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Rejection of Disturbance with Nonlinear Dynamics

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Abstract

Asymptotic tracking and disturbance rejection is a desirable performance in many applications. Linear feedback control based on internal model principle achieves asymptotic tracking for linear system with known linear exogenous signal dynamics. This paper investigates the case of rejecting exogenous chaotic signals with known nonlinear dynamics for linear systems in the discrete time domain. Feedback controllers based on the internal model principle and predictive internal model control respectively are proposed and investigated in this paper. Both control algorithms are based on inversion of the linear system. It is shown that asymptotic tracking performance is achieved when perfect plant inversion is possible and it cannot be achieved with either algorithm when inversion errors from unmodeled dynamics or plant nonminimum phase zeros exist. The closed loop stability and performance rely on the relative size of the linear system inversion errors to the exogenous signal's local growth rate.

1. Introduction

Tracking and rejection of exogenous signals is of major concern in feedback control design. The exogenous signals can often be modeled as unknown deterministic signals with known signal generating dynamics. For linear systems with linear disturbance dynamics, this problem has been studied by Davison (1976) and Francis and Wonham (1976) etc. The terminology of "Internal Model Principle" (IMP) was coined by Francis and Wonham (1976), which shows that it is necessary to place the disturbance dynamics in the feedback control loop for achieving asymptotic tracking. Integral control and repetitive control are examples of IMP type controllers. Recently more research efforts have been concentrated on developing similar concepts for nonlinear systems. Isidori and Byrnes (1990) discussed the local output regulation of nonlinear systems and gave necessary and sufficient conditions for solvability of the problem. Isidori (1997) further extended the results to the semiglobal output regulation problem, where the initial condition of the disturbance dynamics is confined in a bounded set. Huang and Lin (1991) proposed a kth-order robust nonlinear servomechanism design, and discussed the necessary and sufficient conditions for the existence of the kth-order servomechanism. All these works have focused on nonlinear systems with linear and in some cases slightly nonlinear disturbance dynamics.

Another class of controller design for disturbance rejection is internal model control (IMC). Garcia and Morari (1982, 1985, 1986) provided a unifying review on internal model control and further extended it to multivariable systems and nonlinear systems. A crucial step in applying internal model control is system inversion. Unstable zeros pose constraints on system inversion performance. Robustness to modeling errors is always a concern for internal model control. Various methods have been proposed to enhance system robustness. Morari and Zafiriou (1989) provided a detailed discussion on the robustness issue of IMC. Tsypkin (1993) proposed a robust internal model control. The so called "absorption principle" was proposed, which essentially embedded the disturbance signal model in the internal model control structure for asymptotic tracking performance.

This paper considers discrete time linear systems with chaotic disturbances, where the signal dynamics are known but the initial conditions are unknown. The chaotic signal generated by unstable nonlinear dynamics is sensitive to the initial conditions. The signal may converge to fixed points or periodic limit cycles with periods depending on the initial conditions. In this case, the repetitive control with a period identification scheme (Tsao and Nemani 1992, Tsao and Qian 1993) may be applied. In other cases, the chaotic signal may not converge to periodic orbits at all. This paper considers the control design for this latter case.

Two control algorithms are proposed in this paper: The nonlinear internal model principle (NIMP) scheme is designed by incorporating nonlinear disturbance model in the feedback loop. The predictive internal model control (PIMC) scheme is proposed by predicting the future disturbance based on the disturbance model in the internal model control structure. It will be shown that these two similar schemes are identical in the case that system is minimum phase.

The rest of this paper is organized as follows. Section2 describes the system and disturbance models; Section 3 presents the nonlinear internal model principle control; Section 4 presents the predictive internal model control; Section 5 presents the robust stability and performance analysis of both schemes; Section 6 presents simulation examples followed by conclusions in Section 7.

2. Problem Description

Consider following single input single output discrete-time linear time invariant causal system:

$$A_m(q^{-1})y(k) = B_m(q^{-1})u_m(k) + C_m(q^{-1})d_m(k) \qquad (1)$$

where $u_m(k)$ and $y(k)$ are input and output respectively. d_m is bounded disturbance. $A_m(q^{-1})$ and $B_m(q^{-1})$ are coprime. $A_m(q^{-1})$ and $C_m(q^{-1})$ are monic. $C_m(q^{-1})$ is stable.

