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Robust Repetitive Controller Design with Improved Performance

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Abstract

This paper presents the design of a digital repetitive controller with improved performance via discrete-time μ-synthesis technique. The performance improvement at the fundamental frequencies is obtained by a modified low-pass $q(z,z^{-1})$ filter structure and robust stability is obtained through μ-synthesis design methodology. The new structure of $q(z,z^{-1})$ filter is motivated by efforts to square the sensitivity function which has very small values at the fundamental frequencies. This paper includes a robust repetitive controller design example for an electrohydraulic system for the noncircular cam turning process. Simulation and experimental results demonstrate the improved performance of the proposed design.

1 Introduction

Repetitive controllers are used in control applications where input reference signals and/or disturbances are periodic and their periods are known. Some of the repetitive control applications in literature are computer disk drive systems [1, 2], robot manipulators [4], material testing systems [6, 7], noncircular turning [9, 10, 11] and so on.

The internal model principle states that a periodic signal generator is required in the feedback loop to asymptotically track a periodic reference by the output of a closed-loop system and this periodic signal generator includes a big time delay term corresponding to the period of references. A "prototype" discrete-time repetitive controller design has been proposed by Tomizuka *et al.* [8], based on zero phase error tracking control (ZPETC). A zero phase lowpass filter $q(z,z^{-1})$ was introduced for robust stability to the same structure [9, 10]. Guo [2] proposed to use $S(z)/(S(z)+R(z))$ to substitute for the $q(z,z^{-1})$ in the prototype repetitive controller structure. Frequency shaping of the sensitivity function was utilized to reject both the repeatable and non-repeatable runout by choosing $S(z)$ and $R(z)$. A discrete-time two-parameter robust repetitive controller design by using structured singular values was proposed [3]. In this method, the high order delay term in the periodic signal generator is treated as a fictitious uncertainty and discrete-time μ-synthesis is applied.

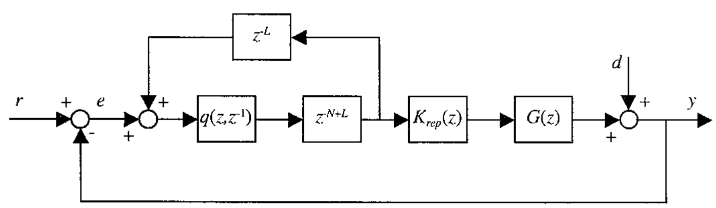

Figure 1: ZPETC-type Repetitive Controller Structure

Figure 1 shows the traditional ZPETC-type repetitive controller structure, where $G(z)$ is a stable plant, $K_{rep}(z)$ is a repetitive controller. $K_{rep}(z)$ is the approximate inverse of plant $G(z)$. The periodic signal generator consists of $q(z,z^{-1})$, z^{-N+L}, and z^{-L}, where N is the period of the periodic signal and L is the sum of the plant delay and the controller delay which comes from inversion of unstable zero part in plant. General descriptions about repetitive controller structure can be found in [5]. Note that the sensitivity function S_o of ZPETC-type repetitive controller structure is equal to

$$\begin{aligned} S_o &= \frac{1-q(z,z^{-1})z^{-N}}{1-q(z,z^{-1})z^{-N}+q(z,z^{-1})z^{-N+L}K_{rep}(z)G(z)} \\ &\approx 1-q(z,z^{-1})z^{-N} \qquad (1) \\ &\quad \text{if } z^{+L}K_{rep}(z)G(z)\approx 1, N \gg L. \end{aligned}$$

Let T be the sampling time and substitute $z=e^{j\omega T}$, then it is obvious that as long as the magnitude of lowpass filter $q(z,z^{-1})$ is unity, the magnitude of sensitivity function S_o is almost zero at integer multiples of $\frac{1}{NT}$(Hz) and reaches two between these frequencies. We call these frequencies the fundamental frequencies.

