Adaptive Zero Phase Error Tracking Algorithm for Digital Control

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This paper describes an adaptive feedforward controller to let the output of a plant with stable and unstable zeros track a time varying desired output. The dynamics of the closed loop system consisting of the plant and the feedback controller are assumed unknown or slowly varying due to changes on the plant parameters. In the control scheme proposed in this paper, the feedforward controller is adaptive while the feedback controller is fixed under the assumption that the closed loop system remains stable at all times. With a few samples of future reference input data available, the preview action of the adaptive feedforward controller cancels the phase lag caused by the closed loop dynamics and attains the zero phase error tracking performance (i.e., the plant output is in phase with any sinusoidal desired output) asymptotically.

1 Introduction

The design of adaptive controllers for nonminimum phase systems is not straightforward because the stability consideration prohibits direct applications of self-tuning and model reference adaptive control algorithms which cause pole-zero cancellations. Clarke (1984) conducted an extensive review and simulation test on existing adaptive control methods for nonminimum phase systems and concluded that with appropriate modifications and by using as much prior information of the process as possible adaptive control is viable even for nonminimum phase systems. However, while avoiding cancellations of unstable zeros, the tracking quality is usually sacrificed in most of the modified algorithms because the unstable zeros remain in the reference model transfer function from the desired output to the plant output and they cause significant phase errors over a broad range of frequencies.

This paper describes an adaptive feedforward controller to let the output of a plant with stable and unstable zeros track a time varying desired output without any phase error. The dynamics of the closed loop system consisting of the plant and feedback controller are assumed unknown or slowly varying due to parameter changes in the plant. It is further assumed that the closed loop system with a nonadaptive feedback controller remains stable at all times. For achieving a good tracking performance, the feedforward controller is made adaptive. This assumption certainly needs some prior knowledge of the plant to design a robust feedback controller.

The adaptive feedforward controller is based on the zero phase error tracking controller proposed by Tomizuka (1985) which compensates the phase error caused by the dynamics of uncancellable zeros. A normalized least squares algorithm is used for identifying plant parameters. The identified parameters are used for calculating the parameters of the feedforward controller and successively the feedforward control inputs. Therefore, the proposed adaptive algorithm is an indirect adaptive control scheme.

The remainder of this paper is organized as follows. Section 2 describes the zero phase error algorithm and the parameter adaptation algorithm (P.A.A.). Section 3 analyzes the system stability and the convergence of the plant output to the reference signal in the deterministic sense. Section 4 presents the simulation results and verifies the validity of the algorithm. Conclusion and future works are given in Section 5.

2 Feedforward Controller for Tracking Time Varying Signal

2.1 Zero Phase Error Tracking Controller. Figure 1 depicts the overall structure of the tracking control scheme proposed by Tomizuka (1985). The closed loop system, which
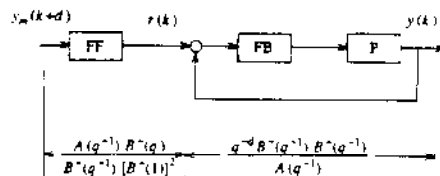
P: Plant.

FB: Feedback Controller.

FF: Feedforward Controller

Fig. 1 Zero phase error tracking system

consists of a plant and a feedback controller, is asymptotically stable and is described by

$$\frac{y(k)}{r(k)} = \frac{q^{-d}B(q^{-1})}{A(q^{-1})} = \frac{q^{-d}B^{-}(q^{-1})B^{+}(q^{-1})}{A(q^{-1})} \qquad (2.1)$$

where q^{-1} is a one step delay operator, $r(k)$ and $y(k)$ are the reference input and the plant output, respectively, $A(q^{-1}) = 1 + a_1 q^{-1} + \ldots + a_n q^{-n}$, $B(q^{-1}) = b_0 + b_1 q^{-1} + \ldots + b_m q^{-m}$, $B(1) \neq 0$, and $B(q^{-1})$ is factorized into $B^{-}(q^{-1})$ and $B^{+}(q^{-1})$, which will be explained below. The condition $B(1) \neq 0$ implies that the closed loop system has a nonzero static gain and is true in almost all the control problems. The zero phase error feedforward controller (ZPETC) is

$$r(k) = \frac{A(q^{-1})B^{-}(q)}{B^{+}(q^{-1})[B^{-}(1)]^2} y_m(k+d) \qquad (2.2)$$

where $y_m(k+d)$ is a d-step advanced bounded desired output signal, and $B^{-}(q)$ is obtained by replacing q^{-1} in $B^{-}(q^{-1})$ by q. Notice that ZEPTC cancels all the closed loop poles and zeros contained in $B^{+}(q^{-1})$. Therefore, $B^{-}(q^{-1})$ defined in equation (2.1) must include all the plant zeros outside and on the unit circle of the complex plane and some of those inside the unit circle.

