2002 Japan-USA Symposium
on Flexible Automation
Hiroshima, Japan, July 14-19, 2002

AN INTEGRATED FEEDFORWARD ROBUST REPETITIVE CONTROL DESIGN FOR TRACKING NEAR PERIODIC TIME VARYING SIGNALS

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ABSTRACT

This paper addresses the tracking of near periodic time varying signals by proposing an integrated approach for simultaneously designing previewed feedforward, feedback, and repetitive control in a unified framework. The design problem is formulated as a μ-synthesis problem, in which the distance between a specified reference model and the achievable tracking performance with feedforward compensation is minimized in terms of the structure singular value. Desired upper bounds of disturbance rejection as well as tracking performance are conveniently incorporated in the proposed controller design. The reference signal is assumed to be previewed with a certain length and is near periodic. This structural knowledge about the reference signal characteristics is reflected in a μ-synthesis framework as a feedforward block and a periodic signal generator. The periodic signal generator enables the control system to track the periodic component of the reference while the previewed feedforward action compensates for the slow changes in magnitude and phase of the periodic signal. Such type of reference signals has applications in turning of non-circular profiles with tapers and twists, and camless engine valve actuators for variable valve timing operations. The proposed design is applied to an electrohydraulic actuator to generate a cam-shape profile that varies in both the angular θ-direction and the axial x-direction. Experimental results demonstrate its effectiveness.

1 INTRODUCTION

One of the most important problems in control is the tracking problem, where the system output is designed to be as close as possible to a reference signal. In many control systems, reference inputs and/or disturbances invariably include a significant periodic component with a known period. The internal model principle proposed by Francis and Wonham (1975) states that a periodic signal generator is required in the feedback loop to asymptotically track a periodic reference by the output of a closed-loop system. Various discrete-time repetitive control design techniques (Tomizuka *et al.*, 1989; Shaw and Srinivasan, 1993; Tsao and Tomizuka, 1994; Kim and Tsao, 2001) based on the internal model principle have been proposed in literature. This type of discrete-time repetitive controller is easy to design and implement, and shows a rapid convergent speed. The major interest in repetitive control is tracking (and/or rejecting) the periodic reference (and/or disturbance) signals.

Some of published research in the discrete-time domain considered both periodic and non-periodic signals. Tenney and Tomizuka (1996) used a disturbance observer to estimate and cancel the disturbances. To reduce the effects of non-periodic disturbances, they also proposed an "adaptive error saturation" algorithm in which the knowledge of the non-periodic signals' magnitude is required to determine an appropriate saturation limit. Guo (1997) used $S(z)/(S(z)+R(z))$ to replace the low-pass filter $q(z,z^{-1})$ in the "prototype" repetitive control structure proposed by Tomizuka *et al.* (1989) and then, frequency shaping of the sensitivity function by choosing $S(z)$ and $R(z)$ was utilized to reject both the repeatable and non-repeatable runout in the disk drive servo control problem. A robust repetitive control design method by using structured singular values was proposed by Li and Tsao (1998; 2001). In this method, the high order delay term in the periodic signal generator is treated as a fictitious uncertainty so that μ-synthesis technique, which is applied to a generalized plant $P(z)$ including a performance weighting function $W_p(z)$, produces a reasonably low order controller for implementation. Since the upper bound of the magnitude of the sensitivity function may be captured by $1/W_p(z)$, the selection of $W_p(z)$ provides a way to reject non-periodic disturbances.

There are many published research results on the design of linear two-degree-of-freedom controllers. A general two-degree-of-freedom control receives the commands and feedbacks separately and process them independently. Hoyle *et al.* (1991) and Limebeer *et al.* (1993) proposed a two-degree-of-freedom extension of the H_∞ loop-shaping design by McFarlane and Glover (1992) to enhance the model-matching properties of the closed-loop. Prempain and Bergeon (1998) used the Youla parametrization of two-degree-of-freedom controllers and proposed a two-step design procedure. In the first step, a model-matching approach was proposed to set the desired nominal tracking objectives, while in the second step, μ-synthesis technique was used to achieve the robust performance objectives. Shaked and de Souza (1995) proposed a game theory H_∞ approach to solve the tracking problem of a causal (measured on-line) or non-causal (known in advance) reference signal. Even though these two-degree-of-freedom control design methodologies are theoretically elegant, it is not easy to apply them to build a repetitive control system because it is not clear how to enforce them to have the internal model of the periodic reference which contains a long delay in the controller structure.