The main idea of this paper is to make the sensitivity S_o given at Eq. (1) squared by using a modified $q(z,z^{-1})$ filter structure, so that we can obtain much smaller sensitivity function magnitude at the fundamental frequencies. Namely, the new sensitivity function S_{mod} is to be

$$S_{mod} = S_o^2 \approx [1-q(z,z^{-1})z^{-N}]^2. \qquad (2)$$

This paper is organized as follows. Section 2 presents the proposed robust repetitive controller design method. Section 3 shows a design example for an electrohydraulic actuator system for the noncircular cam turning process and Section 4 contains concluding remarks.

2 Proposed Robust Repetitive Controller Design

Repetition of a continuous time periodic signal with period of T(sec) amounts to sampling its spectrum at integer multiples of $\frac{1}{T}$(Hz) with magnitude scaling by $\frac{1}{T}$. In repetitive control problems, reference signal r and/or disturbance d are assumed to be periodic with known periods. It means that by the sampling theorem the frequency spectrums of these signals appear only at the fundamental frequencies. Since the error e can be described by $e = S_o r$ and the sensitivity function S_o given at Eq. (1) knocks out every frequency component of reference r with its deep notches at the fundamental frequencies, the corresponding error e becomes small for the reference with known period.
To improve nominal performance of repetitive controller at the fundamental frequencies, it is desirable to make the magnitude of sensitivity function S_o smaller at the fundamental frequencies. The approximate squared sensitivity S_{mod} at Eq. (2) will apparently have much deeper notches at the fundamental frequencies and it is equivalent to,

$$S_{mod} \approx 1 - \underbrace{q(z,z^{-1})[2 - z^{-N}q(z,z^{-1})]}_{\text{Modified } q(z,z^{-1}) \text{ filter structure}} z^{-N}. \qquad (3)$$

By comparing Eq. (1) and (3), we can see that replacing the original $q(z,z^{-1})$ filter with the modified structure gives the squaring effect on the original sensitivity function S_o. Even though theoretically we could make the nth power of sensitivity function by the similar approach, the power greater than two may not work in practical sense because of the stability problem. Thus, we discuss only ${S_o}^2$ case in this paper.

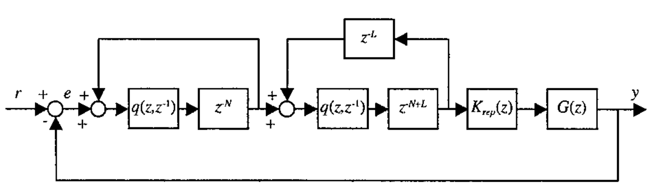

Figure 2: Periodic Signal Generators Connected in Series

Simple repetitions of the periodic signal generator may seem to accomplish the squaring effect on the sensitivity function if all of the delay loops have the same period. Let's consider two cases. The periodic signal generators are connected in series and parallel in Figure 2 and Figure 3, respectively. The corresponding sensitivity functions, S_{serial}

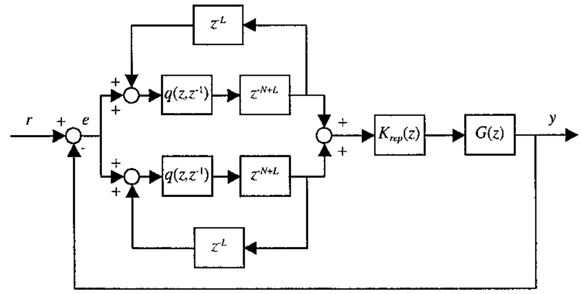

Figure 3: Periodic Signal Generators Connected in Parallel

and $S_{parallel}$ are given in Eq. (4) and (5).