Suppose the disturbance satisfies following nonlinear model:

$$d_m(k+1) = \psi(d_m(k)) \qquad (2)$$

where ψ is a nonlinear memoryless function.

A stabilizing feedback controller is first designed for the original plant,

$$S(q^{-1})u_m(k) = -R(q^{-1})y(k) + u(k)$$

Plug it into system (1), we have

$$A_m(q^{-1})y(k) = \frac{B_m(q^{-1})}{S(q^{-1})}(u(k) - R(q^{-1})y) + C_m(q^{-1})d_m(k)$$

$$y(k) = \frac{B(q^{-1})}{A(q^{-1})}u(k) + \frac{C(q^{-1})}{A(q^{-1})}d_m(k) \quad (3)$$

where $A(q^{-1}) = S(q^{-1})A_m(q^{-1}) + B_m(q^{-1})R(q^{-1})$ and $C(q^{-1}) = S(q^{-1})C_m(q^{-1})$ are stable. $B(q^{-1}) = B_m(q^{-1})$.

For notation convenience, we rewrite system (3) in the following form:

$$y(k) = Pu(k) + d(k) \quad (4)$$

where $P = \frac{B(q^{-1})}{A(q^{-1})}$, $d(k) = F(q^{-1})d_m(k) = \frac{C(q^{-1})}{A(q^{-1})}d_m(k)$.

Assume S is monic and stable, then F is proper stable and inversely stable. Therefore the disturbance $d(k)$ satisfies following nonlinear model:

$$d(k+1) = F\psi(\frac{1}{F}d(k)) \quad (5)$$

Remark: If the original system (1) is not stable, we can first stabilize it by designing a feedback controller. Also note both input and output disturbances can be represented in the form of system (1). So the control schemes we are going to propose in following sections can be used for rejecting either input or output disturbances.

3. Nonlinear Internal Model Principle Control

As shown in Fig. 1, the nonlinear internal model principle control law is as follows:

$$u(k) = -G_c z(k), \; z(k) = F\psi^L(\frac{1}{F}(z(k-L) + y(k))) \quad (6)$$

where ψ is defined in (2), G_c is a stable controller to be designed in section 3.1.

Plug the control law into system (3), we have

$$y(k) = d(k) - PG_c F\psi^L(\frac{1}{F}(z(k-L) + y(k)))$$

$$y(k) = d(k) - PG_c F\psi^L(\frac{1}{F}(z(k-L) - PG_c z(k) + d(k)))$$

$$y(k) = d(k) - PG_c F\psi^L(\frac{1}{F}((q^{-L} - PG_c)z(k) + d(k)))$$

If $PG_c = q^{-L}$, we have

$$y(k) = d(k) - q^{-L}F\psi^L(\frac{1}{F}d(k))$$

$$y(k) = d(k) - d(k)$$

$\therefore y(k) \to 0$ asymptotically, the output disturbance is rejected in a deadbeat fashion.

Remark: As shown in Fig. 1, disturbance model has been placed in the feedback loop. Besides this, to achieve perfect disturbance rejection, we also need to design a inverse controller.

3.1 Inverse Controller Design

To achieve perfect disturbance rejection, we need to design a inverse control by solving following model matching problem:

$$J = \inf_{G_c \in l_1} \left\| z^{-L} - PG_c \right\|_\infty \quad (7)$$

If P is a minimum phase system, we can design $G_c = \frac{q^{-L}}{P}$ by direct inversion. If the plant is a non-minimum phase system, delays are needed to improve the inversion performance.

Remark: A feature of the proposed nonlinear internal model principle control is the insertion of multiple delays into the system by putting more disturbance models in the feedback loop. The number of delays directly determines the achievable inversion performance in (7).

3.2 Extension to Disturbance in Vector Form

In many cases, the dynamic model of disturbance is in vector form.

