Proceedings of the American Control Conference
Philadelphia, Pennsylvania • June 1998

Identification and Control of Electrohydraulic Actuator Modeled as a Linear Periodic System

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Abstract

In tracking of periodic signals, the nonlinear dynamics of electrohydraulic actuators are modeled as linear periodic systems perturbed along periodic input and output trajectories. System identification of linear periodic systems is first performed and then periodic repetitive controllers are designed and implemented within the valid range of the linear approximation. Successive iterations of the identification and control are then performed to drive the system output towards the desired output trajectory.

1. Introduction

Noncircular turning for camshaft machining is a process, which uses fast response high-force actuators like electrohydraulic actuators and high performance motion control to make the cutting tool attached to the actuator track specified periodic cam profiles. The design of such high performance precision motion control requires the use of accurate electrohydraulic models. While the hydraulic systems are nonlinear, it is desirable to use a linear input/output approach for modeling, identification, and control. This is because that the main system nonlinearities for the electrohydraulic actuator for this application reside in the servovalve, for which the nonlinear model parameters are difficult to determine and internal state variables cannot be measured [7].

It is well known that a periodic linear system results from linearizing a nonlinear system along a periodic state trajectory that corresponds to a certain input trajectory. Utilizing this fact, linear periodic input/output models of linearization around some nominal input and output trajectories can be identified. However, controllers designed using the identified linear models will only be effective for small perturbations around the nominal trajectories. Since the input trajectory that corresponds to the desired output trajectory is unknown, more than one iteration of this identification and controller design procedure may be required so that the nominal input and output trajectories will converge to the desired ones. This paper proposes a formal successive iteration procedure, which involves periodic system identification, periodic repetitive control design, and control performance evaluation, to address the periodic trajectory tracking problem for the nonlinear electrohydraulic systems.

Repetitive controllers are useful for asymptotic tracking/regulation of systems with exogenous periodic signals. Previous work in repetitive control of periodic systems can be found in [6] and [2]. Omata et al. [6] developed a sufficient stability condition and a control design method based on *l-2* optimization. Hanson and Tsao [2] developed a necessary and sufficient stability condition and a control design method based on gain scheduling of a zero phase error compensator described in Tsao and Tomizuka [8].

The rest of this paper is organized as follows. The experimental system is described briefly. The identification methodology and control design strategy for the linear periodic system are described, and are performed in two successive iterations. The identified periodic model for each iteration is shown to provide the best match of the corresponding experimental data. The tracking performance is improved with each iteration by using the appropriate periodic repetitive controller and feedforward input.

2. System Description

The experimental system consists of a two-stage flow control servovalve and a double-ended actuator, operated at a supply pressure of 18.6 MPa (2700 psi). The system input is a voltage corresponding to current input to the servovalve torque motor, and the output is the actuator displacement measured by a laser displacement linear encoder with 0.6 micron resolution. An analog proportional control loop (with gain = 1) is applied to the open loop system to create a stabilized plant with input signal "v_{3m}."

This plant model has the form

$$P_o'(s) = \frac{x_{am}(s)}{v_{3m}(s)} = \frac{-f_0 s^5 + f_1 s^4 + f_2 s^3 + f_3 s^2 + f_4 s + f_5}{s^8 + g_1 s^7 + g_2 s^6 + g_3 s^5 + g_4 s^4 + g_5^3 + g_6 s^2 + g_7 s + g_8} \quad (1)$$

which incorporates the actuator and servovalve model [7]. In order to facilitate the subsequent system identification and controller implementation, the consistency of the electrohydraulic system response must be enhanced. This is accomplished by using an appropriate inner-loop H^∞ feedback controller [4]. The closed-loop system with this controller is now considered the plant, with input "in_{xam}" as shown in figure 1:

$$P_o(s) = \frac{x_{am}(s)}{in_{xam}(s)}, \quad (2)$$

where $P_o(s)$ will be 18^{th}-order due to $P_o'(s)$ and the 10^{th}-order H^∞ feedback controller [4].