$$\begin{aligned} S_{serial} &= \frac{1}{1 + [\frac{q(z,z^{-1})z^{-N}}{1-q(z,z^{-1})z^{-N}}]^2 z^{+L} K_{rep}(z) G(z)} \\ &\approx \frac{[1 - q(z,z^{-1})z^{-N}]^2}{[1 - q(z,z^{-1})z^{-N}]^2 + [q(z,z^{-1})z^{-N}]^2} \\ &\quad \text{if } z^{+L}K_{rep}(z)G(z) \approx 1, N \gg L. \end{aligned} \qquad (4)$$

$$\begin{aligned} S_{parallel} &= \frac{1}{1 + 2[\frac{q(z,z^{-1})z^{-N}}{1-q(z,z^{-1})z^{-N}}] z^{+L} K_{rep}(z) G(z)} \\ &\approx \frac{1 - q(z,z^{-1})z^{-N}}{1 + q(z,z^{-1})z^{-N}} \\ &\quad \text{if } z^{+L}K_{rep}(z)G(z) \approx 1, N \gg L. \end{aligned} \qquad (5)$$

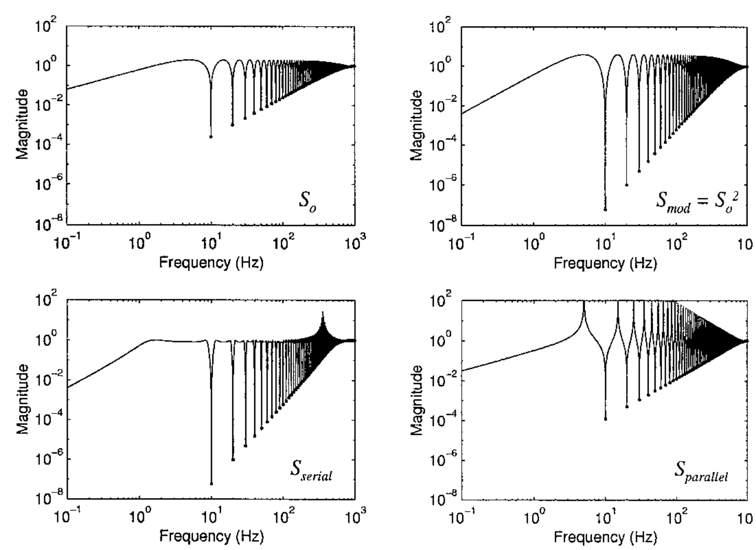

Figure 4: Plots of Approx. of S_o, S_{mod}, S_{serial} and $S_{parallel}$

Figure 4 shows the magnitude plots of the sensitivity functions described in Eq. (1), (3), (4) and (5) with a 2kHz sampling rate, $N = 200$, $q(z,z^{-1}) = [0.25z + 0.5 + 0.25z^{-1}]$. S_{serial} has not only the desired square term in the numerator but also other terms in the denominator. The denominator produces a big peak in the high frequency range even though it makes nice sensitivity shape in the low frequency range. In this particular numerical example, $|S_{serial}|$

reaches 27 at around 361Hz and it may be very difficult (or impossible) to find a stabilizing controller $K_{rep}(z)$ for the serial connection. Since $q(z,z^{-1})$ is a lowpass filter, it starts decreasing its magnitude from unity after a certain pass band and $q(z,z^{-1})z^{-N}$ makes a spiral toward the origin in complex plane. The magnitude of denominator approaches zero if $q(z,z^{-1})z^{-N}$ passes by close to the critical point $z = (0.5 + j0.5)$. Even $|S_{serial}|$ can be infinite, if $q(z,z^{-1})z^{-N} = (0.5 + j0.5)$ as shown in Figure 5. $S_{parallel}$ is far away from the desired square form. Therefore, these serial and parallel structures are not appropriate to be used.

