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Disturbance Rejection for Nonlinear Systems

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Abstract

Tracking or rejection of unknown exogenous signals with known generating dynamics is of major concern in feedback control design. Linear feedback control based on the internal model principle achieves asymptotic performance for linear systems with linear exogenous signal dynamics. This paper presents a control design based on the internal model principle to track or reject nonlinear exogenous signal dynamics for nonlinear systems. Necessary condition to achieve asymptotic disturbance rejection based on the proposed control structure is first derived. It is shown that the necessary condition becomes sufficient for linear systems with linear disturbance dynamics. Inspired by the unique structure of the necessary condition, sufficient conditions are then proposed. Simulations of a nonlinear plant with chaotic disturbance show the effectiveness of the proposed scheme.

1. Introduction

Globally asymptotic tracking and/or rejection of exogenous signals is a desirable performance in many control systems. The exogenous signals can often be modeled as unknown deterministic signals with known signal generating dynamics. For linear systems with linear disturbance dynamics, this problem has been studied by Davison (1976) and Francis and Wonham (1976) etc.. The terminology of "Internal Model Principle" (IMP) was coined by Francis and Wonham (1976), which shows that it is necessary to place the disturbance dynamics in the feedback control loop for achieving asymptotic tracking or disturbance rejection. Integral control and repetitive control are examples of IMP type controllers. Recently more research efforts have been concentrated on developing similar concepts for nonlinear systems. Isidori and Byrnes (1990) discussed the local output regulation of nonlinear systems and gave necessary and sufficient conditions for solvability of the problem. The so called "nonlinear regulator equation" is proposed and discussed. This approach is a natural extension of linear internal model principle to the nonlinear settings. However the nonlinear regulator equation, which is a partial differential equation (PDE) is usually difficult to solve, and in many cases, the solution of the nonlinear regulator equation may not exist at all. Khalil (1994) presented a robust servomechanism output feedback control design based on high gain observer and saturation for feedback linearizable systems. Combining this control design and the local output regulation results, Isidori (1997) addressed the semiglobal output regulation problem, where the initial condition of the disturbance dynamics is confined in a known bounded set. Huang and Lin (1991) and Huang (1995, 1998) proposed a kth-order robust nonlinear servomechanism design, and discussed the existence conditions of the kth-order servomechanism. A k-fold exosystem is built based on the linearized disturbance model. Then a feedback controller including the k-fold exosystem is designed based on the linearized plant model. It is shown that the controlled system output is bounded by the disturbance up to the kth order. However this is a local result which requires the initial conditions of the plant and disturbance close enough to the origin. Also to reject the disturbance to the k-th order, one need to use a very complex exosystem model in the controller design. Ding (2001) presented a global output regulation result for a class of minimum phase nonlinear systems with linear disturbances. Sun and Tsao (1999, 2001) addressed the asymptotic disturbance rejection problems for linear system with nonlinear disturbance dynamics, especially chaotic disturbances in the discrete and continuous time domains respectively. Sufficient conditions for achieving global asymptotic performance were presented. System robust stability and performance under unmodeled dynamics and model matching errors were derived and demonstrated through simulations.

This paper considers the asymptotic tracking or disturbance rejection problem for single-input-single-ouput nonlinear systems from the input-output viewpoint. The objective is to find a control synthesis method to asymptotically reject unknown exogenous nonlinear disturbances for nonlinear systems. A feedback control structure which incorporates the nonlinear disturbance model in the feedback loop is adopted. Necessary condition is first derived for asymptotic disturbance rejection. It is shown that the necessary condition becomes sufficient for linear systems with linear disturbance dynamics provided that the closed loop system is asymptotically stable. Inspired by the unique structure of the necessary conditions, sufficient conditions to achieve asymptotic disturbance rejection are then proposed. Our results differ from the previous output regulation results in three aspects. First, we do not address the problem of existence of control for asymptotic tracking. Instead, we make it an assumption and focus on the controller design problem for a specific feedback control structure. Second, we present the global asymptotic output regulation results for SISO nonlinear systems with nonlinear disturbance. Thus, the plant and disturbance initial conditions need not to stay in a small neighborhood of the origin. Finally, the disturbance model needs not to be neutrally stable and thus includes chaotic disturbance signals.