The discrete-time plant model is

$$P_o(q^{-1}) = \frac{x_{am}(q^{-1})}{in_{xam}(q^{-1})} = \frac{q^{-d} B(q^{-1})}{A(q^{-1})} \quad (3)$$

$$A(q^{-1}) = 1 - A_1 q^{-1} - \dots - A_n q^{-n}$$

$$B(q^{-1}) = B_0 + B_1 q^{-1} + \dots + B_m q^{-m}, \; B_0 \neq 0,$$

where the discrete-time delay operator is designated by q^{-1}.

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3. Problem Formulation

Figure 2 shows a reference cam profile, which contains 200 points (i.e. N = 200). For the equivalent spindle speed of 600 RPM, the corresponding sampling frequency is 2,000 Hz. Initially, repetitive controllers and feedforward controllers are designed from a linear time-invariant model, which corresponds to perturbations about the zero actuator position [3]. These controllers are used to track three reference trajectories with maximum amplitudes of 9.6, 10.5, and 11.4 mm, the largest being shown in figure 2. The tracking performance diminishes for the larger trajectories, as the maximum errors are 180 microns, 290 microns, and 410 microns, respectively, and the RMS errors are 44 microns, 69 microns, and 95 microns, respectively. Hence, the controllers designed from the time-invariant model do not provide adequate tracking for the largest amplitude reference, indicating the existence of system non-linearities.

Due to the periodic nature of this tracking application, the electrohydraulic system is modeled as a linear periodic system, with the models obtained from linearization about nominal input and output trajectories. A progressive identification and control approach is used and illustrated in figure 3. The two-dimensional space in figure 3 can be considered in two ways, as a function space for the input and output trajectories, and as a performance space, where the vertical axis represents the tracking errors and the horizontal axis represent the iterations. The point (in_{xam}^*, x_{amref}) in the function space represents perfect tracking of the reference trajectory and its corresponding input. The point ($in_{xam}^{(1)}$, $x_{am}^{(1)}$) represents the first input trajectory and the corresponding output response. The linear periodic model for motion near ($in_{xam}^{(1)}$, $x_{am}^{(1)}$), valid within radius R_1, is used for controller design. The implementation of these first controllers moves the nominal performance to the point ($in_{xam}^{(2)}$, $x_{am}^{(2)}$), which signifies improved tracking of the reference trajectory. Next, perturbed data around these input and output trajectories are obtained, and the identification and controller design is repeated. The nominal trajectories are then moved to point ($in_{xam}^{(3)}$, $x_{am}^{(3)}$), which represents further improved tracking. With each successive iteration, the controllers designed from the corresponding linear periodic model are used to improve the tracking of the reference trajectory, until the tracking error converges to zero.

For each iteration, the N-periodic nominal input trajectory

$$\overline{in}_{xamNOM} = \{ in_{xamNOM}(1), \ldots, in_{xamNOM}(N) \} \qquad (4)$$

and the corresponding nominal output trajectory

$$\bar{x}_{amNOM} = \{ x_{amNOM}(1), \ldots, x_{amNOM}(N) \} \qquad (5)$$

is obtained. Defining the input and output deviations from the nominal trajectories

$$\Delta in_{xam} = in_{xam} - in_{xamNOM} \qquad (6)$$

and

$$\Delta x_{am} = x_{am} - x_{amNOM}, \qquad (7)$$

the perturbed model at each point in the trajectory is expressed as

$$P_{o\Delta}(q^{-1}) = \frac{\Delta x_{am}(q^{-1})}{\Delta in_{xam}(q^{-1})} = \frac{q^{-d} B_\Delta(q^{-1})}{A_\Delta(q^{-1})} \qquad (8)$$

$$A_\Delta(q^{-1}) = 1 - a_1 q^{-1} - \ldots - a_n q^{-n}$$

$$B_\Delta(q^{-1}) = b_0 + b_1 q^{-1} + \ldots + b_m q^{-m}, \; b_0 \neq 0,$$

4. Periodic System Identification

Due to the inherent nonlinearity of the electrohydraulic system and the large desired range of motion, it is expected that the perturbed model (8) will vary along each periodic trajectory. The strategy for system identification is to determine the perturbed model corresponding to each point in the trajectory.

