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Control of Linear Motor Machine Tool Feed Drives for End Milling : Robust MIMO Approach

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Abstract

In this work, a MIMO H_∞ controller for the linear motor machine tool feed drives has been designed to reduce tracking errors induced by cutting forces for end milling. The controller is designed using normalized coprime factorization method for the dynamic model of the linear motor system including constant in-line and cross coupling force gain, since the feedback cutting force can be considered as the product of the constant gain and the moving velocity of each axis.

Analysis of the structured singular value shows that the designed controller has good robust performance despite wide variations of the cutting force and physical parameters. It is directly implemented on a linear motor X-Y table which is mounted on a milling machine to have cutting experiments via a DSP board. Experimental results verified effectiveness of the proposed scheme to suppress the effects of the cutting force in the high feed rate.

1. Introduction

High speed machining is getting more desirable to improve productivity. Directly driven feed drives eliminate backlash and structural flexibilities due to gear reduction mechanism. It seems that linear motors can be used as good machine tool feed drives due to high acceleration and direct driving. The feed drive and the cutting process in the end milling are no longer decoupled, because the liner motor system has no gearing mechanism. While the elimination of gearing gives benefit of high speed tracking, the cutting forces are directly reflected to the motors due to the direct coupling and have an even more severe effect on tracking accuracy. The higher the stiffness becomes, the better the linear motor can bear the external disturbances.

Alter and Tsao[1] investigated the use of linear motors as feed drives for the turning process. They showed that the system stability is primarily dependent on the interaction of the cutting process and the feed drive servo loop in a direct drive.

The cutting force in the end milling process is primarily periodic with DC component due to the rotating flutes. The frequency of the periodic force is mostly outside of the closed loop system bandwidth. Accounting for the average force component would be sufficient to reduce tracking errors due to cutting. The average forces in the in-line and cross directions was obtained by integrating the instantaneous forces over the duration of the cutting passes [2]. McNab[3] addressed the control of linear motors machine tool feed drives for the end milling process, and showed that 2 axes end milling process is a MIMO system rather than two SISO system with inclusion of in-line force and cross force feedback.

A MIMO H_∞ controller for X-Y table system using linear motors as machine tool feed drives in the end milling process is suggested to improve tracking accuracy in this research. The dynamic model for designing the controller is considered as 2 input 2 output system coupled by the constant force gain for the cross force feedback. The controller with high gain is designed to increase stiffness of the system and reduce tracking error by the cutting forces. The feedback force gains are considered to have parametric uncertainties and therefore robust controller is designed using loop shaping method with normalized coprime factorization. Robustness of stability and performance for parametric changes of the force gains and the physical parameters including unmodeled power amplifier dynamics is examined using structured singular value. Simulation and experimental results verify the closed loop system maintains its robustness in spite of wide changes of

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force gains.

2. Modeling of the linear motor system

Neglecting the dynamics of the electrical parts in the linear motor X-Y table, its velocity model can be represented as 1st-order differential equation for each axis

$$\dot{v}_i + \frac{b_i}{m_i} v_i + \frac{F_{Coul,i}}{m_i} sign(v_i) = \frac{k_i}{m_i} u_i \qquad (1)$$

where v is velocity, m is mass, b is viscous damping, k is input gain, F_{Coul} is Coulomb friction and i=x, y, respectively.

There have been a lot of researches on the cutting force model for end-milling operation, but they are complicated for control purpose[4]. The cutting force is basically composed of 2 frequency components, the DC component and another at the tooth passing frequency. Accounting for the average force component would be sufficient to reduce tracking errors due to cutting, since the cyclic component amplitude of the tool force does not exceed 20 % of its averaged value[2]. The average forces in the in-line and cross directions was obtained by integrating the instantaneous forces over the duration of the cutting passes. The average forces are expressed

$$F_{il} = k_{il} v_{il} \qquad (2)$$

$$F_c = k_c v_c \qquad (3)$$

where F_{il} and F_c are the forces in the in-line and cross directions, and k_{il} and k_c are in-line cutting and cross coupling force gain, respectively.

