi\theta\phi(-\hat{\pi}\theta(z(t))+y(t))$$
$$y(t)=d(t)+\pi\theta\phi(-\hat{\pi}\theta(z(t))+\pi\theta(z(t))+d(t))$$

If condition (7) is satisfied, it implies $\hat{\pi}(s) = \pi(s)$. Combining with condition (8), we have
$y(t) = d(t) + \psi\phi(d(t)) = 0$

(d): Suppose the disturbance model (4) is linear, then $\phi(\cdot)$ becomes a linear operator. For steady state output, plug $d(t) = -\psi\phi(d(t))$ into above derivation, we have following results:
$[1+(\psi(\cdot)-\pi\theta(\cdot))\phi(\cdot)]y(t) = (1+\psi\phi(\cdot))d(t) = 0$ for NIMPC,
$[1+(\hat{\pi}\theta(\cdot)-\pi\theta(\cdot))\phi(\cdot)]y(t) = [1+(1+\Delta_2)\psi\phi(\cdot)]d(t) = -\Delta_2 d(t)$ for PIMC.

Remark: If $0 \le \frac{\phi(\sigma)}{\sigma} \le M$, the Circle Criterion gives the stability condition that:

$\mathrm{Re}(\psi(e^{-j\omega})(\Delta_1+\Delta_2+\Delta_1\Delta_2)) > \frac{-1}{M}$ for NIMPC, and $\mathrm{Re}(\psi(e^{-j\omega})(\Delta_1+\Delta_1\Delta_2)) > \frac{-1}{M}$ for PIMC.

Remark: From the above derivation, we find that the PIMC is more robust than the NIMC. For linear disturbance dynamics, the internal model principle control provides asymptotic tracking performance while the predictive internal model control can't due to the model matching error.

5. Simulation Results

In this section, we will simulate the disturbance rejection performance of the proposed schemes. A linear system with unmodeled dynamics and a non-minimum phase zero is used in the simulations.

$$\pi(s) = \hat{\pi}(s)(1+\frac{1}{s+\alpha}) \tag{14}$$

where the nominal model $\hat{\pi}(s)$ is:

$$\hat{\pi}(s) = \frac{s-\beta}{(s+7)^2} \text{ with } \beta > 0 \tag{15}$$

The Duffing's equation is used as the disturbance model:

$$\ddot{d} + \delta\dot{d} - d + d^3 = \gamma\cos(t+t_0) \tag{16}$$

where $d(0)$, $\dot{d}(0)$ and t_0 are unknown.

This model represent the chaotic motion of a nonlinear spring, damper and mass system. It is a good approximation of many chaotic mechanical systems, for example, the chaotic motion of the spindle due to rolling bearing defect. The chaotic motion is very sensitive to the initial conditions and the values of δ and γ. In the following simulations, we choose $\delta = 0.25$ and $\gamma = 0.4$. It is easy to verify that the Jacobian matrix of (16) at origin has unstable eigenvalue.

The block diagrams of NIMP control and PIMC to reject the chaotic disturbance are shown in Fig. 3 and 4 respectively. Now we need to design $\psi(\cdot)$, $\theta(\cdot)$ and $\phi(\cdot)$.

We choose $\psi(s) = \frac{-1}{s+a}$.

First, we need to find $\theta(s)$ to satisfy condition (7). Since $\pi(s)$ is a non-minimum phase system, direct inversion will result in an unstable controller. Instead we solve Eq. (9) to find the optimal approximate controller.

$$\theta(s) = \frac{(s+7)^2}{(a+\beta)(s+a)} \tag{17}$$

Next we design a nonlinear observer to satisfy condition (8).

$$z(t) = \phi(d(t)) \Leftrightarrow$$
$$\dot{X}(t) = AX(t) + B\sigma\Gamma(d(t)) + K(d(t)-\hat{d}(t)) \tag{18}$$
$$\hat{d}(t) = CX(t)$$
$$z(t) = PX(t)$$

where $X(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \end{bmatrix}$, $A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -\delta & 0 & \gamma \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$,