The rest of this paper is organized as follows. Section 2 describes the plant and disturbance models; Section 3 presents the feedback control design; Section 4 shows the simulation results and Section 5 is the conclusion.

2. Problem Description

Consider the following single input single output nonlinear plant:

$$\dot{x}(t) = f(x,u) \qquad (1)$$
$$y(t) = h(x,u) + d(t)$$

where $x(t) \in R^m$, $u(t) \in R$ and $y(t) \in R$ are the state, input and output signals respectively. $d(t) \in R$ is the unmeasurable bounded disturbance.

Suppose the disturbance satisfies the following nonlinear model:

$$\dot{w}(t) = \mu(w) \qquad (2)$$
$$d(t) = g(w)$$

where $w(t) \in R^n$, $\mu : R \to R^n$ and $g : R^n \to R$.

Rewrite the plant model (1) and the disturbance model (2) into the following input-output representation:

$$y = \pi[u] + d \qquad (3)$$
$$d = \chi[d]$$

where $\pi : L_{pe} \to L_{pe}$ is strictly causal, internally and finite gain stable. $\chi : L_{pe} \to L_{pe}$ is strictly causal and finite gain stable.

Remark: Although the above plant and disturbance models only show the input and output signals explicitly, those mappings are also functions of the initial states. Since we are mainly interested in the signal flow in the feedback loop, and for notation convenience, the dependence on the initial states is not explicitly shown.

The following assumption is made about the plant:

Assumption 1: There exists a unique control signal $u^* \in L_{pe}$ such that $\pi[u^*] = -d$.

Detailed control structures will be outlined in the following section. The control objective is to achieve $\lim_{t\to\infty} y(t) = 0$ for any initial conditions of the plant and the disturbance.

3. Nonlinear Feedback Control Design

As shown in Figure 1, the nonlinear internal model principle control (NIMPC) law is as follows:

$$u = \theta[z], \; z = \phi\{y - \psi[z]\} \qquad (4)$$

where $\psi : L_{pe} \to L_{pe}$ and $\phi : L_{pe} \to L_{pe}$ are strictly causal and finite gain stable, $\theta : L_{pe} \to L_{pe}$ is finite gain stable and $\pi\theta[\cdot]$ is strictly causal.

The motivation behind the control structure (4) is that we need a self-excitation mechanism in the feedback loop which will drive the system to cancel out the persistent but bounded disturbance once the output becomes zero.

As shown in Figure 1, we can divide the closed loop system into two serially connected blocks. One is $\pi\theta[\cdot] - \psi[\cdot]$ and the other is $\phi[\cdot]$. Since both $\pi\theta[\cdot] - \psi[\cdot]$ and $\phi[\cdot]$ are strictly causal, the feedback system is well posed (Dahleh and Diaz-Bobillo, 1995).

Our approach to achieve asymptotic performance for system (3) with the control law (4) is to derive the necessary condition first and then synthesize a set of sufficient conditions based on the solutions of the necessary condition.

3.1 Necessary Conditions For Asymptotic Disturbance Rejection

In this section, we derive the necessary condition to achieve asymptotic disturbance rejection based on system (3) and control law (4). First, we would like to introduce the following definition:

Definition 1: Consider mappings $\eta_1 : L_{pe} \to L_{pe}$ and $\eta_2 : L_{pe} \to L_{pe}$. Suppose $y_1 = \eta_1[u_1]$ and $y_2 = \eta_2[u_2]$, then $\eta_1[u_1] = \eta_2[u_2] \Leftrightarrow \lim_{t\to\infty}(y_1 - y_2) = 0$.