From equations (2.1) and (2.2), the overall transfer function from the desired output y_m to the plant output y becomes

$$\frac{y(k)}{y_m(k)} = \frac{B^{-}(q^{-1})B^{-}(q)}{[B^{-}(1)]^2} \qquad (2.3)$$

It can be easily verified that the frequency response of this transfer function has a zero phase shift for all frequencies and a unity gain at zero frequency (Tomizuka, 1985). The frequency response gain remains close to 1 in a low frequency range. Therefore, if y_m is smooth, i.e., no strong high frequency components, the actual output stays close to the desired output. Specifically, if y_m a signal is of constant level or constant rate of change, $y = y_m$ is achieved.

2.2 Adaptive Zero Phase Error Tracking Controller.

When the plant parameters are unknown, the feedforward controller (2.3) cannot be directly implementable and is made adaptive. To consider a general case, the closed loop system is separated into a known part and an unknown part. The numerator of the known part is further divided into cancellable part and uncancellable part, i.e.,

$$\frac{y(k)}{r(k)} = \frac{q^{-d}B(q^{-1})B_0^{-}(q^{-1})B_0^{+}(q^{-1})}{A(q^{-1})A_0(q^{-1})} \qquad (2.4)$$

where $A(q^{-1}) = 1 + a_1 q^{-1} + \ldots + a_{n_1}q^{-n_1}$ and $A_0(q^{-1})$ is the denominator of the unknown part and known part respectively. $B(q^{-1}) = b_0 + b_1 q^{-1} + \ldots + b_{m_1}q^{-m_1}$ is the numerator of the unknown part, and $B_0^{-}(q^{-1})$ ($B_0^{-}(1) \neq 0$) and $B_0^{+}(q^{-1})$ are the uncancellable and the cancellable portions of the numerator of the known part.

The separation normally applies since the feedback controller is known and the plant dynamics are often partially known. Only the unknown part of the system is made adaptive in the feedforward controller. Referring to the block diagram in Fig. 2, the reference input $r(k)$ to the closed part is determined by

$$r(k) = \frac{A_0(q^{-1})B_0^{-}(q)}{B_0^{+}(q^{-1})[B_0^{-}(1)]^2} u_m(k),$$

$$u_m(k) = \frac{\hat{A}(q^{-1},k)\hat{B}(q,k)}{[\hat{B}(1,k)]^2} y_m(k+d) \qquad (2.5)$$

$$\left(\text{if } \hat{B}(1,k) = 0, \quad u_m(k) = \frac{\hat{A}(q^{-1},k-1)\hat{B}(q,k-1)}{[\hat{B}(1,k-1)]^2} y_m(k+d) \right)$$

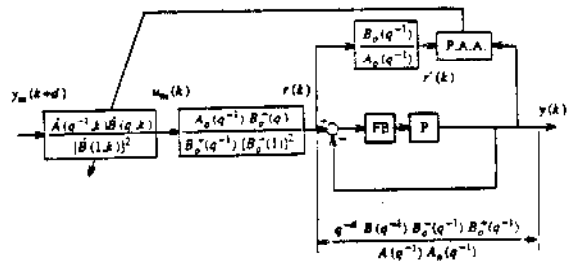where $\hat{A}(q^{-1},k)$ and $\hat{B}(q^{-1},k)$ are given by the parameter

Fig. 2 Adaptive zero phase error tracking system

adaptation algorithm (P.A.A.) described below. Referring to Fig. 2, the reference model for the P.A.A. is,

$$y(k+d) = -A^{*}(q^{-1})y(k+d-1) + B(q^{-1})r'(k) \qquad (2.6)$$

where $A^{*}(q^{-1}) = a_1 + a_2 q^{-1} + \ldots + a_{n_1}q^{-n_1-1}$ and

$$r'(k) = \frac{B_0(q^{-1})}{A_0(q^{-1})} r(k),$$

$(B_0(q^{-1}) = B_0^{-}(q^{-1})B_0^{-}(q^{-1}))$. Equation (2.6) can also be represented as

$$y(k+d) = \theta^T \phi(k) \qquad (2.7)$$

where $\theta^T = (a_1, \ldots, a_{n_1}, b_0, \ldots, b_{m_1})$ and $\phi^T(k) = (-y(k+d-1), \ldots, -y(k+d-n_1), r'(k) \ldots, r'(k-m_1))$.