In this paper, a new integrated feedforward and robust repetitive controller structure is proposed. The proposed method is devised to deal with near periodic time varying reference signals. By near periodic we mean that the signal $r(t)$ has a *a priori* known period τ and that $|r(t)-r(t+\tau)|$ is small in some sense. The periodic signal generator provides the control system with a learning mechanism about the periodic nature of the reference, thus it enables the control system to asymptotically track periodic reference. The preview action by the feedforward block compensates for the slowly changing magnitude and phase of the otherwise periodic signal. The attempt to improve the tracking performance of repetitive control by an additional feedforward control loop can be found in Kim and Tsao (2000). The feedforward part was separately designed based on the zero phase error tracking controller (ZPETC) (Tomizuka, 1987) and used in a plug-in manner. The proposed method in this paper is different in the sense that the feedforward part is directly specified in a μ-synthesis framework and designed with feedback part at the same time.

The rest of this paper is organized as follows: Section 2 presents the proposed robust repetitive control design methodology incorporated with preview action. Section 3 shows a design example for an electrohydraulic actuator. Experimental results show the efficiency of the proposed method. Finally, conclusions are given in Section 4.

2 ROBUST REPETITIVE CONTROL WITH INTEGRATED FEEDFORWARD CONTROL

Figure 1 shows the "prototype" ZPETC-type discrete-time repetitive control system structure (Tomizuka *et al.*, 1989), where $G(z)$ is a stable plant, $K_{rep}(z)$ is a repetitive controller. $K_{rep}(z)$ is built by the zero phase error tracking algorithm to approximately inverse the plant $G(z)$, such that $z^{+L}K_{rep}(z)G(z) \approx 1$. The periodic signal generator is a positive feedback loop consisting of $q(z,z^{-1})$, z^{-N+L}, and z^{-L}, where N is the period of the periodic signal and L is the sum of the plant delay and the controller delay which comes from the inversion of unstable zero part in the plant. The period of the periodic signal is much longer than the sampling rate, thus the z^{-N+L} has a very high order in general. The internal model of repetitive control is a periodic signal generator which can introduce an infinite large feedback gain at the fundamental frequency and its harmonics. A (non-causal) zero-phase low-pass filter $q(z,z^{-1})$ is often used to improve the system stability by turning off the learning mechanism of the periodic signal generator in the high frequency range. The non-causal part of $q(z,z^{-1})$ may be absorbed in the adjacent long delay z^{-N+L}.

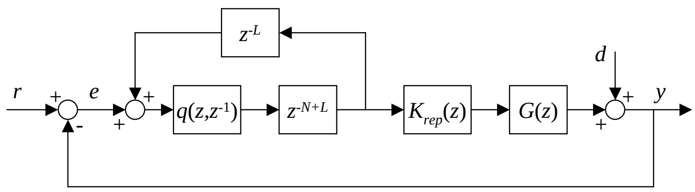

Figure 1. BLOCK DIAGRAM OF ZPETC-TYPE REPETITIVE CONTROL SYSTEM

The sensitivity function which is the closed-loop transfer function from the reference r to the error e or equivalently from the output disturbance d to the output y reveals the main characteristic of the repetitive control system. Note that the sensitivity function S_o of ZPETC-type repetitive controller structure is equal to

$$\begin{aligned} S_o &= \frac{1-q(z,z^{-1})z^{-N}}{1-q(z,z^{-1})z^{-N}+q(z,z^{-1})z^{-N+L}K_{rep}(z)G(z)} \\ &\approx 1-q(z,z^{-1})z^{-N} \\ &\quad \text{if } z^{+L}K_{rep}(z)G(z)\approx 1,\, N \gg L\,. \end{aligned} \tag{1}$$

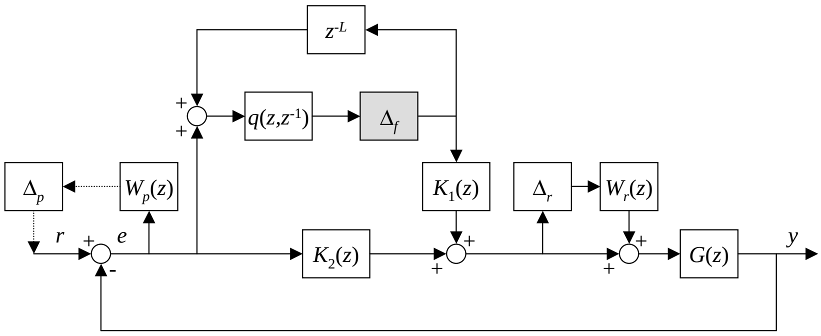

Figure 2. BLOCK DIAGRAM OF TPRRC. THE z^{-N+L} IS REPLACED BY A FICTITIOUS UNCERTAINTY Δ_f AND μ-SYNTHESIS IS APPLIED TO THE CLOSED-LOOP SYSTEM TO DESIGN $[K_1(z), K_2(z)]$.