The notation for system identification is presented. Define the vectors

$$\varphi^T(k-1) = [\Delta x_{am}(k-1) \ldots \Delta x_{am}(k-n) \;\; \Delta in_{xam}(k-1) \ldots \Delta in_{xam}(k-m)] \qquad (9)$$

$$\theta^T(k) = [-a_1(k) \ldots -a_n(k) \quad b_0(k) \ldots b_m(k)] \qquad (10)$$

It can be seen that $\Delta x_{am}(k) = \varphi^T(k-1)\,\theta(k)$, and the goal of the system identification is to use the input-output data to estimate $\theta(k)$ $\forall$ k.

The identification framework first is described for a linear time-invariant system, i.e. $\theta(k) = \theta_o$, and then is expanded for the linear periodic system. Using the direct least squares algorithm, the estimate for θ_o, i.e. $\hat{\theta}$, computed from "m" sets of data is

$$\hat{\theta} = \left\{ \sum_{k=1}^{k=m} [\varphi(k-1)\,\varphi^T(k-1)] \right\}^{-1} \sum_{k=1}^{k=m} [\varphi(k-1)\;\Delta x_{am}(k)] \qquad (11)$$

and the prediction error verifies the accuracy of the identified model:

$$e_{xamPRED}(k) = \Delta x_{am}(k) - \varphi^T(k-1)\,\theta_o, \;\; k = 1 \,..\, m. \qquad (12)$$

Here, a total of "m_{cycle}" cycles of perturbed data around the nominal input trajectory and corresponding output trajectory are used to determine the best model $\hat{\theta}_j$ for j = 1, 2, … N:

$$\hat{\theta}_j = \left\{ \sum_{k_c=1}^{k_c=m_{cycle}} [\varphi_{kc}(j-1)\varphi_{kc}^T(j-1)] \right\}^{-1} \sum_{k_c=1}^{k_c=m_{cycle}} [\varphi_{kc}(j-1)\,\Delta x_{amkc}(j)] \qquad (13)$$

For each $\hat{\theta}_j$, the same interval is used on the input trajectories and output trajectories for "m_{cycle}" cycles. For the identification of this experimental system, $m_{cycle} = 100$.

The prediction error is computed for each of the identified models:

$$e_{xamPREDj}(k) = \Delta x_{amj}(k) - \varphi_j^T(k-1)\,\hat{\theta}_j(k), \;\; j = 1, 2, \ldots N \qquad (14)$$

over the "m_{cycle}" cycles. Both the average prediction error and standard deviation are computed at each location over the "m_{cycle}" cycles.

5. Periodic Repetitive Control

Periodic repetitive controllers are designed from each identified linear periodic model, since repetitive controllers improve tracking of the periodic reference trajectory. The discrete-time repetitive controller formulated in [8] is used, specifically

$$\Delta in_{xamrep}(k) = Q(q,q^{-1})[\Delta in_{xamrep}(k-N) + q^{-N+L} M(q^{-1}) e_{\Delta xam}(k)]$$

$$M(q^{-1}) = \frac{A_\Delta(z^{-1})(z^{-m} B_\Delta(z))}{b}, \qquad (15)$$

$$L = d + m$$

$$b = (|b_0|| + \ldots + |b_m|)^2,$$

The signals Δin_{xamrep} (k) and $e_{\Delta xam}(k)$ represent the discrete-time repetitive control signal and tracking error, respectively. In addition, $Q(q,q^{-1})$ is a low-pass, zero-phase filter which is included for robust stability (i.e. $Q(q,q^{-1}) = [0.25q \;\; 0.5 \;\; 0.25q^{-1}]^4$). Under this formulation, each repetitive controller is a finite impulse response (FIR) filter. Repetitive controllers are designed for each of the N perturbed models, resulting in a periodic repetitive controller. The repetitive controllers are implemented with a gain scheduling scheme. Figure 1 illustrates how the perturbed signals are obtained from the nominal input trajectory and output trajectory.

