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Adaptive Tracking Control by System Inversion

Zongxuan Sun and Tsu-Chin Tsao

Department of Mechanical And Industrial Engineering
University of Illinois at Urbana-Champaign
Urbana, IL 61801, USA

Abstract

This paper presents a discrete time adaptive inversion scheme for linear systems and its usage in adaptive feedforward and feedback controllers. Two parameter adaptation algorithms (PAA) were used in the proposed schemes. The first PAA applies the extended bias-eliminating least-squares (EBELS) algorithm for plant estimation to ensure convergence to a tuned model under colored noise and unmodeled dynamics. The second PAA applies EBELS or least mean squares (LMS), respectively for two adaptive inversion schemes, to estimate a finite impulse response (FIR) plant inverse filter. A feedforward and a repetitive feedback control algorithms are designed respectively by including the adaptive inverse filter.

1. Introduction

Most feedforward and some feedback tracking control designs, e.g. repetitive control, involve the determination of a stable inverse of plant dynamics. The tracking control performance relies on the accuracy of the plant inversion, which in turns depends on the accuracy of the plant model estimation and the plant inversion method. Adaptive inversion is a useful technique to obtain an accurate inverse model, which is less sensitive to plant uncertainties and variations but also adjusts itself to plant parameter changes.

In indirect adaptive inversion schemes, a plant model is first estimated and the plant inversion is either calculated or estimated by a second parameter estimation scheme. Since the on-line computation of plant inversion at every time step may be quite computationally intensive, Tsao and Tomizuka (1988, 1989, 1994) proposed the use of the zero phase error filter, which is computationally simple and efficient, as the approximation of delayed inverse plant in the indirect adaptive feedforward tracking controller. Projection parameter adaptation schemes with filtering, dead zones, and parameter space constraints were employed in the PAA to achieve robust adaptive plant estimation against bounded noise and unmodeled dynamics without assuming persistency excitation condition. Sun and Tsao (1997) further applied similar method in indirect adaptive feedback control, where an internal model was also included for asymptotic tracking performance. In particular, the adaptive zero phase error inversion was used in adaptive repetitive control. The system stability and the relations of the tracking error to the plant estimation error were provided respectively for both the adaptive feedforward tracking and the adaptive repetitive control schemes.

The idea of using a second PAA to estimate delayed plant inverse can mostly be found in the adaptive inverse control proposed by Widrow and his associates. Widrow introduced the adaptive inverse feedforward control (1985, 1986) using Least Mean Squares adaptation scheme. Widrow and Walach (1996) further discussed the Filtered-X algorithm and disturbance canceller using adaptive inverse control in a feedback loop. However, there is a lack of stability analysis when systems experience colored noise and unmodeled dynamics. When the plant parameter estimation is biased, the adaptive inverse controller may not be accurate any more.

When the bound of the noise is too large for the relative dead zone approach to be effective or the colored noise makes the LMS scheme invalid, the above mentioned adaptive inversion methods cannot be applied. However, if the persistent excitation condition is met, recent results on eliminating parameter estimation bias for input/output signals corrupted by colored bounded noise (Feng and Zhang 1995, Zhang and Feng 1997) can be utilized to ensure parameter convergence to a tuned model. This paper treats the adaptive feedforward and adaptive repetitive control problems using the extended bias-eliminating least-squares (EBELS) algorithm to solve the bias problem associated with bounded colored noise. Stability and robustness analysis are presented and discussed along with simulation results.

2. System Model Description

Consider the system model as follows:

$$y(k) = Pu(k) + d(k)$$
$$y(k) = G_0 u(k) + \eta(k) + d(k) \tag{1}$$

where the tuned model is $G_0 = \dfrac{B(q^{-1})}{A(q^{-1})}$ (2)

$$A(q^{-1}) = 1 + a_1 q^{-1} + \cdots + a_n q^{-n} \tag{3}$$
$$B(q^{-1}) = b_0 + b_1 q^{-1} + \cdots + b_m q^{-m} \tag{4}$$

$A(q^{-1})$ and $B(q^{-1})$ are coprime. $\eta(k)$ and $d(k)$ denotes the unmodeled dynamics and colored noise respectively. q^{-1} is the delay operator.