$C = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}^T$, $P = \begin{bmatrix} a \\ 1 \\ 0 \\ 0 \end{bmatrix}^T$, $K = \begin{bmatrix} 4.75 \\ 7.1625 \\ 14.869 \\ 6.875 \end{bmatrix}$

$$\bar{d}(t) = \Gamma(d(t)) = \begin{cases} 1.6, & d(t) > 1.6 \\ d(t), & 1.6 \ge d(t) \ge -1.6 \\ -1.6, & d(t) < -1.6 \end{cases} \tag{19}$$

and $\sigma(\bar{d}(t)) = \bar{d}(t) - \bar{d}^3(t)$

In the above observer design, A itself is not strictly Hurwitz since states $x_3(t)$ and $x_4(t)$ are included to estimate the external driving force $\gamma\cos(t+t_0)$. K is designed so that $A-KC$ is strictly Hurwitz. Then it is easy to verify that condition (8) is satisfied.

Remark: Note the optimal controller (17) is not proper since the plant model (15) has a relative degree 1. To implement the controller, derivatives of the input signal $z(t)$ (see Fig.3 and 4) are required, which are available from observer (18).

To evaluate the stability of the closed loop control system, we need to divide the system into a linear block and a static nonlinear block. The linear block is:

$$T = \frac{T_1}{1-T_1T_2} \times T_3$$

where

$T_1 = \pi(s)\theta(s) - \psi(s)$ and $\|T_1\|_\infty \le \frac{2a+\beta+a\alpha}{a\alpha(a+\beta)}$ for NIMPC

$T_1 = \pi(s)\theta(s) - \hat{\pi}(s)\theta(s)$ and $\|T_1\|_\infty \le \dfrac{2a+\beta}{a\alpha(a+\beta)}$ for PIMC

$T_2 = P(sI - A + KC)^{-1}K$

$T_3 = P(sI - A + KC)^{-1}B$

The static nonlinear block is $\sigma\Gamma(d(t))$ and the bound is $M = 1.56$ based on (19).

Choose $a = 0.1$, $\alpha = 200$ and $\beta = 10$, we have

For NIMPC:

$$\| T \|_\infty \le \left\| \frac{1}{1 - T_1 T_2} \right\|_\infty \frac{2a+\beta+a\alpha}{a\alpha(a+\beta)} \|T_3\|_\infty \le 0.4212 < \frac{1}{M} = 0.641$$

For PIMC:

$$\| T \|_\infty \le \left\| \frac{1}{1 - T_1 T_2} \right\|_\infty \frac{2a+\beta}{a\alpha(a+\beta)} \|T_3\|_\infty \le 0.1428 < \frac{1}{M} = 0.641$$

Therefore Theorem 1 predicts global asymptotic stability for both NIMPC and PIMC. Figure 5 and 6 show the chaotic disturbance with its estimate, the derivative of the disturbance with its estimate generated by model (16) with $d(0) = 0.8$, $\dot{d}(0) = 0.9$ and $t_0 = 0.7854$ respectively. Figure 7 shows the asymptotic disturbance rejection performance of the NIMPC and PIMC respectively.

6. Conclusions

This paper addresses the control synthesis problem to achieve global output regulation for linear systems with nonlinear disturbance dynamics, particularly chaotic disturbances. The proposed control schemes differ from previous results in two aspects. One is that the initial conditions of the plant and the disturbances are not required to stay in a small neighborhood of the origin, nor do we assume the bound of the initial conditions is known. The other is that the disturbance model is allowed to contain unstable dynamics, which actually represents the case of chaotic disturbances. Two specific algorithms have been proposed: nonlinear internal model principle control and predictive internal model control. The effects of the disturbance growth rate, system unmodeled dynamics, and non-minimum phase plant model matching errors on the system stability and performance are derived.