The feedforward control input shown in figure 1 is obtained from off-line simulation of the periodic repetitive controller on the identified periodic plant model, with the intention of reducing the tracking error shown in figure 2. These are the system vectors, which contain N points each:

1) $\bar{r}_{\Delta xam}$ (reference inputs) = $\bar{r}_{xam} - \bar{x}_{amNOM}$
2) $\overline{\Delta in}_{xam}$ (system inputs) = $\overline{in}_{xam} - \overline{in}_{xamNOM}$
3) $\overline{\Delta x}_{am}$ (system outputs) = $\bar{x}_{am} - \bar{x}_{amNOM}$
4) $\bar{e}_{\Delta xam}$ (tracking errors) = $\bar{r}_{\Delta xam} - \overline{\Delta x}_{am}$

The closed-loop stability for both simulation and implementation must be assessed for each N-periodic repetitive controller design, since the resulting controller is not necessarily stabilizing. Specifically, the linear periodic system must be lifted to a higher dimensional time-invariant system, so that the stability of the periodic repetitive control system can be determined by the necessary and sufficient condition [2]:

$$\rho(\tilde{A}_{CL}) < 1. \tag{16}$$

Since the linearized models are only valid for small perturbation around the nominal trajectories, the actual valid ranges, as represented by the circles in figure 3, were determined experimentally by accessing the tracking performance for a range of perturbed reference trajectories. The experimental results are analyzed in two ways: the actuator motion with respect to the scaled perturbed reference trajectory, and the actuator motion with respect to the full reference trajectory of figure 2. In both cases, the tracking results are quantified by the maximum error and by the root-mean-square (RMS) error.

6. Results for First Iteration

The N-periodic nominal output trajectory from figure 2 and the corresponding nominal input trajectory are used as the nominal trajectories for the first iteration. The identification methodology is applied to the perturbed system around these nominal trajectories. A uniformly distributed random input signal with maximum amplitude of 300 microns, are sampled and held at each value for seven sampling intervals in order to allow the hydraulic system to respond sufficiently.

For this first iteration, 18^{th}-order models (8) are fit at each of the 200 points. This identified periodic model provides better prediction errors than the time-invariant model, as demonstrated by figure 4, which shows the averaged prediction error and its standard deviation at each point on the trajectories (j = 1 ... N). The biggest difference can be seen near the peak amplitude on the reference cam trajectory (i.e. j = 80), which also corresponds to the position of greatest acceleration.

The periodic repetitive controllers (14) are implemented via gain scheduling, in addition to the feedforward input obtained from the repetitive control simulation. The goal is to reduce the nominal tracking error of figure 2, which is the difference of the nominal output trajectory from the desired output. This nominal error is used as the perturbed reference trajectory for the periodic repetitive control simulation (for finding the feedforward input) and implementation. Stability of the periodic control system is confirmed from the calculation of the lifted system matrix spectral radius, where $\rho(\tilde{A}_{CL}) = 0.98$ for this case.

The best results are obtained with the perturbed reference trajectory scaled by 50 percent, which are shown in figure 5. The improved tracking error with respect to the full reference trajectory is shown in figure 7. As seen in figure 9, the tracking performance worsens for larger scaling of the perturbed reference trajectory. This indicates that the system nonlinearity has deviated from the linear periodic behavior around the nominal trajectories. Another iteration of this procedure around the new input and output trajectories, which are now closer to the desired ones, would further improve the performance. For this first iteration, the hydraulic pump and system variations contribute 12 microns to each RMS error.