Remark 1: In system model (1), the unmodeled dynamics $\eta(k)$ can be multiplicative, additive or both. Note the bound for unmodeled dynamics will vary corresponding to the tuned model structure one chooses. In many tracking control applications, the upper bound for unmodeled dynamics can be determined experimentally once the tuned model is fixed. So by choosing a good tuned model, the upper bound of unmodeled dynamics can be suppressed. Generally one can't suppress the bound for noise once the hardware of a system is fixed.

Assumption 1: System (1) is a stable system, i.e. $A(q^{-1})$ is strictly stable, η is bounded by u and some constant.

Remark 2: We assume the original open loop system is stable or can be stabilized by a fixed controller. This is a general case in

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many tracking control applications. Also note there is no assumptions made on the zero dynamics of the system model.

3. Parameter Adaptation Algorithm

Our goal of plant estimation is to identify a tuned model instead of the true model because of the existence of unmodeled dynamics. With both unmodeled dynamics and colored noise in system model (1), the tuned model will be biased if we use ordinary least squares. Recent results on eliminating estimation bias for input output signals corrupted by colored bounded noise (Feng and Zhang 1995, Zhang and Feng 1997) can be utilized for plant estimation to ensure convergence to the tuned model. The basic idea behind the extended bias-eliminating least-squares (EBELS) algorithm is that by inserting some known parts into the estimated system, we can asymptotically estimate the bias caused by noise and then eliminate it. If we take the unmodeled dynamics $\eta(k)$ as a kind of noise which is correlated with both input and output in the identification process, we can then eliminate its bias effect together with the bias caused by colored noise by using EBELS algorithm.

Assumption 2: Persistent excitation is ensured in the adaptive system.

Define $\theta = [-a_1, \cdots, -a_n, b_0, \cdots, b_m]$ (5)

$$\hat{\theta} = [-\hat{a}_1, \cdots, -\hat{a}_n, \hat{b}_0, \cdots, \hat{b}_m] \quad (6)$$

$$\hat{P} = \frac{\hat{B}(q^{-1})}{\hat{A}(q^{-1})} \quad (7)$$

By the properties of the extended bias eliminating least squares algorithm, we have following lemma:

Lemma 1: $\lim_{N \to \infty} \hat{\theta}(N) = \theta$. w.p.1.

Proof: See Zhang and Feng (1997).

***Remark** 3:* In the identification process, we take the unmodeled dynamics $\eta(k)$ as a kind of noise because in most applications, it is impossible to distinguish unmodeled dynamics and noise from experimental data. Generally $\eta(k)$ is correlated with $u(k)$, so now both input and output are corrupted by noise. The Extended Bias Eliminating Least Squares (EBELS) ensures an accurate estimate of the tuned model, i.e. $\hat{P} \to G_0$. Also notice that it can be used in either open loop or closed loop because this estimation algorithm doesn't have any requirements on the noise model.

4. Adaptive Feedforward Tracking Control

Widrow and Walach (1996) proposed an adaptive feedforward inverse control scheme (as shown in Fig. 1 with $F = 1$). Filtered-X algorithm was used to online adapt controller parameters based on the estimated plant model. With no unmodeled dynamics and noise, the adaptive feedforward inverse control can achieve perfect tracking performance. For the case that Figure 1 is at 'a' position, the estimated plant and therefore the adaptive inverse controller may be biased if there exist unmodeled dynamics or colored noise. Another approach is to put the switch at 'b' position as shown in Fig. 1 . In this case, the LMS adaptation gain may need to be very small to ensure convergence. One way to alleviate this problem is to introduce a low pass filter F in the Filterd-X algorithm. The Modified Filterd-X algorithm is

$$\hat{C}(k+1) = \hat{C}(k) + 2a\varepsilon(k)F\tilde{X}(k) \quad (8)$$

where

$\tilde{X}(k) = [\tilde{x}(k), \tilde{x}(k-1), \cdots, \tilde{x}(k-M+1)]^T$, $\tilde{x}(k) = \hat{P}y_{ref}(k)$,

ε is the error signal as shown in Fig. 1, a is the adaptation gain, F is a low pass filter, and the adaptive inverse controller $\hat{C}$ is a FIR filter with length M .