Suppose $\bar{d}(k) = \begin{bmatrix} d(k) \\ d(k-1) \\ \vdots \\ d(k-m) \end{bmatrix} = \psi\left(\begin{bmatrix} d(k-1) \\ d(k-2) \\ \vdots \\ d(k-m-1) \end{bmatrix}\right)$

As shown in Fig. 2, we have

$$y(k) = d(k) - PG_c MF\psi^L(\frac{1}{F}(\bar{z}(k-L) + Ny(k)))$$

where $M = [1,0,\cdots 0] \in R^{(m+1)\times 1}$, $N = [1, q^{-1}, q^{-2} \cdots q^{-m}]^T$

$\bar{z}(k-1) = [z(k-1), z(k-2), \cdots z(k-m-1)]^T$

Therefore

$$y(k) = d(k) - PG_c[1,0,\cdots,0]F\psi^L\frac{1}{F}\left(\begin{bmatrix} z(k-L) \\ z(k-L-1) \\ \vdots \\ z(k-L-m) \end{bmatrix} + \begin{bmatrix} y(k) \\ y(k-1) \\ \vdots \\ y(k-m) \end{bmatrix}\right)$$

$$y(k) = d(k) - PG_c[1,0,\cdots,0]F\psi^L\frac{1}{F}\left(\begin{bmatrix} z(k-L) \\ z(k-L-1) \\ \vdots \\ z(k-L-m) \end{bmatrix} + \begin{bmatrix} d(k) - PG_c z(k) \\ d(k-1) - PG_c z(k-1) \\ \vdots \\ d(k-m) - PG_c z(k-m) \end{bmatrix}\right)$$

If $PG_c = q^{-L}$, we get

$$y(k) = d(k) - q^{-L}[1,0,\cdots,0]F\psi^L\frac{1}{F}\left(\begin{bmatrix} d(k) \\ d(k-1) \\ \vdots \\ d(k-m) \end{bmatrix}\right)$$

$$y(k) = d(k) - d(k) = 0$$

4. Predictive Internal Model Control

In this section, we will propose a new disturbance rejection scheme based on internal model control and disturbance prediction: Predictive Internal Model Control (PIMC). The block diagram of PIMC is shown in Fig. 3 and the control law is as follows:

$$u(k) = -G_c z(k) \quad (8)$$
$$z(k) = F\psi^L(\frac{1}{F}(\hat{P}G_c z(k) + y(k)))$$

Plug Control Law (8) into system (3), we have

$$y(k) = d(k) - PG_c F\psi^L(\frac{1}{F}(\hat{P}G_c z(k) + y(k)))$$
$$y(k) = d(k) - PG_c F\psi^L(\frac{1}{F}(\hat{P}G_c z(k) - PG_c z(k) + d(k)))$$

If $PG_c = \hat{P}G_c = q^{-L}$,

$$y(k) = d(k) - q^{-L}F\psi^L(\frac{1}{F}d(k))$$
$$y(k) = d(k) - d(k) = 0$$

Remark: Note the proposed predictive internal model control is different from ordinary internal model control because the disturbance model is inserted into the closed loop to predict future disturbance. Actually it is a integration of internal model control and internal model principle control. If $PG_c = q^{-L}$, the predictive internal model control scheme is exactly the same as nonlinear internal model principle control.

Remark: Similarly to nonlinear internal model principle control proposed in section 3, the predictive internal model control can also insert multiple delays into the feedback loop which makes an approximate inversion of non-minimum phase system possible. Also note that extensions of PIMC to disturbance in vector form can be obtained in similar fashion as the NIMPC.

5 Robust Stability and Performance Analysis for NIMPC and PIMC

In sections 3 and 4 we have shown that deadbeat disturbance rejection can be achieved by the proposed schemes based on the assumption of accurate model and perfect plant inversion. The effects of the unmodeled dynamics and the plant model inversion error on both schemes are analyzed next. Denote

Δ_1: Unmodeled dynamics of plant, i.e. $P = \hat{P}(1+\Delta_1)$

Δ_2: Plant inversion error, i.e. $\hat{P}G_c = q^{-L}(1+\Delta_2)$

Theorem 1 : Suppose $\psi(0) = 0$ and $\frac{|\psi(\sigma)|}{|\sigma|} \le M$ for some finite positive number M.