7. Results for Second Iteration

The identification methodology is applied again to the perturbed system around the nominal trajectories corresponding to the responses from the first iteration shown in figure 5. Perturbed input data similar to the previous iteration, except now a maximum amplitude of 100 microns are used.

As shown in figure 6, the new identified periodic model provides smaller prediction errors than the periodic model from the first iteration.

Following the identical procedure in the previous iteration, the periodic repetitive controllers (14) are implemented via gain scheduling, in addition to the feedforward input obtained from the repetitive control simulation, the goal being to further reduce the error from the first iteration. Stability is again checked with $\rho(\tilde{A}_{CL}) = 0.97$ for the periodic repetitive control system.

The best result is obtained with the new perturbed reference trajectory scaled by 50 percent, which shown in figure 8. The further improvement in tracking error with respect to the full reference trajectory is shown in figure 7. As seen in figure 9, the performance again worsens for larger scaling of the perturbed reference trajectory. For this second iteration, the hydraulic pump variations contribute 6 microns to each RMS error, due to the fact that the hydraulic system varies with time.

Table 1 indicates improved performance, in terms of maximum error and cumulative error, for the full reference trajectory of figure 2, using the periodic repetitive controllers and feedforward inputs for the two iterations. The improved tracking of this periodic reference signal validates the modeling of the electrohydraulic system as a linear periodic system, which is valid for a certain amplitude range around nominal input and output trajectories. Conceivably more iterations could have been applied to further improve the tracking performance up to the system's noise levels or hardware limits.

8. Conclusions

The linear periodic systems have been obtained from linearizing the electrohydraulic system around the nominal input and output trajectories corresponding to the reference trajectory, and the correct identification methodology has been developed and utilized. The identification and controller design has been performed in two successive iterations. In each case, the identified periodic model provides the best agreement with experimental data. The repetitive controllers and feedforward inputs designed from these linear periodic models provide improved tracking of the reference trajectory.

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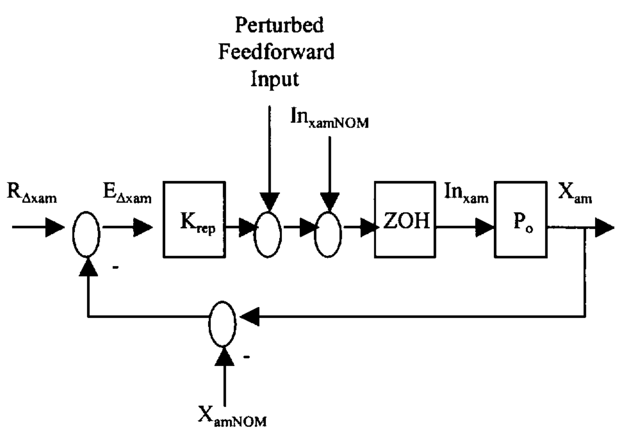

Figure 1. Control System Block Diagram

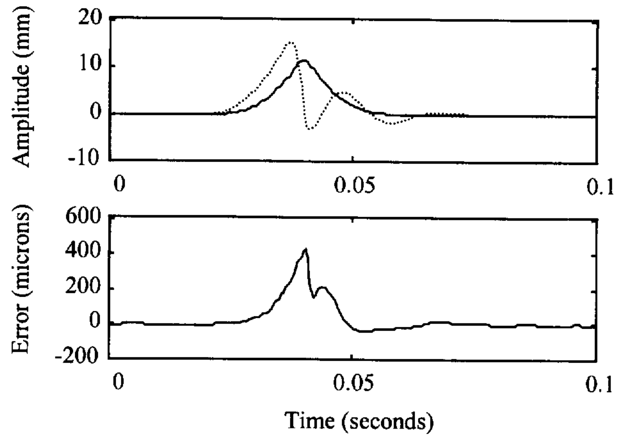

Figure 2. Nominal responses from time-invariant controllers
Top: Nominal input (dot)
Reference trajectory (solid)
Bottom: Nominal error