Theorem 1: The proposed adaptive feedforward tracking control scheme using EBELS and Modified Filterd-X algorithm can guarantee system stability and the adaptive controller will converge to the inverse model of the original plant.

Proof: Suppose the inverse model of the original plant P is C^*, as shown in Fig. 1 (with switch at position 'b'), we have

$$\varepsilon(k) = (C^* - \hat{C}(k))^T X(k) - d(k),$$

where

$$X(k) = [x(k), x(k-1), \cdots, x(k-M+1)]^T, \; x(k) = Py_{ref}(k)$$

$$\varepsilon(k) = -V^T(k)X(k) - d(k),$$

where $V(k) = \hat{C}(k) - C^*(k)$

$$\varepsilon(k) = -X^T(k)V(k) - d(k),$$

Subtracting C^* from both sides of Eq. (8), we get

$$V(k+1) = V(k) + 2a\varepsilon(k)F\tilde{X}(k)$$

$$V(k+1) = V(k) + 2a(-X^T(k)V(k) - d(k))F\tilde{X}(k)$$

$$V(k+1) = (I - 2aF\tilde{X}(k)X^T(k))V(k) - 2ad(k)F\tilde{X}(k)$$

Assume $d(k)$ is not correlated with $\tilde{X}$, then

$$E[V(k+1)] = E[V(k)] - 2aE[F\tilde{X}(k)X^T(k)]E[V(k)]$$

$$E[V(k+1)] = E[V(k)] - 2a\tilde{R}E[V(k)],$$

where $\tilde{R} = E[F\tilde{X}(k)X^T(k)]$

we transform $\tilde{R}$ into normal form

$$E[V(k+1)] = \tilde{T}(I - 2a\tilde{\Lambda})\tilde{T}^{-1}E[V(k)]$$

$$E[V'(k+1)] = (I - 2a\tilde{\Lambda})E[V'(k)],$$

where $V'(k) = \tilde{T}^{-1}V(k)$

$$E[V'(k+1)] = (I - 2a\tilde{\Lambda})^k E[V'(0)]$$

$$E[V(k+1)] = \tilde{T}(I - 2a\tilde{\Lambda})^k \tilde{T}^{-1}E[V(0)]$$

we choose the adaptation gain $0 < a < \frac{1}{\lambda_{max}}$, where λ_{max} is the maximum eigenvalue of $\tilde{R}$ (also the maximum eigenvalue of $\tilde{\Lambda}$), then

$$\lim_{k\to\infty}(I - 2a\tilde{\Lambda})^k \to 0, \quad \lim_{k\to\infty} E[V(k)] \to 0, \quad \lim_{k\to\infty} \hat{C}(k) \to C^*.$$

Because $\hat{C}$ is a FIR filter and it converges, stability of the adaptive system is guaranteed.

$\therefore \lim_{k\to\infty} y(k) = y_{ref}(k-c)+d(k)$

Remark 4:

- We choose the adaptive inverse controller $\hat{C}$ as a FIR filter so that it won't be biased as long as input $\tilde{X}$ is not correlated with noise.
- If the structure of the tuned model G_0 is carefully chosen, $\hat{P}$ may be very close to P (since $\hat{P} \to G_0$ by EBELS), then $\tilde{R}$ is nearly symmetric. We also know that unmodeled dynamics are mostly concentrated at high frequencies, therefore by appropriately filtering signal $\tilde{X}$, we can guarantee that $\tilde{R}$ is almost symmetric so that it is easy to determine λ_{max} .
- Note in the proof, we have used the independence assumption for the LMS algorithm. Also the convergence of variance can be guaranteed by carefully choosing the adaptation gain.

5. Adaptive Repetitive Control for Tracking Periodic Signals

To track or reject periodic signals, Tomizuka et. al. (1989) proposed a discrete time repetitive control scheme which requires the inverse plant model. In this section, we are going to apply the adaptive inverse control to the repetitive control structure to track or reject periodic signals.