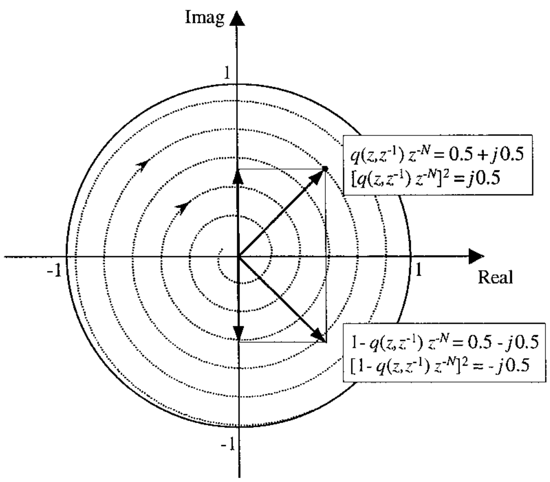

Figure 5: Behavior of the Denominator of S_{serial}

The attempt to improve the repetitive controller performance with a modified lowpass filter structure also can be found in [5]. In the first robustness optimization design step, they design a continuous time $\mathcal{H}_\infty$ repetitive controller, which is corresponding to $K_{rep}(z)$ block in Figure 1, with a predetermined lowpass filter $q_o(s)$. In the following performance improving design step, $q_o(s)$ is replaced by

$$q(s) = q_o(s) + (1 - e^{-\tau s} q_o(s))\, q_o(s)\, q_1(s), \qquad (6)$$

where τ is the period of the periodic signal and then $q_1(s)$ is designed to satisfy some robust stability conditions. If $q_1(s)$ in Eq. (6) is equal to 1 then the modified $q(z,z^{-1})$ filter structure in Eq. (3) is the same as Eq. (6). The proposed method directly inserts the modified $q(z,z^{-1})$ filter structure in the discrete-time μ-synthesis framework instead of having another design step as [5] does with $q_1(s)$ for performance improvement stage. Another advantage of the proposed method is that zero phase lowpass filters can be used in repetitive controller design since it is a discrete-time design scheme.

The zero phase lowpass filter $q(z,z^{-1})$ plays an important role for the system stability in the ZPETC-type repetitive controller design [10]. It is not trivial to guarantee the system stability with the modified $q(z,z^{-1})$ filter structure because it significantly increases the sensitivity function magnitude greater than two between fundamental frequencies. It is shown that the modified $q(z,z^{-1})$ filter structure approximately makes the sensitivity function

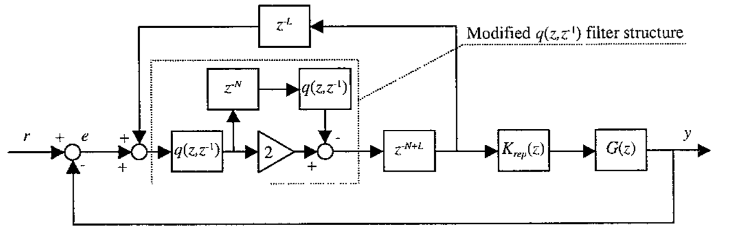

Figure 6: Repetitive Controller w/ Modified q Filter

S_o squared and its configuration is depicted in Figure 6. The repetitive controller $K_{rep}(z)$ in this figure may be obtained through ZPETC-type design method or discrete-time μ-synthesis technique which will be discussed later but in either case, much more conservative lowpass $q(z,z^{-1})$ filter will be needed to ensure the system stability.

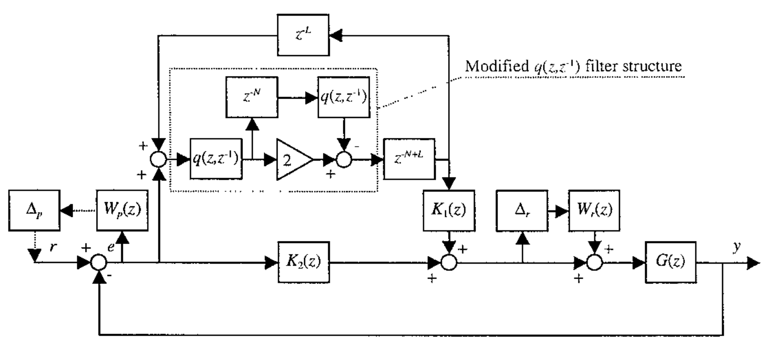

Figure 7: Robust Repetitive Controller Design Structure

The proposed robust repetitive controller design structure is shown in Figure 7. $W_r(z)$ and $W_p(z)$ are input multiplicative uncertainty weighting function and performance weighting function, respectively. Δ_r and Δ_p are corresponding uncertainties such that each $||\Delta||_\infty \leq 1$. To relieve the burden for the system stability on one controller, two degrees-of-freedom controller, $K_1(z)$ and $K_2(z)$ are to be designed via discrete-time μ-synthesis technique. In this block diagram, the plant, $G(z)$ can be unstable.