Theorem 1: Consider the nonlinear system (3) and the control law (4), it is necessary to satisfy the following condition to achieve asymptotic disturbance rejection:

$$(-\pi\theta)\phi(-\psi)[z^*] = \chi(-\pi\theta)[z^*] \tag{5}$$

where z^* will be defined in the proof.

(b) If the plant and disturbance dynamics are linear, the above condition is also sufficient provided that the closed loop system is asymptotically stable.

Proof:

(a) As shown in Figure 2, assume asymptotic disturbance rejection has been achieved, i.e. $y(t) \equiv 0$, along with assumption 1, we have:

$$\pi[u^*] = -d \tag{6}$$

$$u^* = \theta[z^*] \tag{7}$$

$$z^* = \phi(-\psi)[z^*] \tag{8}$$

Combining (6), (7) and (8), we get

$$d = -\pi\theta\phi(-\psi)[z^*] \tag{9}$$

From (3), (6) and (7), we get

$$d = \chi(-\pi\theta)[z^*] \tag{10}$$

Comparing (9) and (10), we have

$$(-\pi\theta)\phi(-\psi)[z^*] = \chi(-\pi\theta)[z^*]$$

(b): Suppose the system and disturbance dynamics are both linear, then $\pi[\cdot]$, $\theta[\cdot]$, $\psi[\cdot]$ and $\phi[\cdot]$ become linear mappings. The necessary condition (5) becomes:

$$-\psi\phi(-\pi\theta)[z^*] = \chi(-\pi\theta)[z^*]$$

So $d = -\psi\phi[d] = \chi[d]$ (11)

Combining it with the system model (3) and control law (4), we have the following results:

$$\{1 + \psi\phi - \pi\theta\phi\}[y] = \{1 + \psi\phi\}[d] = 0$$

So $\lim_{t\to\infty} y(t) = 0$ provided that the closed loop system is asymptotically stable.

Remark: We name condition (11) as the generalized disturbance model. The physical meaning of this condition is that disturbance dynamics are incorporated into the feedback loop to achieve asymptotic performance. This is consistent with the well known Internal Model Principle.

Remark: It is worth to point out that repetitive control which is widely used to track or reject periodic signals can be viewed as a special case of the proposed control design. For that case, $\phi[\cdot]$ becomes the identity mapping and $\psi[\cdot]$ is the delay mapping.

3.2 Sufficient Conditions for Asymptotic Disturbance Rejection

In this section we will first find solutions for the necessary condition (5) and then propose the sufficient conditions based on them. Inspired by condition (11), to include the disturbance dynamics in the feedback loop, we design $\psi[\cdot]$ and $\phi[\cdot]$ such that:

$$-\psi\phi[\cdot] = \chi[\cdot] \tag{12}$$

Then condition (5) becomes:

$$\pi\theta\phi(-\psi)[z^*] = \psi\phi(-\pi\theta)[z^*] \tag{13}$$

The above condition is nothing but swapping between the mappings. Obviously it will be satisfied automatically for linear time invariant (LTI) systems, but not for general nonlinear systems. To utilize this unique structure, we propose the following condition:

$$\psi[\cdot] = \pi\theta[\cdot] \tag{14}$$

Obviously, condition (5) is then satisfied. With this, we propose the following sufficient conditions to achieve asymptotic performance.

Theorem 2: Consider system (3) and control laws (4), (12), and (14), we have the following:

(a): The closed loop system is input output L_p -stable if

$$\|\pi\theta - \psi\|_p \times \|\phi\|_p < 1 \tag{15}$$

(b): Asymptotic disturbance rejection is achieved if conditions (12) and (14) are satisfied.