5.1 Adaptive Repetitive Control Scheme I

The closed loop adaptive repetitive control system is shown in Fig.2 (with switch at position 'a' and $F=1$). Uncertainties of the adaptive system are mainly from two sources. One is the unmodeled dynamics of the original plant; the other is due to the plant inversion error which may rise because of limited length of the FIR filter and the adaptation time. Both uncertainties could deteriorate the performance and stability of the adaptive closed loop system. To enhance system robustness, the repetitive control law is designed as follows:

$$u(k) = \hat{C}(k)\frac{Qq^{-L+C}}{1-Qq^{-L}}(y_{ref} - y) \tag{9}$$

where Q is a low pass filter, L is the period of the periodic signals and C is some positive constant smaller than L.

The adaptation law for inverse controller is as follows:

$$\hat{C}(k+1) = \hat{C}(k) + 2a\varepsilon(k)X(k) \tag{10}$$

where $X(k) = [x(k), x(k-1), \cdots, x(k-M+1)]^T$, $x(k) = \hat{P}\omega(k)$, ω is white noise, ε is the error signal and a is the adaptation gain.

Theorem 2: The closed loop adaptive system is BIBO stable. By appropriately choosing Q , the closed loop transfer function will converge closely to a pure delay.

Proof: Assume the original plant is $P = \hat{P}(1+\Delta_1)$, the closed loop characteristic polynomial becomes:

$$M = 1 - Qz^{-L} + Qz^{-L+C}\hat{C}P = 1 - Qz^{-L} + Qz^{-L+C}\hat{C}\hat{P}(1+\Delta_1)$$

Denote $\hat{C}\hat{P} = z^{-C}(1+\Delta_2)$, where Δ_2 is due to the plant inversion error.

Then $M = 1 - Qz^{-L} + Qz^{-L}(1+\Delta_2)(1+\Delta_1)$

so we can design Q such that

$\| Q(\Delta_1 + \Delta_2 + \Delta_1\Delta_2) \|_\infty < 1$

Thus $|z|<1$, also $\hat{C}$ converges, then the closed loop system stability can be guaranteed (see Goodwin, et. al., 1986).

5.2 Adaptive Repetitive Control Scheme II

Adaptive repetitive control scheme I is conservative because the adaptive inverse controller $\hat{C}$ only approaches the inverse of estimated plant model $\hat{P}$ instead of P. So we need to design Q to improve system robustness. This is the trade off between system robustness and performance. In this section, we are going to present an unbiased adaptive repetitive control scheme which will result in better performance.

The closed loop adaptive system is shown in Fig. 2 (with switch at position 'b'). The repetitive control law is as follows:

$$u(k) = \hat{C}(k)\frac{q^{-L+C}}{1-q^{-L}}(y_{ref} - y) \tag{11}$$

The goal of the adaptation law for $\hat{C}$ is to approach the inverse of P. As shown in Fig. 2 (with switch at position 'b'), we closed the loop for inverse controller parameter adaptation. Now we use the actual output to adjust the controller parameters which will minimize tracking error variance. Since we have two closed loops in the system, during the adaptation for $\hat{C}$, both input X and output $\bar{y}$ will be corrupted by noise and unmodeled dynamics. The extended bias eliminating least squares is used again to get the unbiased inverse controller C^* .

Theorem 3: The closed loop adaptive system is BIBO stable. The closed loop transfer function will converge to a pure delay, i.e. $\lim_{k\to\infty} y(k) = y_{ref}(k-L) + d(k)$.

Proof : From Fig. 2 (with switch at position 'b'), we know the reference model is

$\tilde{y}(k) = Fq^{-c}\bar{u}(k)$

$\tilde{y}(k) = FPC^*\bar{u}(k)$

$\tilde{y}(k) = FG_0C^*\bar{u}(k) + F\eta(k)$

where $C^* = \dfrac{q^{-c}}{P}$ is the exact inverse model of P, F is a low pass filter to be designed such that $F\eta(k) \to 0$.

The actual output $\bar{y}$ is

$\bar{y}(k) = FP\hat{C}\bar{u}(k) + Fd(k)$

$\bar{y}(k) = FG_0\hat{C}\bar{u}(k) + F\eta(k) + Fd(k)$

where G_0 is the tuned model.