It is a well known fact that the order of controller designed by μ-synthesis is higher than that of total system. For example, a noncircular cam turning application with a sampling frequency of 2kHz and a spindle speed of 600rpm requires $N = 200$, so if μ-synthesis is applied directly to the block diagram in Figure 7, the order of final controller will be well beyond 400. Practically speaking, it is almost impossible to include such big delay blocks in a plant description for a discrete-time repetitive controller design by μ-synthesis. To cope with the high order problem, the big delay terms are replaced with fictitious uncertainty blocks in μ-synthesis design [3]. This drastically reduces the order of augmented plant for controller synthesis and hence generates a low order controller.

Figure 8 shows the μ-synthesis design block diagram. The

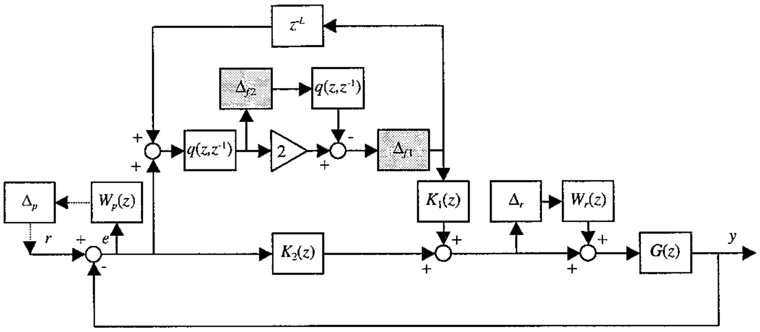

Figure 8: μ-Synthesis Design Block Diagram

final augmented block structure $\hat{\Delta}$ is

$$\hat{\Delta} = \begin{bmatrix} \Delta_{f1} & 0 & 0 & 0 \\ 0 & \Delta_{f2} & 0 & 0 \\ 0 & 0 & \Delta_r & 0 \\ 0 & 0 & 0 & \Delta_p \end{bmatrix}. \tag{7}$$

Assume nominal stability such that $M = F_l(P,[K_1,K_2])$ is (internally) stable, robust performance is obtained if

$$\mu_{\hat{\Delta}}(M) < 1. \tag{8}$$

See [12] for details. It should be noted that the inverse of performance weight $1/W_p(z)$ does not describe nominal performance of repetitive controller at the fundamental frequencies, but represents the required upper bound on the sensitivity function. This weight can be used for disturbance rejection in the low frequency range but we can not enforce a demanding $W_p(z)$ in Figure 7, otherwise it will fight with the demanding modified $q(z,z^{-1})$ filter structure to influence on the final controller characteristics and we won't get desired performance at the fundamental frequencies.

Due to the fictitious uncertainties, $\mu_{\hat{\Delta}}(M)$ value at Eq. (8) contains some amount of conservatism. It is fine to have $\mu_{\hat{\Delta}}(M)$ value greater than one during *D-K* iteration as long as robust stability test with real delays z^{-N+L} and z^{-N}, i.e. Figure 7 configuration, shows $\mu_{\Delta_r}(M_{11}) < 1$ after getting a controller from μ-synthesis. Robust performance test in Figure 7 configuration is not as important as robust stability test because the primary purpose of the repetitive controller design is to get better performance at the fundamental frequencies.

3 Design Example: Simulation and Experimental Results

In this section, we present a design example for an electrohydraulic system for the noncircular cam turning process. We use a 2kHz sampling rate and assume $L = 10$ and $N = 200$ (spindle speed of 600rpm) fixed. Figure 9 shows the Bode plots for $G(e^{j\omega})$ used to model the electrohydraulic system.