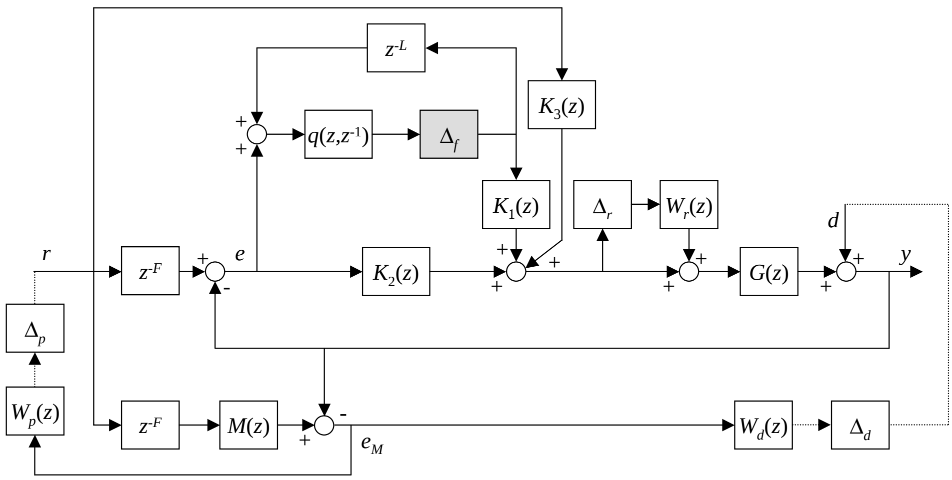

Figure 3. BLOCK DIAGRAM OF THE PROPOSED METHOD. THE CONTROLLER BLOCK $[K_1(z), K_2(z), K_3(z)]$ USES THE REFERENCE OF F-STEPS AHEAD AS WELL AS THE FEEDBACK ERROR SIGNAL.

Let T be the sampling time and substitute $z = e^{j\omega T}$, then it is easy to see from Eq. (1) that as long as the low-pass filter $q(z, z^{-1})$ is close to unity, the magnitude of sensitivity function S_o is close to zero at integer multiples of $\frac{1}{NT}$ (Hz) that is called the fundamental frequency. The magnitude may be as large as 2 between these frequencies. If non-periodic disturbances whose major frequency components are quite different from the fundamental frequency and its integer multiples, come into the closed-loop system equipped with the ZPETC-type repetitive control, the disturbances will be amplified rather than rejected because the magnitude of sensitivity function is greater than one in the frequency range where the disturbances appear.

μ-synthesis is a powerful control design technique for a system subjected to structured, linear fractional transformation (LFT) perturbations. While designing a controller via μ-synthesis, it is easy to enforce the robustness (both stability and performance) of the system by providing appropriate weighting functions. It is a well known fact that modern control design methods such as H_∞ synthesis and μ-synthesis produce controllers of order at least equal to the plant, and usually higher because of the inclusion of weighting functions. With regards to computational complexity and practical implementation, the order of z^{-N+L} in the periodic signal generator is too high to be directly included in a discrete-time μ-synthesis formulation. Li and Tsao (1998; 2001) introduced a fictitious uncertainty Δ_f replacing z^{-N+L} and applied μ-synthesis technique to design a robust repetitive controller. Let's call this scheme two parameter robust repetitive control (TPRRC). The control design structure is shown in Figure 2, where $W_r(z)$ is an input multiplicative uncertainty and $W_p(z)$ is a performance weighting function. $K_1(z)$

and $K_2(z)$ are controller blocks to be designed via discrete-time μ-synthesis. $K_1(z)$ is supposed to stabilize the positive feedback loop and $K_2(z)$ is supposed to improve the robust performance of the closed-loop system. Since the $1/W_p(z)$ describes the desired upper bound of the sensitivity function in this structure, non-periodic disturbances also can be rejected by choosing an appropriate performance weighting function.

To further enhance the tracking performance especially when the reference slowly changes in its magnitude and phase, we propose a new repetitive control design structure shown in Figure 3. We also replace z^{-N+L} with Δ_f and apply the discrete-time μ-synthesis to this closed-loop system. In tracking control, it is often desirable to incorporate preview action to compensate for the dynamic delay of the plant. This means that a finite number of future desirable output is available and that the feedforward controller needs not be causal. Applying the model matching approach for previewed feedforward action (Tsao, 1994), the feedforward block is represented as $K_3(z)$ in the closed-loop system. The number of "look ahead" signal corresponds to the length of delay z^{-F} i.e. the reference of F-steps ahead enters into $K_3(z)$, equivalently the feedback controller block uses the F-steps delayed reference signal with respect to $K_3(z)$. $M(z)$ is the desired closed-loop transfer function selected by the designer to introduce time-domain specifications into the design process. $M(z)$ is often called the reference model and a zero-phase low-pass filter with unity gain may be a good candidate for it in the tracking problem. When a high bandwidth low-pass filter is chosen for $M(z)$, it is observed that the final result is similar to the case that e is directly used instead of e_M for the input signal to $W_d(z)$ and $W_p(z)$. Notice that one may choose non-causal $M(z)$, as long as $z^{-F}M(z)$ is causal.