Each axis of the linear motor X-Y table including the cutting force in the milling process is modeled as

$$\dot{v}_x + \frac{b_x}{m_x} v_x + \frac{F_{Coul,x}}{m_x} sign(v_x) = \frac{k_x}{m_x} u_x + k_{il} v_x - k_c v_y \qquad (4)$$

$$\dot{v}_y + \frac{b_y}{m_y} v_y + \frac{F_{Coul,y}}{m_y} sign(v_y) = \frac{k_y}{m_y} u_y + k_{il} v_y + k_c v_x \qquad (5)$$

The dynamic model for the 2 axes has 2 input 2 output structure coupled by the cross coupling force

3. H_∞ Loop Shaping Controller Design

This approach makes use of an uncertainty description based on additive perturbation to a normalized coprime factorization of the plant[5]. It is particularly attractive in that the optimal ∞-norm γ can be found without recourse to the γ-iteration which is normally required to solve H_∞ problems. Even though μ–synthesis can ensure robust performance with $\mu \le 1$, the order of the resulting controller is considerably high and γ-iteration should be done to find γ to satisfy the existence of the solution. The H_∞ Loop Shaping method gives a controller near to μ=1 without γ-iteration. Robust stability and robust performance for the controller are examined by μ–analysis in the later section, even though the controller is designed by the H_∞ loop shaping method.

Obtaining the error signal for the external disturbance force reflected on the entry side of the linear motor system,

$$e = (I + GK)^{-1} G f \qquad (6)$$

where e is an error, K is a controller and f is a disturbance, respectively.The norm of the error is $\|e\| \le \|1/\underline{\sigma}(K)\| \|f\|$.

A controller with high gain in low frequencies is desirable to reduce tracking errors . A loop compensator should be selected so that the final feedback controller would have high gain. But a high gain in the controller may result in wide bandwidth of the system, which may allow that the frequency of the cyclic component of the cutting force by the rotating spindle exists inside of the system bandwidth. Therefore, there should be a compromise between a gain selection and a system bandwidth so that the feedback system does not have effects of the cyclic forces of the rotating tool. $W_s = (S+110)/(s+1500) diag\{990, 660\} I_{2\times 2}$ in this design gives the target loop a high gain in the low frequencies region and -20db/dec roll-off rate near the cut-off frequency as shown in Fig. 1.

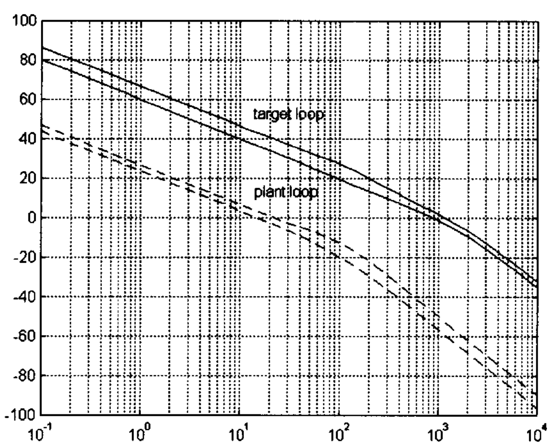


Fig. 1 The open loop and target loop of the system

The proportional gains for both the axes are selected so that they have the similar response time. The introduction of the zero into the loop compensator may increase damping of the system to compensate for deficiency of the mechanical damping in the linear motor axis. The open loop of the linear motor system went up by the loop compensator with high gain, compared with the plant open loop and the target loop has considerably high amplitude in the low frequencies. The order of the designed MIMO controller is 8, but is reduced to 4 by Schur balanced truncation method to reduce the execution time in the DSP board.

4. Analysis of Robustness

Robust stability is highly desirable in this system, because its dynamic model has some parameters with wide variation. Actual values for physical parameters *m*, *b*, k_{il} and k_c are not known exactly, but are believed to be lie in known intervals. Especially, the cutting force gains are highly dependent on axial depth, radial depth, cutting angle etc. and have greater variations than the other parameters. Therefore it is of no use to obtain and compensate for their exact values. Robust control approach which allows their variations in predetermined intervals will be more reasonable for real implementation.