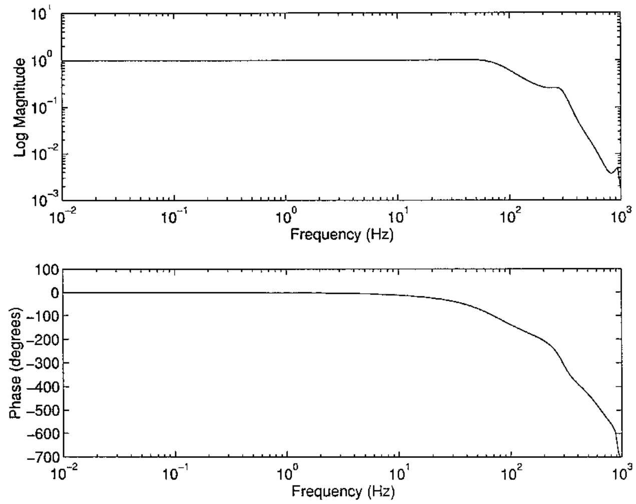

Figure 9: Bode Plot of Electrohydraulic System, $G(z)$

The form of zero phase lowpass filter $q(z,z^{-1})$ is selected to be

$$q(z,z^{-1}) = (0.25z + 0.5 + 0.25z^{-1})^n. \tag{9}$$

All poles are located at $z = 0$ and all zeros are at $z = -1$. Since the point $z = -1$ corresponds to frequency $\omega = \pi/T$, the amplitude response becomes more attenuated at around Nyquist frequency by zeros when n becomes large. About the $q(z,z^{-1})$ filter form for the repetitive controller design, refer to [10].

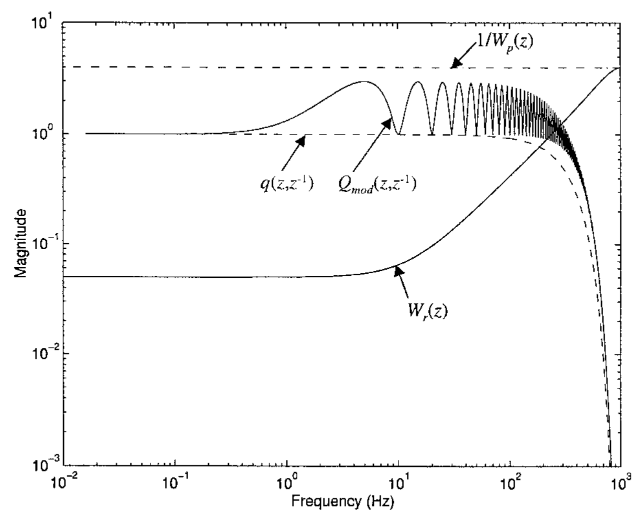

Figure 10: Magnitude Bode Plot of Filters and Weights

Figure 10 shows the magnitude Bode plots for $W_r(z)$, $1/W_p(z)$, $Q_{mod}(z,z^{-1}) = q(z,z^{-1})[2 - z^{-N}q(z,z^{-1})]$ and $q(z,z^{-1})$ used in this design example and we set $n = 3$ in Eq. (9). $1/W_p(z)$ is simply set to be 4 for the upper bound on the sensitivity function in order to put more emphasis on nominal performance improvement. The modified $q(z,z^{-1})$ filter structure, $Q_{mod}(z,z^{-1})$ shows unity gain at each fundamental frequency up to around 200Hz range.

The proposed robust repetitive controller design method was applied to the electrohydraulic system with the above mentioned design parameters. The final controller gave $\mu_{\Delta_r}(M_{11}) = 0.886$, so the system was robustly stable with some margin. For comparison, two parameter robust repetitive control(TPRRC) design method [3] was applied to the same system with the same parameters. The only difference is that TPRRC uses a lowpass $q(z,z^{-1})$ filter instead of the modified $q(z,z^{-1})$ filter structure, $Q_{mod}(z,z^{-1})$, in Figure 7. The zero phase lowpass filter in TPRRC was chosen to provide almost same level of robustness. A filter $q(z,z^{-1}) = [0.045z + 0.91 + 0.045z^{-1}]$ was selected and the designed controller from μ-synthesis showed $\mu_{\Delta_r}(M_{11}) = 0.881$.