Let $n(k)$, $\phi(k)$ and $y(k)$ be respectively defined by $n(k) = \max(\|\phi(k)\|, 1)$,

$$\phi(k) = \frac{\phi(k)}{n(k)} \quad \text{and} \quad y(k) = \frac{y(k)}{n(k)} .$$

Then, the normalized least squares parameter adaptation algorithm for adjusting the parameters in the feedforward controller is

$$\hat{\theta}(k) = \hat{\theta}(k-1) + \frac{F(k-1)\phi(k-d)e^0(k)}{1 + \phi^T(k-d)F(k-1)\phi(k-d)} \qquad (2.8)$$

where $e^0(k) = y(k) - \hat{\theta}^T(k-1)\phi(k-d) = -\tilde{\theta}^T(k-1)\phi(k-d)$ ($\tilde{\theta}(k) = \hat{\theta}(k) - \theta$) is the *a priori* adaptation error. The adaptation gain $F(k)$ is adjusted by

$$F^{-1}(k) = \lambda(k)[F^{-1}(k-1) + \phi(k-d)\phi^T(k-d)],$$

$$0 < \lambda(k) \leq 1, \quad F(0) = F^T(0) > 0$$

or

$$F(k) = \frac{1}{\lambda(k)}\left[F(k-1) - \frac{F(k-1)\phi(k-d)\phi^T(k-d)F(k-1)}{1 + \phi^T(k-d)F(k-1)\phi(k-d)} \right] \qquad (2.9)$$

where $\lambda(k)$ is the forgetting factor.

3 Stability and Convergence Analysis

In the nonadaptive case, the tracking performance of ZPETC is specified by (2.3). In this section, the overall system with the adaptive ZPETC is shown to be bounded-input, bounded-output stable and its asymptotic convergence properties are obtained to understand the tracking performance.

The P.A.A. stated in Section 2 has the following properties: if $\lambda(k)$ is selected such that $0 < m \leq \lambda(k) \leq 1$, and $F(k) < M_1$, ($k > 0$), then

(i) $\lim_{k\to\infty} e^0(k+d) = \lim_{k\to\infty} -\tilde{\theta}^T(k+d-1)\phi(k) = 0 \qquad (3.1)$

(ii) $\lim_{k\to\infty} \hat{\theta}(k) = \theta_\infty \qquad (3.2)$

(iii) $\|\hat{\theta}(k)\| \leq M_2 < \infty \qquad (3.3)$

The assumption on $\lambda(k)$ can be satisfied by setting $\lambda(k) = 1$ or by letting the trace of $F(k)$ constant. The latter is accomplished by setting

$$\lambda(k) = 1 - \frac{1}{trF(0)} \frac{\phi^T(k-d)F^2(k-1)\phi(k-d)}{1+\phi^T(k-d)F(k-1)\phi(k-d)} \quad (3.4)$$

For $\lambda(k) = 1$, the adaptation gain is time decreasing. The adaptation gain with a constant trace is preferred for time varying closed loop dynamics. The proof for the above results is only a slight modification of Lozano and Goodwin (1985) and is omitted here.

Based on (3.1), (3.2) and (3.3) the asymptotic relation of y and y_m can be obtained as follows. First, consider an auxiliary signal y_m^* which is defined by

$$\hat{A}(q^{-1},k)y_m^*(k+d) = \hat{B}(q^{-1},k)r'(k) \quad (3.5)$$

or

$$y_m^*(k+d) = -\hat{A}^*(q^{-1},k)y_m^*(k+d-1) \\ + \hat{B}(q^{-1},k)r'(k) \quad (3.6)$$

Subtracting (3.6) from (2.6), we obtain after some manipulations:

$$\hat{A}(q^{-1},k)(y(k+d)-y_m^*(k+d)) = -\tilde{\theta}^T(k)\phi(k) \\ (\tilde{\theta}(k) = \hat{\theta}(k)-\theta) \quad (3.7)$$