Proof:

(a): As shown in Figure 1, we can divide the closed loop system into two serially connected blocks. One is $\pi\theta[\cdot] - \psi[\cdot]$ and the other is $\phi[\cdot]$. Since the closed loop system is well posed, by the small gain theorem, it is input output L_p -stable if $\|\pi\theta-\psi\|_p \times \|\phi\|_p < 1$.

(b): The steady state output of the closed loop system is:

$$y = d + \pi\theta\phi\{-\psi[z] + y\}$$
$$y = d + \pi\theta\phi\{-\psi[z] + \pi\theta[z] + d\}$$

If condition (12) and (14) are satisfied, we have
$y = d + \psi\phi[d] = 0$

Remark: Conditions (12) and (14) are two independent conditions. Condition (12) is to build the disturbance observer, while condition (14) is to find the stabilizing feedback controller through model matching. We can design them separately and $\psi(\cdot)$ is used to connect these two conditions.

Remark: Although we claim that conditions (12) and (14) are sufficient conditions, they are the only obvious solutions to the necessary condition (5), so they are actually close to be necessary.

Remark: To improve system robustness, we can introduce a low pass filter $Q[\cdot]$ and place it before $\psi[\cdot]$ and $\theta[\cdot]$. Then condition (15) becomes $\|\pi\theta Q-\psi Q\|_p \times \|\phi\|_p < 1$.

4. Simulation Results

In this section, we will simulate the disturbance rejection performance of the proposed scheme. Consider the following nonlinear plant with chaotic disturbance:

$$\begin{aligned} \dot{x}_1 &= -x_1 + a\sin x_2 \\ \dot{x}_2 &= x_1^2 - x_2 + u \\ y &= x_2 + d \end{aligned} \tag{16}$$

The chaotic disturbance satisfies the Duffing's equation:

$$\ddot{d} + \delta\dot{d} - d + d^3 = \gamma\cos(t + t_0) \tag{17}$$

where $d(0)$, $\dot{d}(0)$ and t_0 are unknown, δ and γ are known constants.

This model represents the chaotic motion of a nonlinear spring, damper and mass system. It is a good approximation of many chaotic mechanical systems, for example, the chaotic motion of the spindle due to rolling bearing defect. The chaotic motion is very sensitive to the initial conditions and the values of δ and γ. In the following simulations, we choose $\delta = 0.25$ and $\gamma = 0.4$. It is easy to verify that the Jacobian matrix of (17) at origin has unstable eigenvalue.

The feedback control laws are described in (4) and shown in Figure 1. Based on theorem 2, we need to design $\psi[\cdot]$, $\theta[\cdot]$ and $\phi[\cdot]$ to satisfy conditions (12) and (14) to achieve asymptotic disturbance rejection.

Step 1: Design $\psi[\cdot]$ and $\theta[\cdot]$ to satisfy condition (14).

Let $u = -x_1^2 - z$ and plug it into (16):

$$\dot{x} = -x_1 + a\sin x_2$$
$$\dot{x}_2 = -x_2 - z$$
$$y = x_2 + d$$

So the mapping from z to y becomes $\frac{-1}{s+1}$ and consequently we choose:

$\psi(s) = \frac{-1}{s+1}$, where s is the Laplace operator.

Step 2: Next we design an asymptotic nonlinear disturbance observer to satisfy condition (12).

$$\begin{aligned} & z = \phi[d] \Leftrightarrow \\ & \dot{\xi} = A\xi + B\sigma\Gamma[d] + K[d - \hat{d}] \\ & \hat{d} = C\xi \\ & z = P\xi \end{aligned} \tag{18}$$

where $\xi = \begin{bmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \\ \xi_4 \end{bmatrix}$, $A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -\delta & 0 & \gamma \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$,

$C = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}^T$, $P = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}^T$, $K = \begin{bmatrix} 4.75 \\ 7.1625 \\ 14.869 \\ 6.875 \end{bmatrix}$,

$\bar{d} = \Gamma[d] = \begin{cases} 1.6, & d > 1.6 \\ d, & 1.6 \ge d \ge -1.6 \\ -1.6, & d < -1.6 \end{cases}$ and $\sigma[\bar{d}] = \bar{d} - \bar{d}^3$.