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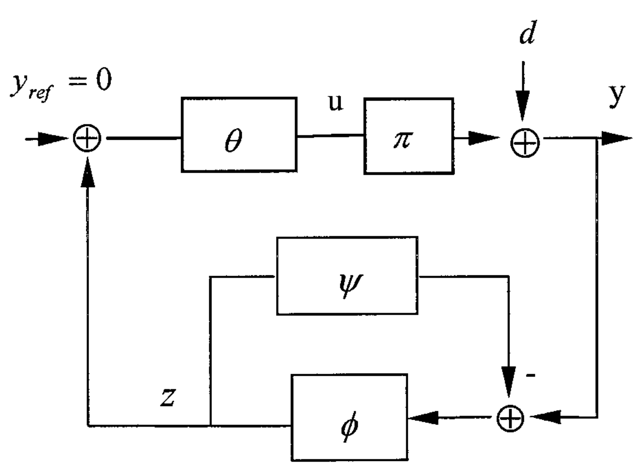

Fig. 1 Block Diagram for the Nonlinear Internal Model Principle Control

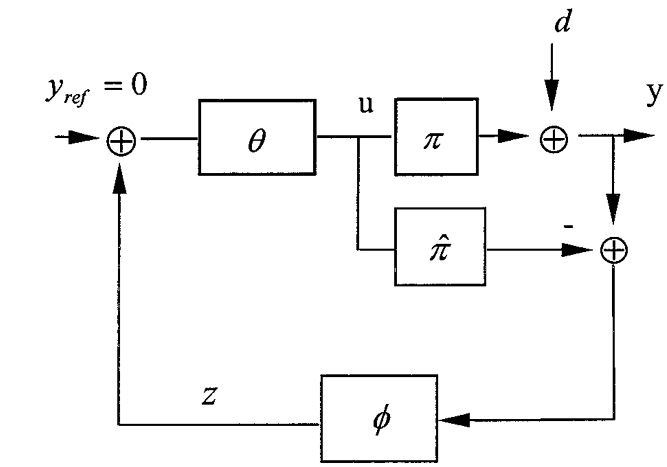

Fig. 2 Block Diagram of Predictive Internal Model Control

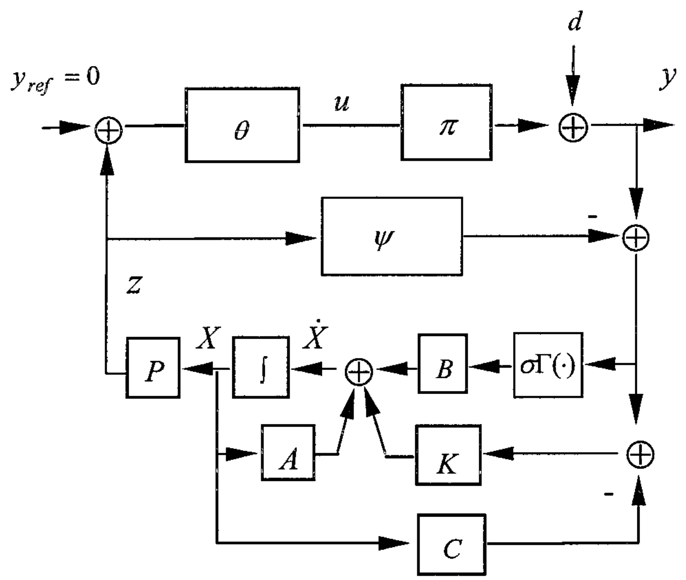

Fig. 3 Block Diagram of NIMP Control for Duffing's Equation