Expressing actual values of all the parameters with multiplicative uncertainties,

$$\begin{aligned} m_{a,x} &= m_x(1+\alpha_{mx}\delta_{mx}) \\ b_{a,x} &= b_x(1+\alpha_{bx}\delta_{bx}) \\ m_{a,x} &= m_x(1+\alpha_{mx}\delta_{mx}) \\ b_{a,y} &= b_y(1+\alpha_{by}\delta_{by}) \\ k_{il,a} &= k_{il}(1+\alpha_1\delta_1) \\ k_{c,a} &= k_c(1+\alpha_2\delta_2) \end{aligned}$$

where perturbation $|\delta_i| \le 1$. α_i is a constant to determine the limit of the known interval of actual parameter values. $1/m_x$ can be represented as LFT in δ_{mx} [6].

$$\frac{1}{m_{a,x}} = \frac{1}{m_x(1+\alpha_{mx}\delta_{mx})} = \Gamma_l(M_{mx}, \delta_{mx}) \quad \text{with}$$

$$M_{mx} = \begin{bmatrix} 1/m_x & -\alpha_{mx}/m_x \\ 1 & -\alpha_{mx} \end{bmatrix}.$$

The same will be done on the parameters of the y axis. Define $z = [z_{bx}\ z_{mx}\ z_{x1}\ z_{x2}\ z_{by}\ z_{my}\ z_{y1}\ z_{y2}]^T$, $y = [y_x\ y_y]^T$, $u = [u_x\ u_y]^T$, $x = [y_x\ v_x\ y_y\ v_y]^T$ and $w = [w_{bx}\ w_{mx}\ w_{x1}\ w_{x2}\ w_{by}\ w_{my}\ w_{y1}\ w_{y2}]^T$ where *x, z, y* and *w* are the system state, the regulated output, the measured output and the exogenous input, respectively. Parametric uncertainties of the system gains k_x and k_y will be included into the unmodeled power amplifier dynamics, which is also considered as multiplicative uncertainty. The perturbed system can be described via the LFT so that all the uncertainty is represented as a nominal system with the unknown parameters. Let G_l be ten-input $(w_{mx}, w_{bx}, w_{my}, w_{by}, w_{x1}, w_{x2}, w_{y1}, w_{y2}, u_x, u_y)$, ten-output $(z_{mx}, z_{bx}, z_{my}, z_{by}, z_{x1}, z_{x2}, z_{y1}, z_{y2}, y_x, y_y)$, four-state nominal system shown in Fig. 4 and the unknown matrix $\Delta = diag\{\delta_{mx}, \delta_{bx}, \delta_{my}, \delta_{by}, \delta_1 I_{2\times2}, \delta_2 I_{2\times2}\}$, referred to as the perturbation, is structured.

The unmodeled power amplifier dynamics for each axes is assumed to be about 30 % below 600 rad/sec frequency, rising to about 100 % at 2000 rad/sec, since it exists in the higher frequency region than the mechanical dynamics. Most of modeling error below 600 rad/sec is assumed to be error of the input gains k_x and k_y. The uncertainty weighting function chosen is a 2 by 2 transfer matrix $W_s = \{(s+600)/(s+2000)\}I_{2x2}$ and the related uncertainties for both the axes are δ_{dx} and δ_{dy}. Robustness of performance as well as stability robustness of the feedback system in the cutting should be examined. Robust performance specification is assumed here that each linear motor axis should, under the parametric and unmodeled uncertainties and the excitation of uncertain exogenous signal, maintain the tracking error to 1/40 of the reference commands below 5 rad/sec. The tracking performance for the command is evaluated using the output sensitivity transfer function $(I+GK)^{-1}$. It is modeled by using a first-order 2×2 weighting matrix $W_p = \{2(s+100)/(s+5)\}I_{2x2}$ to satisfy $\|W_p(I+GK)^{-1}\|_\infty < 1$. To evaluate robust performance, weights for tracking errors of the 2 axes with δ_{px} and δ_{py} are added into the generalized plant. The uncertainty matrix is defined as $\Delta_{all} = diag\{\Delta, \delta_{dx}, \delta_{dy}, \delta_{px}, \delta_{py}\}$ with the augmented generalized plant G_a as shown in Fig. 2[7].