In the closed loop adaptive system, both $\bar{u}(k)$ and $\bar{y}(k)$ are corrupted by noise. By applying the extended bias-eliminating least-squares (EBELS) algorithm to the adaptation of $\hat{C}$, convergence to the tuned model is guaranteed. $\therefore \lim_{K\to\infty} FG_0\hat{C} \to FG_0C^*$, then we can conclude that $\lim_{k\to\infty} \hat{C} \to C^*$. The closed loop transfer function will converge to a pure delay q^{-L}, so system stability is guaranteed and $\lim_{k\to\infty} y(k) = y_{ref}(k-L) + d(k)$.

Remark 5: Note the proposed adaptive repetitive schemes can not only guarantee perfect tracking performance for periodic signals due to the internal model principle but also provide good tracking performance for non-periodic signals since the closed loop transfer function will converge to a pure delay.

6. Simulation Results

The plant model for simulations is $P = \dfrac{1.0q^{-1} - 2.0q^{-2}}{1 - 1.8q^{-1} + 0.9q^{-2}}$ which have unstable zeros at 2 and infinity. Colored noise $d(k) = 0.1w(k) + 0.05w(k-1)$ (where $w(k)$ is white noise with variance 1.0) was added to the output during plant identification. A periodic siganl was used as reference signal to test both adaptive feedforward and adaptive repetitive control schemes. One period of the reference signal and the random colored noise is shown in Fig. 3

For the plant $P = \dfrac{1.0q^{-1} - 2.0q^{-2}}{1 - 1.8q^{-1} + 0.9q^{-2}}$, we run the EBELS and ordinary least squares 10 times and get an averaged estimate $\hat{P}$ respectively.

For EBELS: $$\hat{P} = \frac{1.0398q^{-1} - 2.0290q^{-2}}{1 - 1.7951q^{-1} + 0.8939q^{-2}}$$

For ordinary least squares: $$\hat{P} = \frac{0.9989q^{-1} - 1.1509q^{-2}}{1 - 0.9555q^{-1} + 0.2433q^{-2}}$$

We can see ordinary least squares estimate is seriously biased while EBELS gives a very accurate estimate.

Since plant P has an unstable zero at 2, we chose a 29th order FIR filter as the inverse controller, i.e.

$$\hat{C} = c_0 + c_1q^{-1} + \cdots + c_{29}q^{-29}$$

Denote C^* as the optimal inverse of P, then the inversion errors are $\dfrac{|\hat{C}_k - C^*_k|}{|C^*_k|} * 100\%$, $k = 0, 1, \cdots 29$. Fig. 4 and 5 show the tracking errors and plant inversion errors of adaptive inverse feedforward control using filtered-X algorithm, adaptive feedforward tracking control using modified filtered-X algorithm, adaptive repetitive control scheme I and adaptive repetitive control scheme II respectively. In Fig. 4, the maximum steady state errors (from 1 to 4) are 0.12, 0.07, 0.03 and 0.02 respectively.

7. Conclusion

Adaptive tracking control schemes by system inversion using extended bias-eliminating least-squares (EBLES) is presented in this paper. Modified Filtered-X algorithm is proposed to compensate unmodeled dynamics in feedforward loop to achieve better tracking performance. The adaptive inverse controller is then applied to the repetitive control structure to track or reject periodic signals. Two adaptive repetitive control schemes are presented and compared. Simulation results showed the effectiveness of the proposed adaptive tracking schemes.

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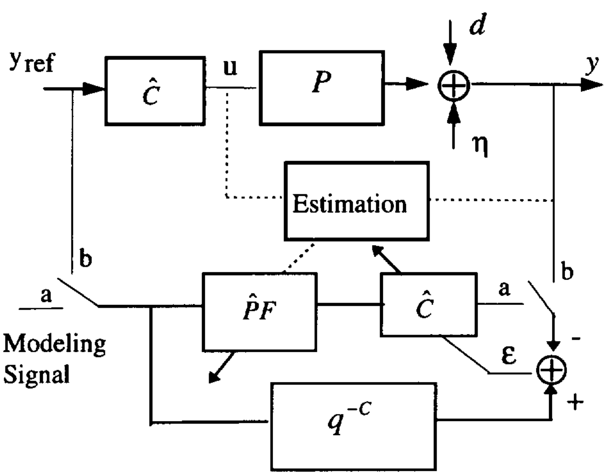