(a): The NIMPC is globally asymptotically stable if $M^L \| \Delta_1 + \Delta_2 + \Delta_1\Delta_2 \|_\infty < 1$ (9)

(b): The PIMC is globally asymptotically stable if $M^L \| \Delta_1 + \Delta_1\Delta_2 \|_\infty < 1$ (10)

Proof: (a): As shown in Fig.1, we can separate the system into a feedback loop with a linear block $q^{-L} - PG_c$ and a memoryless nonlinear block ψ^L. By the Circle Criterion (Khalil, 1996), the feedback system is globally asymptotically stable if

$$\| (z^{-L} - PG_c) \|_\infty < \frac{1}{M^L}$$

Since $z^{-L} - PG_c = z^{-L}(\Delta_1 + \Delta_2 + \Delta_1\Delta_2)$, (9) follows.

(b): Similarly, in view of Figure 3, $\| (PG_c - \hat{P}G_c) \|_\infty < \frac{1}{M^L}$, which implies (10).

Remarks: If $0 \le \frac{\psi(\sigma)}{\sigma} \le M$, the Circle Criterion gives the stability condition that $\mathrm{Re}(e^{-j\omega L}(\Delta_1 + \Delta_2 + \Delta_1\Delta_2)) > \frac{-1}{M^L}$ for NIMPC, and $\mathrm{Re}(e^{-j\omega L}(\Delta_1 + \Delta_1\Delta_2)) > \frac{-1}{M^L}$ for PIMC.

Now let's investigate system performance while uncertainties exist. To improve system performance, we insert a low pass filter Q into the system. For nonlinear internal model principle control, we have:

$$y(k) = d(k) - Qq^{-L}(1+\Delta_1)(1+\Delta_2)F\psi^L\frac{1}{F}(Qz(k-L) + y(k))$$
$$y(k) = d(k) - Qq^{-L}(1+\Delta_1)(1+\Delta_2)F\psi^L\frac{1}{F}(Qz(k-L)$$
$$-Q(1+\Delta_1)(1+\Delta_2)z(k-L) + d(k))$$
$$y(k) = d(k) - Qq^{-L}(1+\Delta_1)(1+\Delta_2)F\psi^L\frac{1}{F}(d(k) - Q(\Delta_1 + \Delta_2 + \Delta_1\Delta_2)z(k-L))$$

If $\| Q(\Delta_1 + \Delta_2 + \Delta_1\Delta_2) \|$ is small enough, $y(k)$ will be close to zero.

For predictive internal model control, we have:

$$y(k) = d(k) - QPG_c F\psi^L\frac{1}{F}(Q\hat{P}G_c z(k) + y(k))$$
$$y(k) = d(k) - Qq^{-L}(1+\Delta_1)(1+\Delta_2)F\psi^L\frac{1}{F}(Q\hat{P}G_c z(k) - QPG_c z(k) + d(k))$$
$$y(k) = d(k) - Qq^{-L}(1+\Delta_1)(1+\Delta_2)F\psi^L\frac{1}{F}(d(k) - Q(\Delta_1 + \Delta_1\Delta_2)z(k-L))$$

If $\| Q(\Delta_1 + \Delta_1\Delta_2) \|$ is small enough, $y(k)$ will be close to zero.

Suppose the disturbance is linear, then ψ becomes a linear operator. Plug $d(k+1) = \psi d(k)$ into above derivation, we have following results:

$(1 - (q^{-L} - PG_c)\psi^L)y(k) = (1 - q^{-L}\psi^L)d(k) = 0$ for NIMPC,

$(1 - (\hat{P}G_c - PG_c)\psi^L)y(k) = (1 - q^{-L}\psi^L - \Delta_2 q^{-L}\psi^L)d(k) = -\Delta_2 d(k)$ for PIMC.