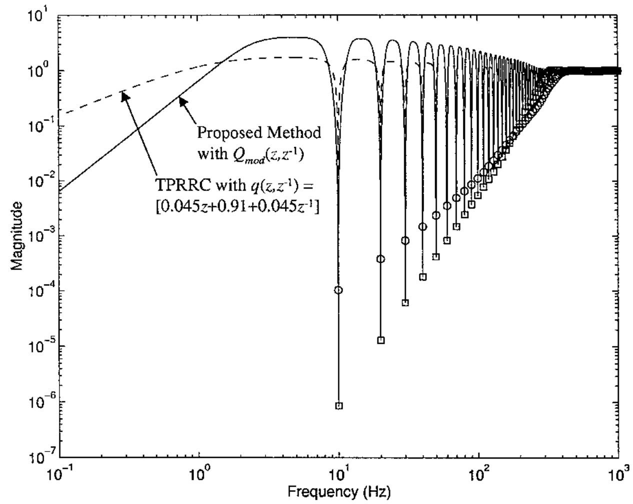

Figure 11: Comparison of Sensitivity Functions I

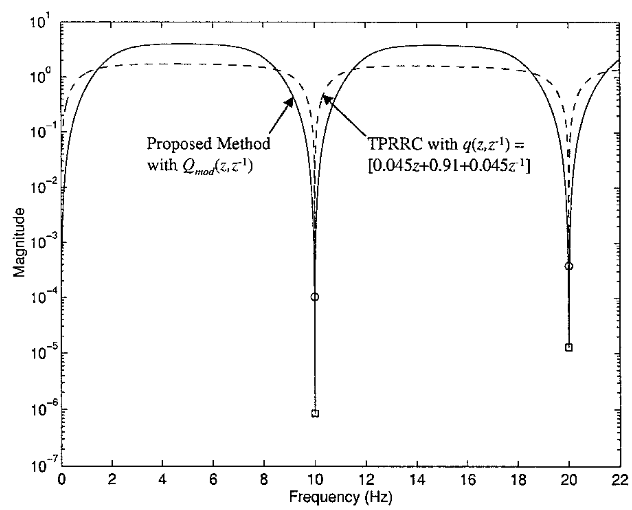

Figure 12: Comparison of Sensitivity Functions II

Sensitivity functions from each case are compared in Figure 11 and 12. The solid line with square marks are from the proposed method with the modified $q(z,z^{-1})$ filter structure and the dashed line with circular marks are from TPRRC. The square and circular marks represent the magnitude of sensitivity functions at the fundamental frequencies. The sensitivity function from the repetitive controller designed with the modified $q(z,z^{-1})$ filter structure shows deeper notches at the fundamental frequencies up to 180Hz, so it represents better nominal performance especially within 180Hz. Figure 12 shows zoomed-in sensitivity function magnitude curves at the first three harmonic frequencies. The solid line from the proposed method exhibits not only deeper but also wider notches than the dashed line from TPRRC. It means the proposed method is more robust for small variations in the period time than TPRRC.