In TPRRC, the inverse of $W_p(z)$ represents not only a tracking performance but also a disturbance rejection bound because disturbances are equivalent to references for a one-degree-of-freedom controller which is driven (for example) by an error signal e. In Figure 3, the feedforward part $K_3(z)$ can not see the existence of the disturbance d and it tends to take upon itself the tracking task specified by $1/W_p(z)$ regardless of disturbance rejection. Therefore, another weighting function $W_d(z)$ is used to notify the feedback part $[K_1(z), K_2(z)]$ of the desired disturbance rejection bound. The inverse of $W_d(z)$ describes the upper bound of disturbance rejection transfer function $(d \to e)$.

When all the uncertain perturbations are pulled out into a block-diagonal matrix, the final augmented block structure of the perturbations $\hat{\Delta}$ is

$$\hat{\Delta} = \begin{bmatrix} \Delta_r & 0 & 0 & 0 \\ 0 & \Delta_f & 0 & 0 \\ 0 & 0 & \Delta_p & 0 \\ 0 & 0 & 0 & \Delta_d \end{bmatrix}. \quad (2)$$

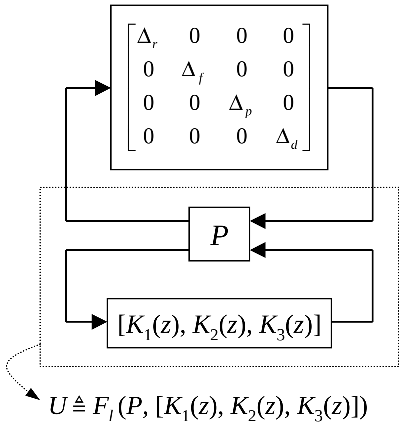

Figure 4. TRANSFORMING TO GENERAL LFT FORM

If we also pull out the controller $K \triangleq [K_1(z), K_2(z), K_3(z)]$, we get the generalized plant P, as shown in Figure 4. Assume that the nominal stability is achieved such that $U = F_l(P, K) \triangleq P_{11} + P_{12}K(I - P_{22}K)^{-1}P_{21}$ is (internally) stable, then robust performance is obtained if the following is satisfied (Zhou *et al.*, 1996):

$$\mu_{\hat{\Delta}}(U) < 1. \quad (3)$$

Due to the introduction of the fictitious uncertainty, $\mu_{\hat{\Delta}}(U)$ value at Eq. (3) contains some amount of conservatism. It is allowable to have $\mu_{\hat{\Delta}}(U)$ value greater than one during D-K iteration as long as robust stability test with the real delay term z^{-N+L} shows $\mu_{\Delta_r}(U_{11}) < 1$ when inserting the controller obtained from the μ-synthesis, where U_{11} is the transfer function from the output to the input of the perturbation Δ_r.

3 DESIGN AND IMPLEMENTATION ON AN ELECTRO-HYDRAULIC ACTUATOR

In this section, we present a design example for an electrohydraulic system used in the noncircular turning process. Figure 5 shows a "two-dimensional" cam-shape profile used in the experiment. This profile could correspond to variable cam timing or lift in contrast to the conventional "one-dimensional" profile, in which the cross section is fixed. θ- and x-directions are defined as shown in the figure. Assume that we want to machine this twisted cam-shape by direct turning process with a constant spindle speed and feed rate. The tool motion should follow the reference given by $r(\theta, x)$, which is why the profile is called two-dimensional. Notice in the figure that the cam profile remains fixed from $x = 0$ to 20 mm (i.e. periodic), then its magnitude and phase change in the range $x = 20 \sim 36$ mm, and finally there is only phase change from $x = 36$ to 60 mm. The cross sectional