In the above observer design, A itself is not strictly Hurwitz since states ξ_3 and ξ_4 are included to estimate the external driving force $\gamma\cos(t+t_0)$. K is designed so that $A - KC$ is strictly Hurwitz. Then it is easy to verify that condition (12) is satisfied. Also note in (18) d is only used to represent the input to $\phi[\cdot]$, it doesn't mean that we can measure the disturbance directly. Instead, we assume the disturbance is unmeasurable and the actual input to $\phi[\cdot]$ in the feedback loop is $y - \psi[z]$ as shown in Figure 1.

Figure 3 shows the chaotic disturbance generated by model (17) with $d(0) = 0.5$, $\dot{d}(0) = 0.8$ and $t_0 = \pi / 5$ respectively. Figure 4 shows the estimate of the disturbance and the estimation error generated by the nonlinear observer (18). Figure 5 shows the regulated output of the plant and the control signal. State signals of the nonlinear plant are shown in Figure 6. As predicted by theorem 2, asymptotic performance has been achieved.

5. Conclusions

This paper presents the control analysis and synthesis to achieve global output regulation for SISO nonlinear plant with nonlinear disturbance dynamics. Under the assumptions of the existence of the control signal for asymptotic tracking, the necessary condition to achieve asymptotic performance for the specifically proposed feedback control structure was derived. A set of sufficient conditions were then presented. Using these conditions, the control design was performed for an example of a nonlinear plant with chaotic disturbance. Simulation results were given to illustrate the control performance.

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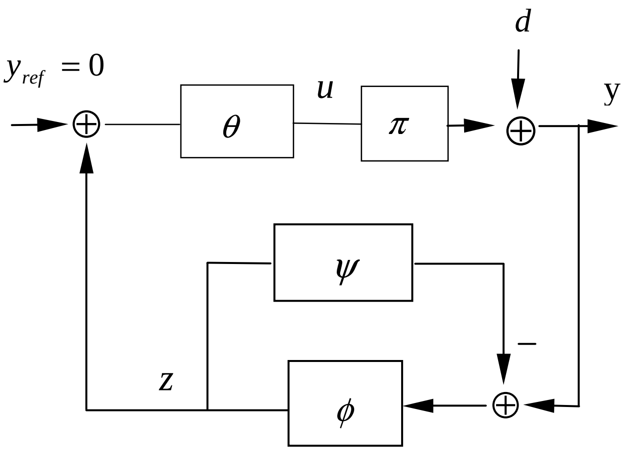

Figure 1. Block Diagram for the Nonlinear Internal Model Principle Control (NIMPC)