If the augmented generalized plant G_a has a structured singular value below 1 in the all the operating frequencies region, it is said to be robust in performance.

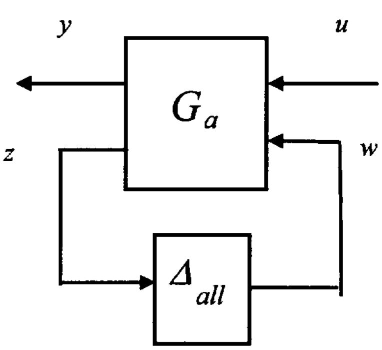


Fig. 2 Extended LFT representation for robust performance

The physical parameters and their uncertainties used in this controller design is summarized in Table 1where k_{il} and k_c are chosen for up milling.

Table 1 Numerical values of the system parameters

Parameters	Nominal value	Uncertainty(%)
k_{il}	-2500	110
k_c	2500	110
m_x	52.3	30
m_y	17	40
b_x	193.5	20
b_y	36	20

The MIMO controller is compared with a SISO PID controller to check its robustness and performance. The PID controller considers the cutting forces as external forces. It is designed to have high gain for stiffness by using conventional loop shaping. The structured singular value is a good measure to check robust stability and performance of the controller. The H_∞ controller shows its performance robustness for the command and disturbances below 5 rad/sec and is superior than the PID as shown in Fig.3.

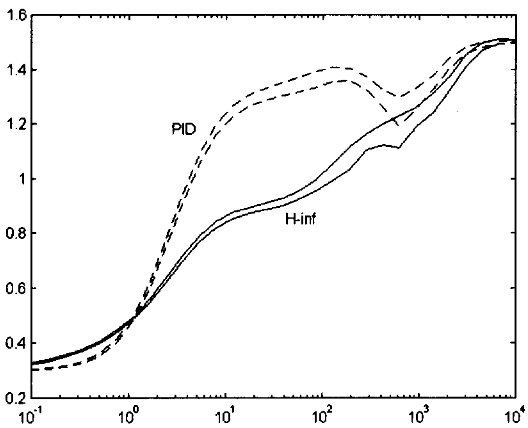


Fig. 3 Structured singular value for robust performance

5. Experimental Results

The linear motors used for the X-Y table are Anorad brushless linear servo motor LEB-S-8 for both the axes and have the peak force of 982N and the continuous force of 349N, respectively. A 1 μm resolution linear encoder is mounted for the X axis which is the lower axis and a 2 μm resolution one is done for the Y axis. A Spectrum TMS320C30 DSP is used to execute the control algorithm written in C language. Control is implemented at a sampling rate of 2 KHz. The linear motor X-Y table was mounted and fixed on the bed of a Mori Seiki SV50 machining center. The vertical axis of the machining center with the rotating spindle moves up and down and its bed is fixed during the cutting. The linear motor X-Y table makes circles and lines to have cutting experiments.

The tracking controller consists of a feedback controller and a zero phase error tracking controller as a feedforward controller. A 2 flute end mill is used as a cutting tool in the cutting experiments and the workpiece is aluminum. A circle with the radius of 28 mm is examined and the flute will cut the inner surface of the circle. The axial depth of cut is 3 mm, the radial depth of cut is 3 mm, the spindle speed is 10000 rpm and the feed rate of the table is 4800 mm/min, respctively. This corresponds to a feed per tooth of 0.24 mm. Roughly estimating the cutting force gains for this cutting condition[3], their absolute values are about 1800 N/(m/sec) and exist inside of the allowable variation range for the designed MIMO controller. Contour error is the distance from the reference trajectory to the output position. The maximum contour errors for both the axes are about 36 and 41 μm, respectively as shown in Fig. 4, in spite of high feed rate of the table.