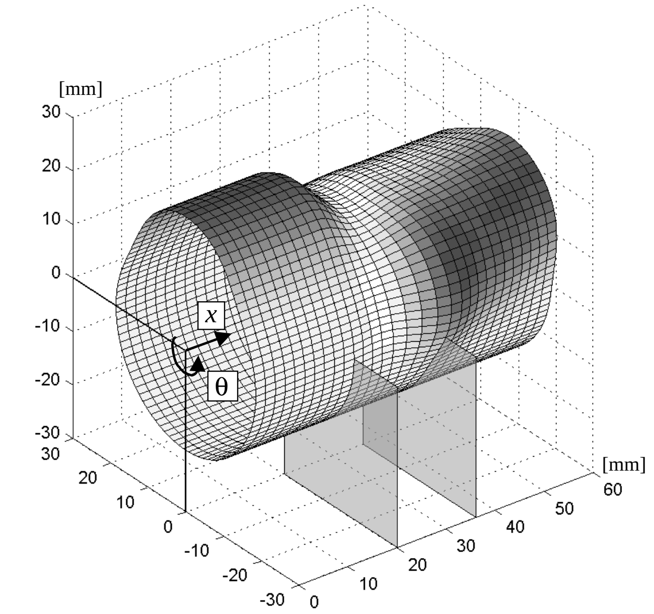

Figure 5. TWO-DIMENSIONAL CAM-SHAPE PROFILE

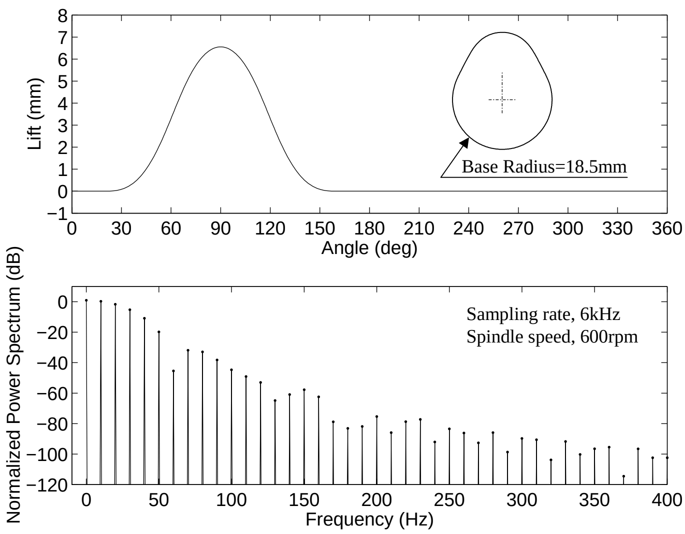

Figure 6. CROSS SECTIONAL VIEW AND NORMALIZED POWER SPECTRUM OF THE TWO-DIMENSIONAL CAM-SHAPE PROFILE AT $x = 0$

view and its normalized power spectrum of the two-dimensional cam-shape profile at $x = 0$ mm are shown in Figure 6. The spectrum appears only at the fundamental frequency and its harmonics because the cam profile is periodic when the spindle rotates at a constant speed. The cross sections at $x = 36$ and 60 mm are rotated 60 and 36 degrees, respectively, clockwise from the cross

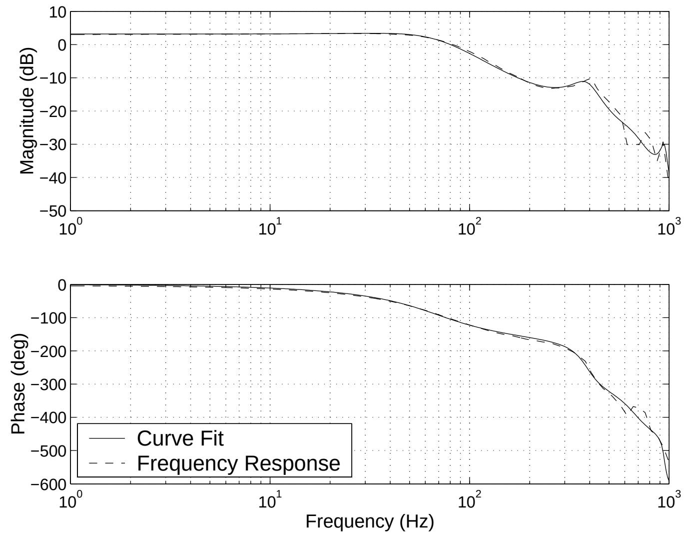

Figure 7. FREQUENCY RESPONSE OF THE ELECTROHYDRAULIC ACTUATOR AND ITS 8TH ORDER MODEL

section at $x = 0$ mm. In the experiment, the real-time reference signal was generated by traversing this cam profile with a spindle speed of 600 rpm and a feed rate of 0.2 mm/sec. The sampling rate of the digital control was 6 kHz.