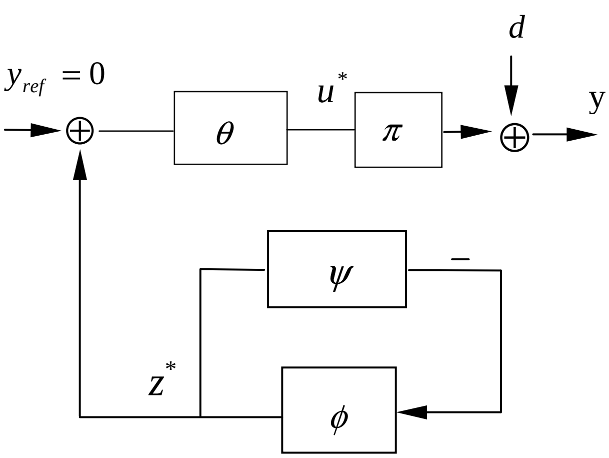

Figure 2. Block Diagram for Asymptotic Disturbance Rejection

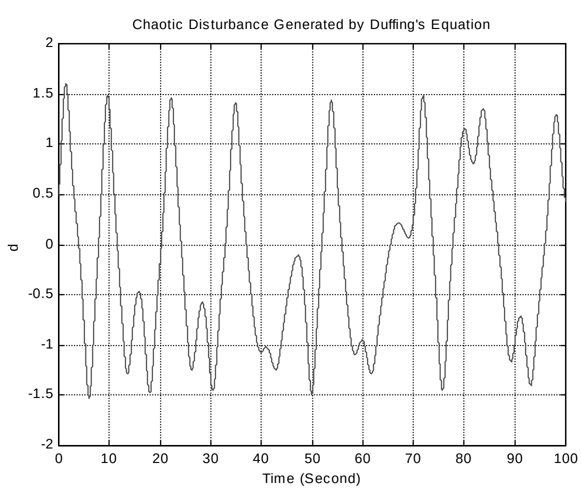

Figure 3. Chaotic Disturbance Generated by Duffing's Equation

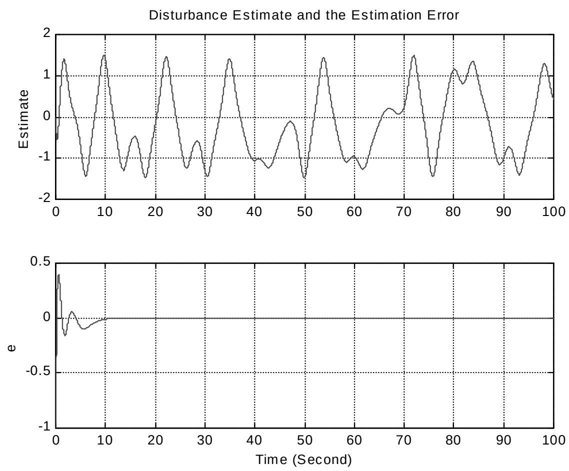

Figure 4. Disturbance Estimate and the Estimation Error

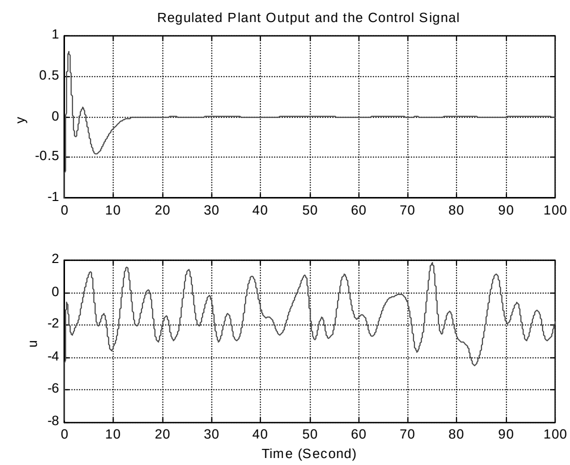

Figure 5. Regulated Plant Output and Control Signal

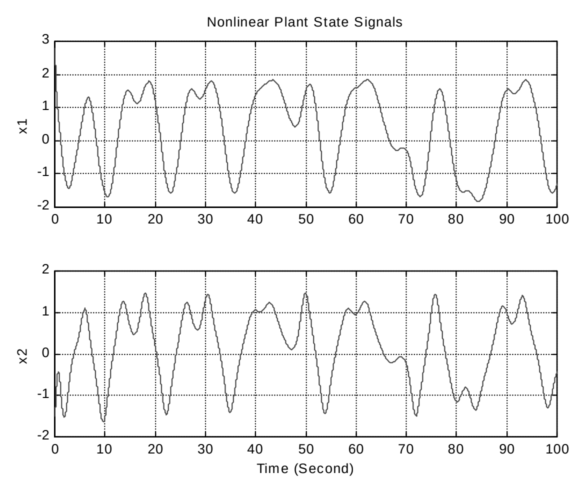

Figure 6. States of the Nonlinear Plant