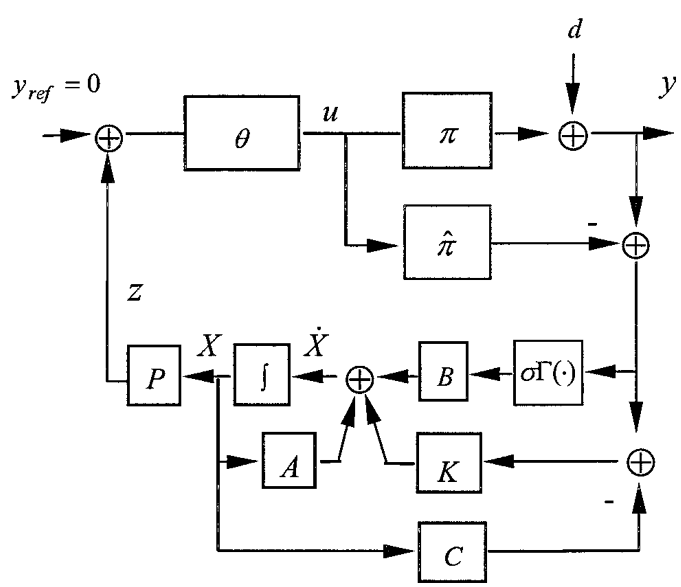

Fig. 4 Block Diagram of PIMC for Duffing's Equation

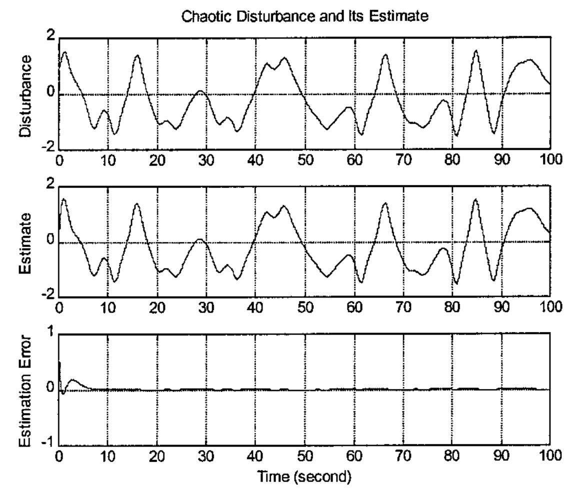

Fig. 5 Chaotic Disturbance and Its Estimate

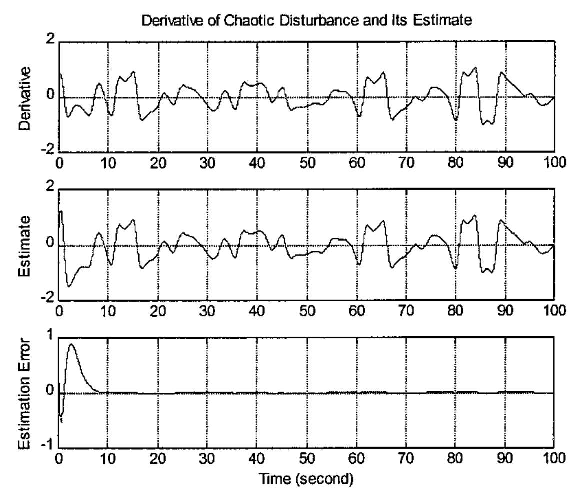

Fig. 6 Derivative of Chaotic Disturbance and Its Estimate

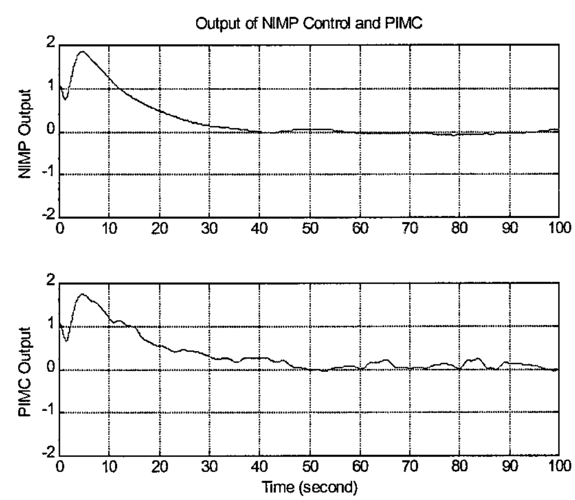

Fig. 7 Actual System Outputs of NIMPC and PIMC With Unknown $d(0)$, $\dot{d}(0)$ and t_0