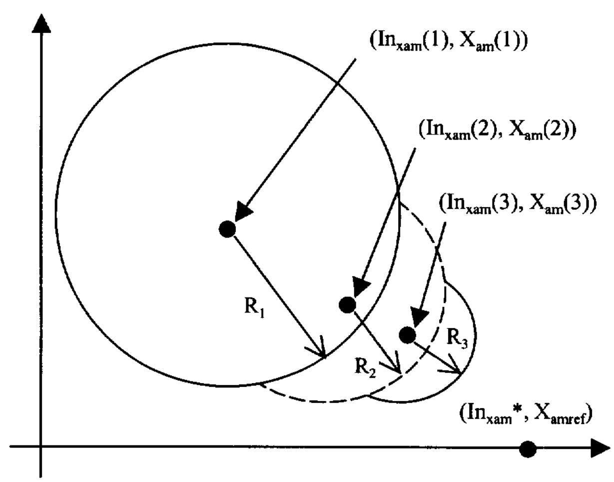

Figure 3. Illustration of Problem Formulation

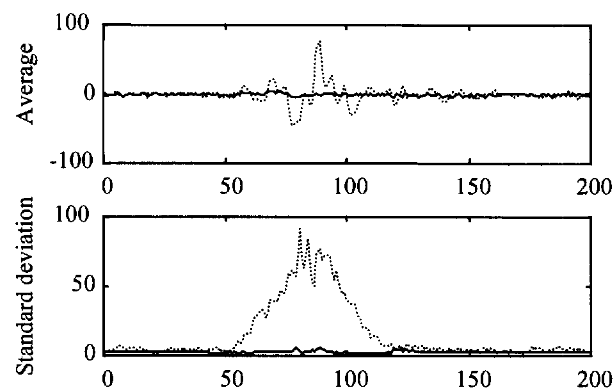

Figure 4. Prediction errors for first iteration in microns:
Periodic model (solid), Time-invariant model (dot)

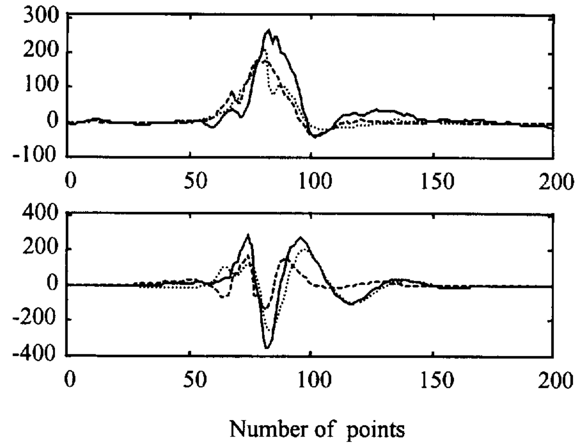

Figure 5. Results for first iteration in microns:

Top: Scaled perturbed reference (dot)
Simulated perturbed output (dash)
Experimental perturbed output (solid)

Bottom: Experimental perturbed input (solid)
Perturbed feedforward input (dot)
Perturbed repetitive input (dash)

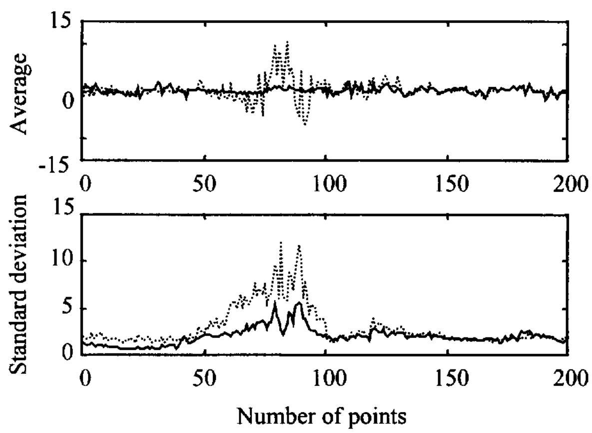

Figure 6. Prediction errors for second iteration in microns: new periodic model (solid), previous model (dot)