The Bode plots of the electrohydraulic actuator are shown in Figure 7. The actuator system has been stabilized by an analog proportional feedback loop and an eighth order nominal plant model is obtained from the frequency response by least squares fit. The actuator motion is measured by a laser encoder having a 0.63 μm resolution. The magnitude Bode plots of $G(z)$, $M(z)$, $q(z,z^{-1})$, $1/W_p(z)$, $1/W_d(z)$, and $W_r(z)$ used in this design example are shown in Figure 8. N was 600 according to the spindle speed and the control sampling rate. L was chosen to be 10. The form of the reference model $M(z)$ is selected to be $M(z) = (0.25z + 0.5 + 0.25z^{-1})^n$. It is a zero phase low-pass filter with all the poles at $z = 0$ and all the zeros at $z = -1$. The bandwidth reduces as n increases. We set n to be 2 and use the same zero phase low-pass filter for $M(z)$ and $q(z,z^{-1})$. Since we use a non-causal $M(z)$, the preview length F should be no less than 2 for this particular $M(z)$, so that $z^{-F}M(z)$ is causal.

Several sets of controllers $[K_1(z), K_2(z), K_3(z)]$ are designed with different preview length (F values). The frequency responses of the tracking error transfer functions ($r \to e$) are compared for $F = 2,4,6,8,10$ in Figure 9. We can see deep notches whose ends are marked at the fundamental frequency and its harmonics. The notches are a distinctive characteristic of repetitive control providing high gain at these frequencies. It clearly shows that tracking error reduces as the preview length F increases. While increasing F theoretically and intuitively continues to reduce tracking errors, experimental improvements were almost same beyond $F = 8$ due to the effects of unmodeled dynamics

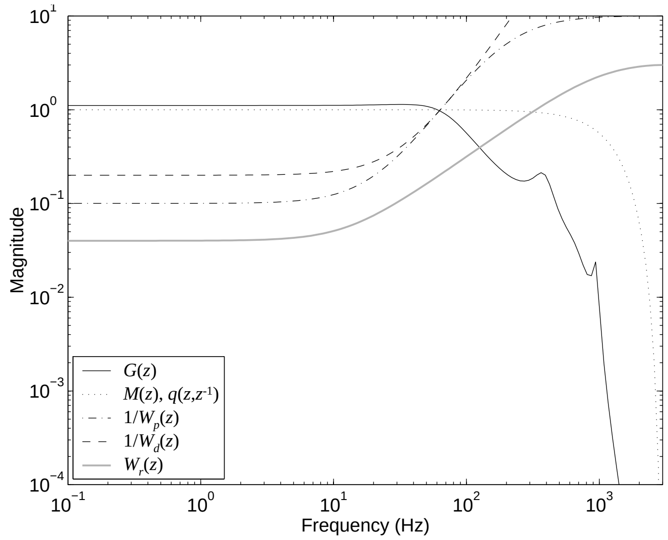

Figure 8. MAGNITUDE BODE PLOTS OF DESIGN PARAMETERS

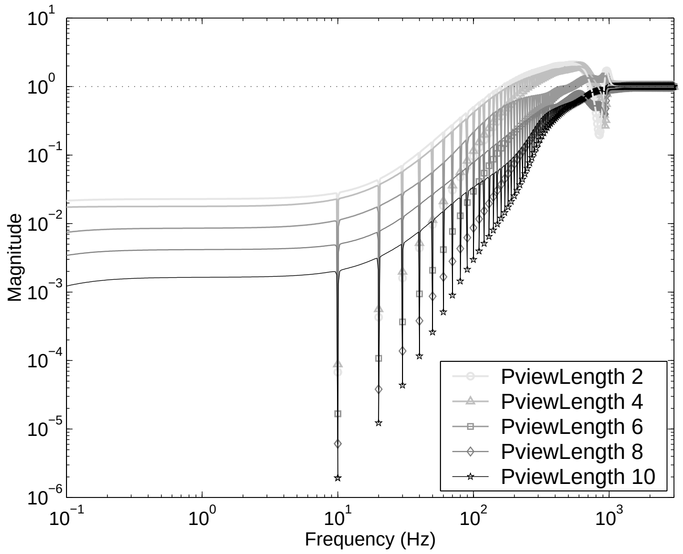

Figure 9. THE EFFECT OF PREVIEW LENGTH $F = 2, 4, 6, 8, 10$. FREQUENCY RESPONSES OF TRACKING ERROR TRANSFER FUNCTIONS SHOW THAT TRACKING ERROR REDUCES AS THE PREVIEW LENGTH INCREASES.

and noise. Notice that all the tracking error sensitivity functions are within the upper bound $1/|W_p(z)|$. Also,the disturbance rejection transfer functions ($d \rightarrow e$) are all bounded by $1/|W_d(z)|$ although the corresponding plots are not presented here.