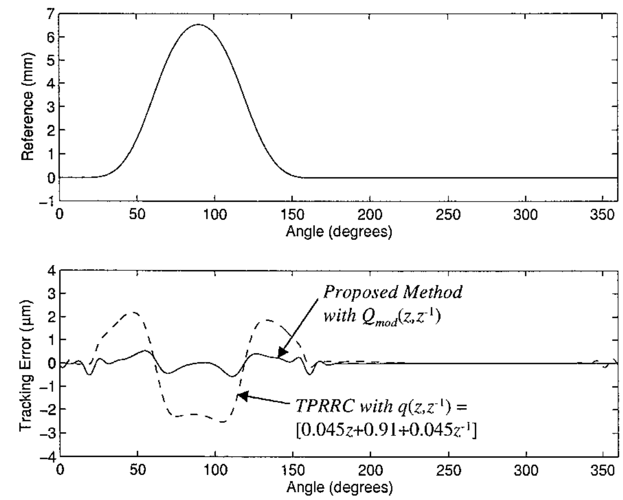

Figure 13: Simulaton Results: Comparison of Steady-State Errors

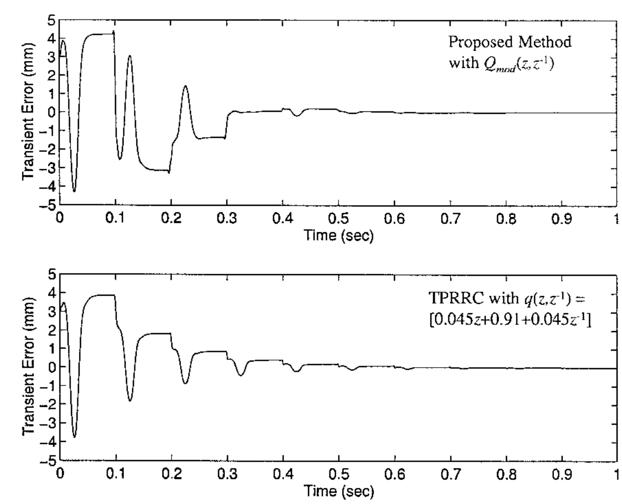

Figure 14: Simulation Results: Comparison of Transient Errors

Figure 13 and 14 show the simulation results with a particular mild cam profile reference signal. The maximum steady-state tracking error of the proposed method with the modified $q(z,z^{-1})$ filter structure is 0.53μm and it is a fourth of the TPRRC tracking error. Even though the controller with the modified $q(z,z^{-1})$ filter structure contains two big delays, the transient error of the proposed method converges to its steady state with almost same rate as TPRRC transient error does. Both methods reach their steady states within one second in simulation.

The controllers were implemented on a TMS320C32 DSP

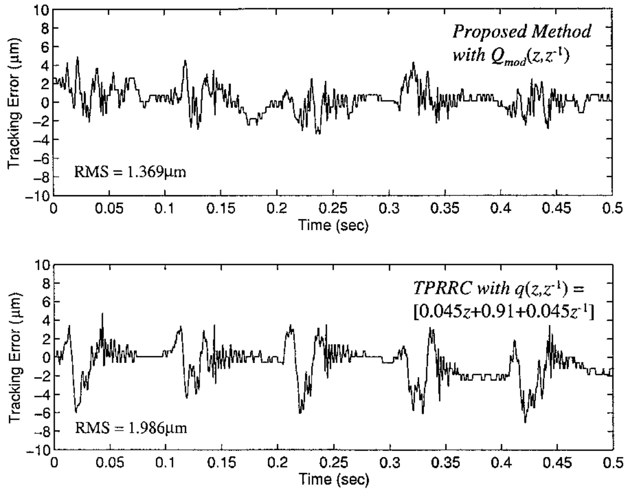

Figure 15: Experimental Results

board and the position feedback signal was collected by a laser encoder with a 0.6μm resolution. The controller from the proposed method had initially the 54th order. The controller order was reduced to the 11th order for implementation. Likewise the 32nd order controller from TPRRC was reduced to a 10th order controller. The same reference signal given in Figure 13 was used for experiments and the experimental results are shown in Figure 15. The shown tracking errors are the errors during five steady-state consecutive cycles from each case. The tracking errors in the experiments are not as small as those in the simulation because there are several other factors in real implementation, such as the model reduction process and a finite word length problem in the DSP board. However, it is still clear that the proposed robust repetitive controller with the modified $q(z,z^{-1})$ filter structure demonstrates better tracking performance over TPRRC.

4 Conclusions

A new discrete-time robust repetitive controller design with better performance was presented. Nominal performance improvement at the fundamental frequencies was obtained with a modified $q(z,z^{-1})$ filter structure and discrete μ-synthesis was used for robust stability of the system. Simulation and experimental results illustrated the performance improvement for an electrohydraulic system for noncircular turning application.

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