Fig. 1 Block Diagram of Adaptive Inverse Feedforward Control

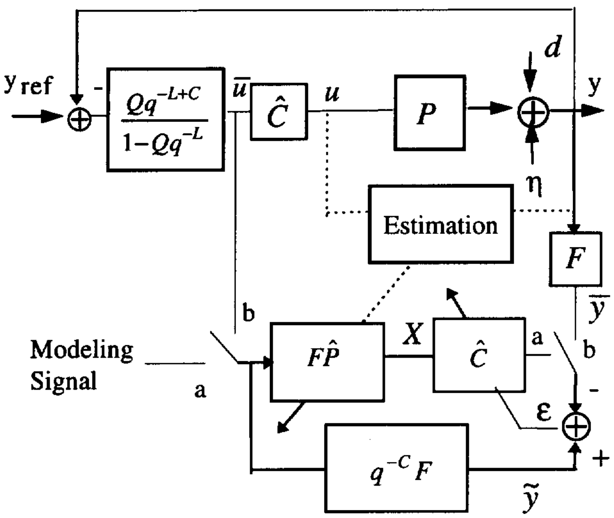

Fig. 2 Block Diagram of Adaptive Repetitive Control Schemes

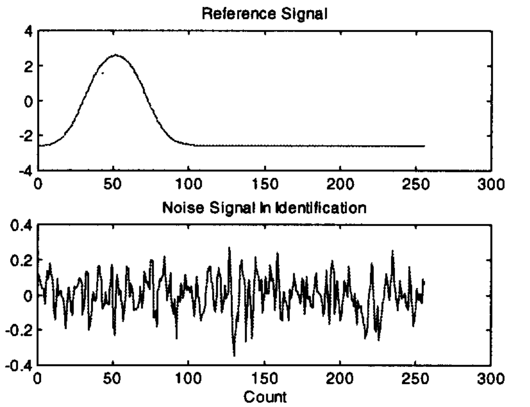

Fig. 3 Reference Signal and Noise in Identification process

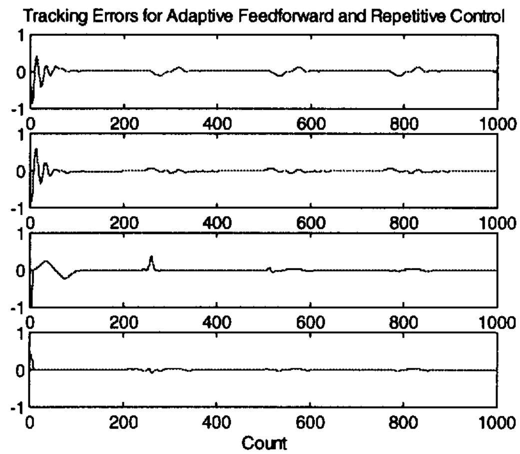

Fig. 4 Tracking errors (from 1 to 4) for adaptive inverse feedforward control using filtered-X algorithm, adaptive feedforward tracking control using modified filtered-X algorithm, adaptive repetitive control scheme I and adaptive repetitive control scheme II respectively.

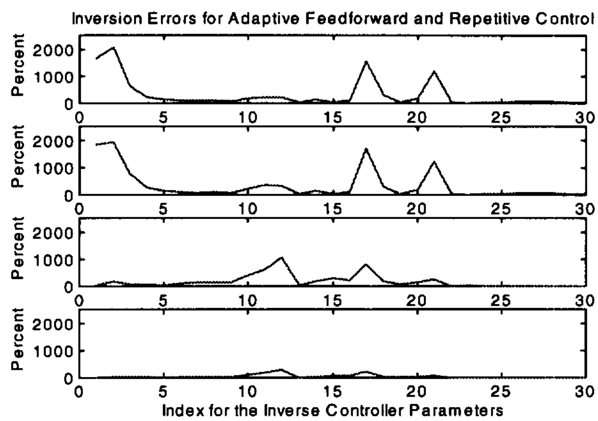

Fig. 5 Inversion errors (from 1 to 4) for adaptive inverse feedforward control using filtered-X algorithm, adaptive feedforward tracking control using modified filtered-X algorithm, adaptive repetitive control scheme I and adaptive repetitive control scheme II respectively.