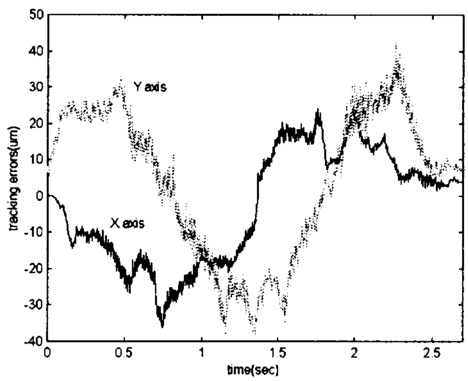
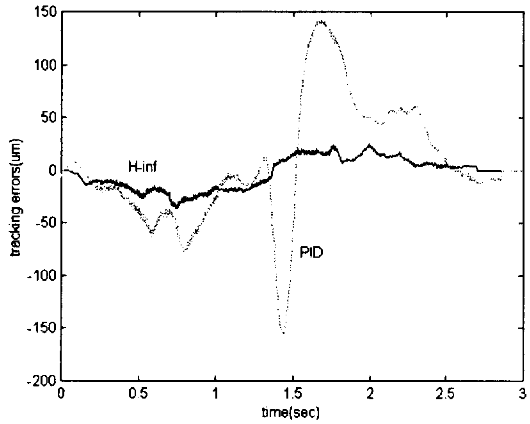
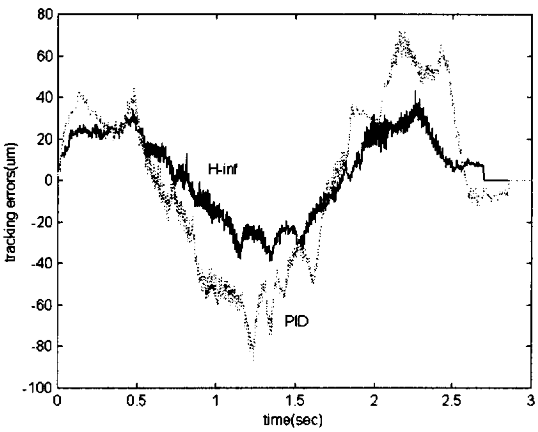


Fig. 4 Tracking errors for the MIMO controller

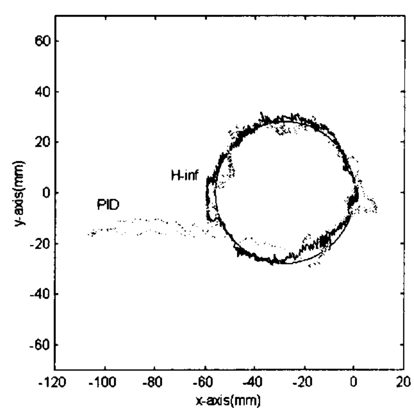
The tracking performances of the MIMO and the PID in the cutting are compared. The inner surface of the circle with the radius of 30 mm is cut for the PID. The cutting tool will be less embedded in the workpiece when it cut the inner surface of the larger circle. The less cutting force may be reflected to the PID than the MIMO which cut the circle with the radius of 28mm. Fig. 5 shows the tracking performance of the MIMO and PID for cutting circles. The contouring error in the Fig. 5(c) is magnified to 300 times its actual size. The MIMO demonstrates considerably better performance for both the axes than the PID. The PID induces larger error at about 1.4 sec for the X-axis and 1.24 sec for Y-axis when the axes reverse their moving directions.



(a) Errors for the X axis



(b) Errors for the Y axis



(c) Errors for the circle

Fig. 5 Comparison of the contouring errors

6. Conclusions

In this work, a MIMO H_∞ controller for the linear motor machine tool feed drives has been designed to reduce tracking errors induced by cutting forces for end milling. The controller was designed using normalized coprime factorization method and by considering constant cutting force gain to give coupling effects between X and Y axis. Simulation results for performance robustness showed that the MIMO controller allows less tracking error than the PID controller. The designed controller was directly implemented on a linear motor X-Y table via a DSP board. The table was mounted on a milling machine to have cutting experiments. Experiments were performed to verify the superior performances of the MIMO controller over the PID controller. The MIMO controller demonstrated good performance in the cutting condition of the high feed rate and high spindle speed, even though the PID brought about considerably large tracking error. The experimental results for the air and real cutting showed robustness of the MIMO controller over wide range of the cutting force and feed rate.

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