Remark: From the above derivation, we find that the PIMC is more robust than the NIMC. For linear disturbance dynamics, the internal model principle control provides asymptotic tracking performance while the predictive internal model control has an error due to the inversion error.

6. Simulation Results

The logistic map equation is used:

$$d(k) = r * d(k-1) * (1 - d(k-1)) \quad (11)$$

This is a discrete time analog of the logistic equation for population growth (see Strogate 1994) . This nonlinear model is very sensitive to the values of r and $d(0)$. For $0 \le r \le 4$ and $0 \le d(0) \le 1$, we have $0 \le d(k) \le 1$. For other initial conditions, $d(k)$ will diverge. Let's fix $d(0) = 0.1$, if $3 \le r \le 4$, $d(k)$ becomes chaotic. Fig. 4 shows the plot of $d(k)$ for $r = 4.0$ which is used throughout the simulations. It can be verified that the bound of the signal local growth rate is M= 4.0.

The plant model used in simulations is

$$P = \hat{P}(1 + \alpha q^{-1}),$$

where the nominal model, having a nonminimum phase zero at 10.0 is

$$\hat{P} = \frac{1.0q^{-1} - 10.0q^{-2}}{1.0 - 1.8q^{-1} + 0.9q^{-2}}$$

and the unmodeled dynamics, when existing, are $\alpha = 0.005$.

Using $\hat{P}$ to solve for the inverse controller in Eq.(7), we find the minimizing H_∞ solution:

$$G_c = -0.1(0.1^{L-2} + 0.1^{L-3}q + \cdots + q^{L-2})(1 - 1.8q^{-1} + 0.9q^{-2})$$

and the minimized inversion error is $J = 0.1^{L-1}$ for $\hat{P}$.

Since $\|\Delta_1\|_\infty \le \alpha$ and $\|\Delta_2\|_\infty \le 0.1^{L-1}$, The (sufficient) stability conditions in Theorem 1 can be checked for the case of different L values. Two sets of simulations have been conducted for $L = 3$ and 4 respectively. Theorem 1 predicts stability for the cases of $L = 3$ and $L = 4$ for $\alpha = 0$ (no unmodeled dynamics) and $L = 3$ for $\alpha = 0.005$. Figures 5 and 6 show the simulation results for $L = 3$ and $L = 4$ respectively. All cases achieve certain degree of disturbance rejection. Without unmodeled dynamics, the disturbance rejection performance will become better for larger L, but it is not true when unmodeled dynamics exist.

7. Conclusions

Nonlinear internal model principle control and predictive internal model control algorithms are proposed and analyzed for rejecting exogenous signals with known nonlinear dynamics, particularly chaotic dynamics. The effects of the disturbance growth rate, system unmodeled dynamics, and nonminimum phase plant inversion error on the system stability are derived.

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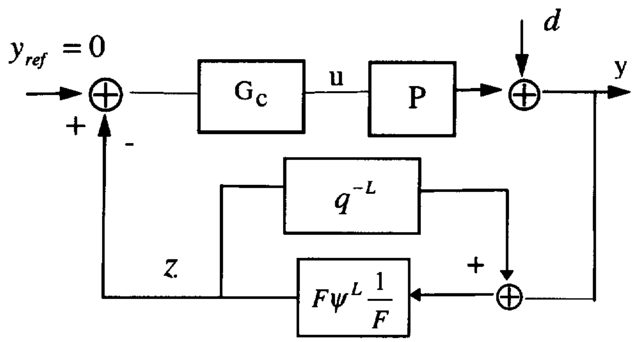