A repetitive control system using the TPRRC structure with the same design parameters described above was also designed for comparison with the proposed controller. Figure 10 shows the resulting tracking error transfer function. By comparing

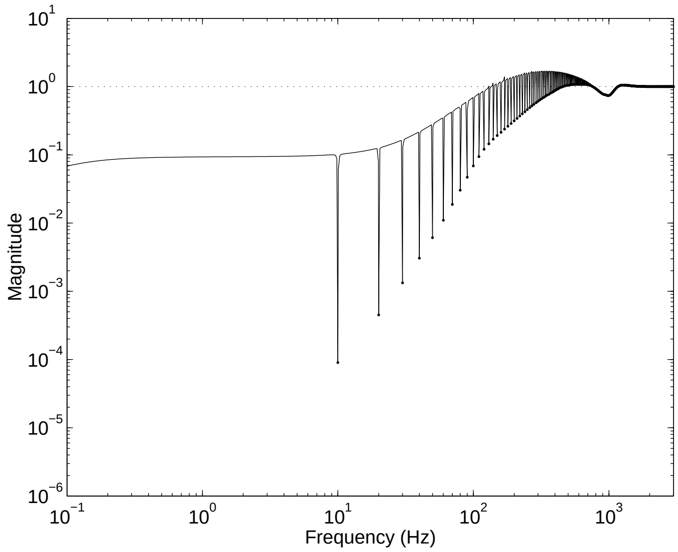

Figure 10. FREQUENCY RESPONSE OF TRACKING ERROR TRANSFER FUNCTION IN TPRRC CASE

Figure 9 and Figure 10, we can see that the proposed method with a preview action gives better tracking performance than TPRRC. This observation is verified in the following experiment. The controller designed by the proposed method with a preview length of 8 showed $\mu_{\hat{\Delta}}(U) = 0.983$ and was initially of 65th order while μ-synthesis. The controller order was reduced to 9th, 8th, and 13th for $K_1(z)$, $K_2(z)$, and $K_3(z)$, respectively, for real-time implementation. Likewise the 45th order controller from TPRRC which showed $\mu_{\hat{\Delta}}(U) = 0.980$ was reduced to a 9th and 8th order for $K_1(z)$ and $K_2(z)$, respectively. They were implemented by a 32 bits floating point digital signal processor (TMS320C32).

Finite wordlength (FWL) truncation error is another important factor in the fast sampling rate digital controller implementation. The 32 bits floating point DSP provides single precision floating point number representation, which has a finite precision of 7 significant decimal digits and a finite range of 10^{-38} to 10^{+38}. The original high order controllers not only consume much computational time but also are vulnerable to FWL errors. Controller order reduction is employed to overcome this situation. The output from $K_3(z)$ can be calculated off-line because it does not use feedback signal. The controller order reduction and numerical truncation effect for $K_3(z)$ was checked by comparing the off-line double precision computation for the initial 65th order $K_3(z)$ with the on-line single precision computation of the reduced order controller. The experimental tracking performance for these two cases was almost the same. This justifies our controller reduction approach.

Using the reference signal generated from the two-dimensional cam-shape profile in Figure 5, the experimental results of the proposed control system are shown in Figure 11 in

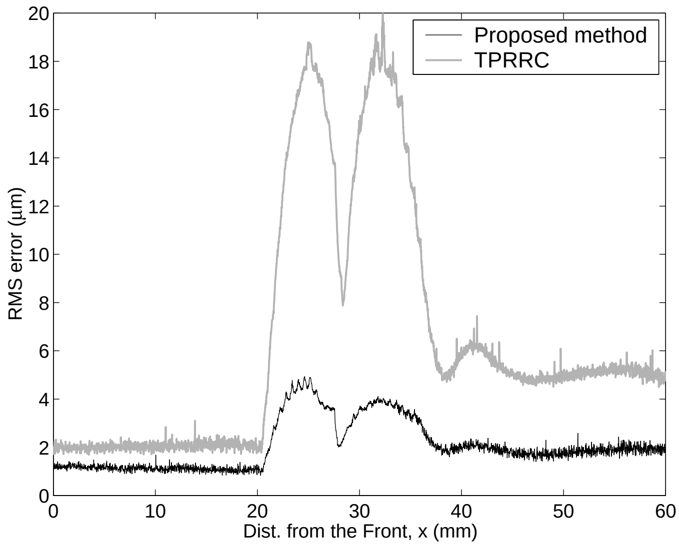

Figure 11. COMPARISON OF RMS ERRORS

terms of RMS errors calculated per spindle revolution. A few leading seconds were given to the actuator system until it reached its steady-state at $x = 0$. Due to the hardware limitations, the whole two-dimensional reference could not be imported into the C32 DSP. The cross sections of Figure 5 were specified at every 0.6 mm from the front and linear interpolation was used to generate the real-time reference for the controller. The effect of the linear interpolation appeared as ripples in the RMS error curve distinctively in the range of $x = 20 \sim 36$ mm. Figure 12 shows the tracking performance at $x = 10, 25, 50$ mm of the two control design methods. The abscissa of each plot represents time in seconds. Note that at $x = 10$ mm the reference is purely periodic, at $x = 25$ mm the magnitude and phase change, and at $x = 50$ mm only the phase changes. As shown by the figure, the proposed method has better tracking performance than TPRRC. The proposed method renders substantial improvement in tracking performance especially when the reference changes and becomes near periodic.