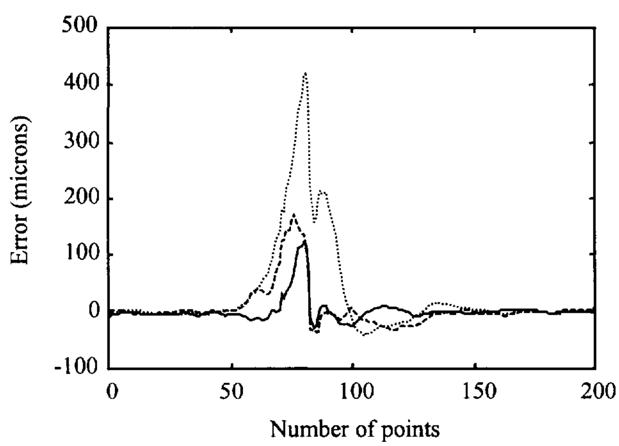

Figure 7. Tracking errors for full reference: nominal (dot) after 1^{st} iteration (dash), after 2^{nd} iteration (solid)

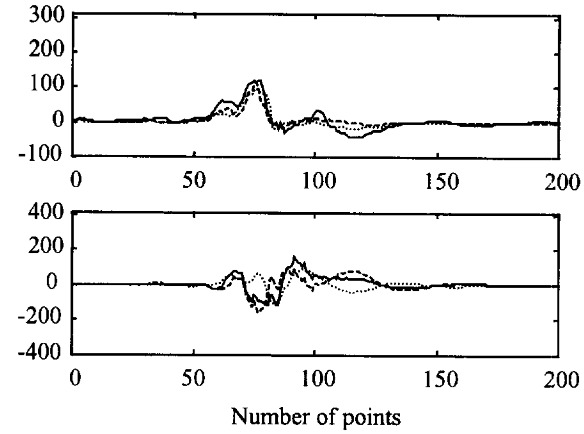

Figure 8. Results for second iteration in microns:

Top: Scaled perturbed reference (dot)
Simulated perturbed output (dash)
Experimental perturbed output (solid)

Bottom: Experimental perturbed input (solid)
Perturbed feedforward input (dot)
Perturbed repetitive input (dash)

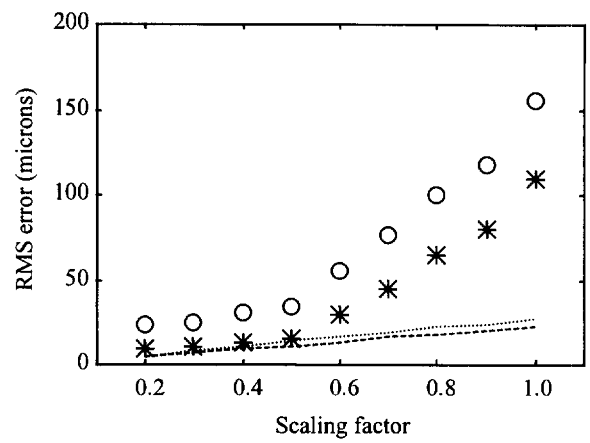

Figure 9. RMS errors for scaled perturbed reference:
Experimental: 1^{st} iteration (o), 2^{nd} iteration (*)
Simulation: 1^{st} iteration (dot), 2^{nd} iteration (dash)

Table 1. Summary of results

	Maximum error (microns)	RMS error (microns)
Nominal response	410	95
After 1^{st} iteration	190	50
After 2^{nd} iteration	125	22