Fig. 1 Block Diagram for the Nonlinear Internal Model Principle Control

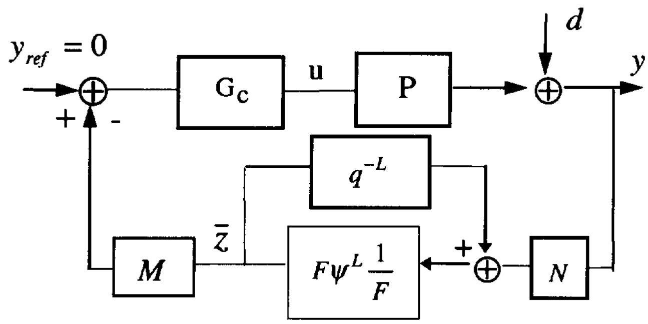

Fig. 2 Block Diagram of NIMPC for Disturbance in Vector Form

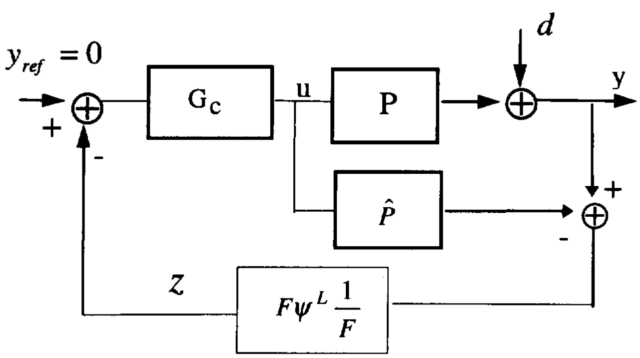

Fig. 3 Block Diagram of Predictive Internal Model Control

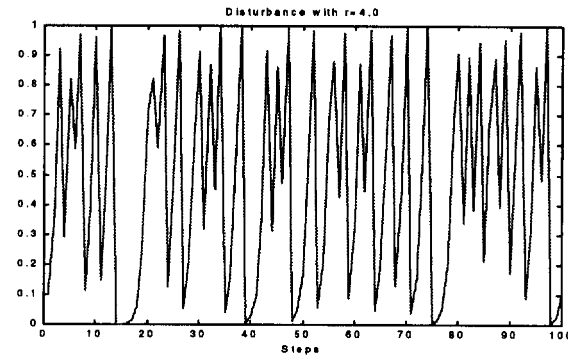

Fig. 4 Chaotic Disturbance used in Simulations.

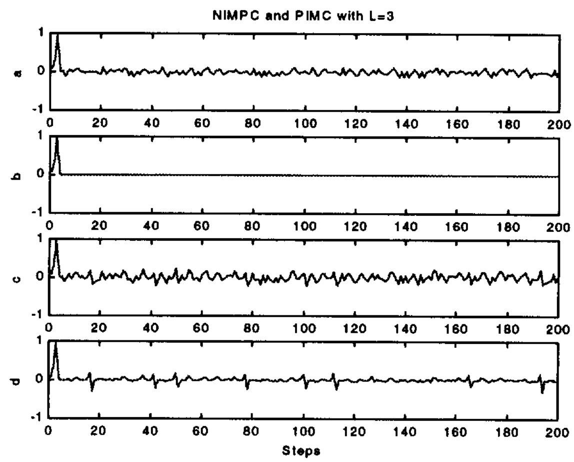

Fig. 5 Disturbance Rejection by Internal Model Principle Control and Predictive Internal Model Control without unmodeled dynamics (a,b) and with unmodeled dynamics (c,d) respectively.

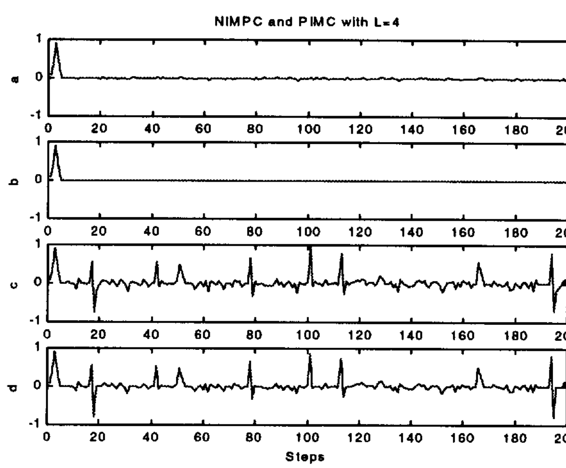

Fig. 6 Disturbance Rejection by Internal Model Principle Control and Predictive Internal Model Control without unmodeled dynamics (a,b) and with unmodeled dynamics (c,d) respectively.