4 CONCLUSIONS

We proposed an integrated previewed feedforward and robust repetitive control design for tracking near periodic reference signals. The entire control design problem has been formulated in the LFT form and solved by μ-synthesis synthesis, In this way, the desired upper bounds of disturbance rejection and tracking error sensitivity function, and the repetitive controller internal model, are explicitly included in the design process. Controller order reduction has been proven essential for the real-time implementation in order to reduce computation time and finite word length truncation error. Experimental results on an electrohydraulic actuator have demonstrated the effectiveness of the design method and implementation technique.

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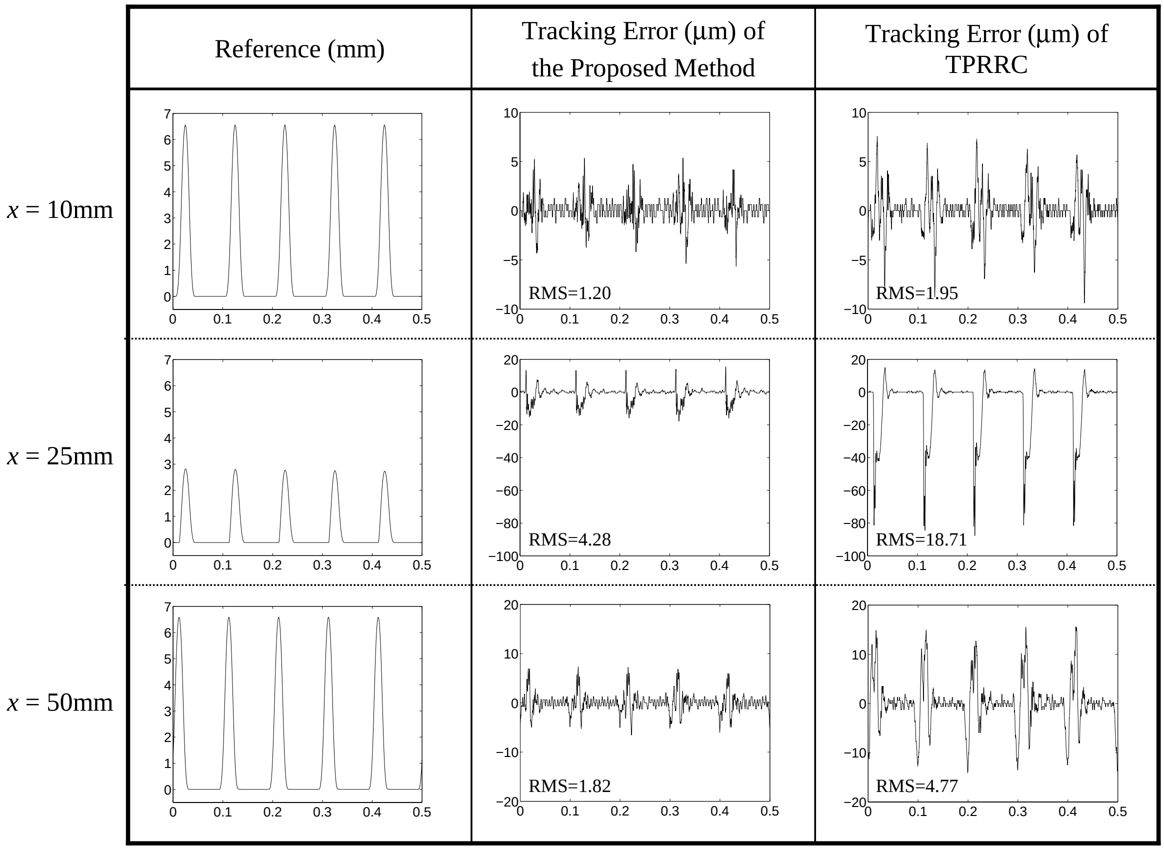

Figure 12. COMPARISONS OF TRACKING PERFORMANCE AT $x = 10, 25, 50$ mm. THE ABSCISSA OF EACH PLOT REPRESENTS TIME IN SECONDS.

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