Comparing (3.7) with (3.1),

$$-\tilde{\theta}^T(k)\phi(k) = [-\tilde{\theta}^T(k+d-1)\phi(k) + (\hat{\theta}(k+d-1) \\ -\hat{\theta}(k))^T\phi(k)]n(k) \quad (3.8)$$

If $n(k)$ is bounded (i.e., $\|\phi(k)\| \le M_3 < \infty$), then from (3.1), (3.2) and (3.8), it follows that

$$\lim_{k\to\infty} -\tilde{\theta}^T(k)\phi(k) = \lim_{k\to\infty} \hat{A}(q^{-1},k)(y(k+d) \\ -y_m^*(k+d)) = 0 \quad (3.9)$$

To show that $n(k)$ is bounded, it suffices to assure that $r'(k)$ is bounded since $A(q^{-1})$ has all the roots inside the unit circle. Note from (2.5) and (2.6) (see Fig. 2 also) that $r'(k)$ is nothing but the moving average of $y_m(k+d)$ except for the stable regressive parts $B_0^*(q^{-1})$ and $A_0(q^{-1})$. Assuming that $y_m(k+d)$ is bounded and $[\hat{B}(1,k)]^2 > \delta > 0$, we conclude that $r'(k)$ is bounded in view of (3.3). Therefore, the system is bounded input (y_m) bounded output (y) stable and the adaptive control algorithm satisfies (3.9).

The asymptotic relation between y and y_m can now be considered. First it should be noted that the following two time varying operations have to be distinguished (Goodwin and Sin, 1984): i.e.,

$$\hat{A}(q^{-1},k)\hat{B}(q^{-1},k)w(k) \equiv \sum_i \sum_j \hat{a}_i(k)\hat{b}_j(k)w(k-i-j)$$

$$= \hat{B}(q^{-1},k)\hat{A}(q^{-1},k)w(k) \quad (3.10)$$

$$\hat{A}(q^{-1},k)\cdot\hat{B}(q^{-1},k)w(k) \equiv \sum_i \sum_j \hat{a}_i(k)\hat{b}_j(k-i)w(k-i-j)$$

$$\ne \hat{B}(q^{-1},k)\cdot\hat{A}(q^{-1},k)w(k) \quad (3.11)$$

However, because all the signals (y_m, u_m, r, r', y) are bounded and by the property (3.2), it is clear that (3.10) and (3.11) are asymptotically indistinguishable: i.e., $\hat{A}(q^{-1},k)\hat{B}(q^{-1},k)w(k) = \hat{B}(q^{-1},k)\hat{A}(q^{-1},k)w(k) = \hat{A}(q^{-1},k)\cdot\hat{B}(q^{-1},k)w(k) = \hat{B}(q^{-1},k)\cdot\hat{A}(q^{-1},k)w(k)$ as $k\to\infty$. Therefore, the following applies:

$$\lim_{k\to\infty} \hat{A}(q^{-1},k)y_m^*(k+d) = \lim_{k\to\infty} \hat{B}(q^{-1},k)r'(k)$$

$$= \lim_{k\to\infty} \hat{B}(q^{-1},k)\cdot\left[\frac{B_0^-(q^{-1})B_0^-(q)}{[B_0^-(1)]^2}\right.$$

$$\left.\cdot\left(\frac{\hat{A}(q^{-1},k)\hat{B}(q,k)}{[\hat{B}(1,k)]^2}y_m(k+d)\right)\right] = \lim_{k\to\infty} \hat{A}(q^{-1},k)$$

$$\cdot\left[\frac{\hat{B}(q^{-1},k)\hat{B}(q,k)}{[\hat{B}(1,k)]^2}\frac{B_0^-(q^{-1})B_0^-(q)}{[B_0^-(1)]^2}y_m(k+d)\right] \quad (3.12)$$

Then, from equations (3.2), (3.9) and (3.12) the asymptotic relation between y and y_m is:

$$\lim_{k\to\infty} A_\infty(q^{-1})(y(k+d)-y_m'(k+d))=0 \quad (3.13)$$

where

$$y_m'(k+d)$$

$$= \frac{B_\infty(q^{-1})B_\infty(q)}{[B_\infty(1)]^2}\frac{B_0^-(q^{-1})B_0^-(q)}{[B_0^-(1)]^2}y_m(k+d) \quad (3.14)$$

Notice that equations (3.14) implies that y_m' is close to y_m in the sense of the zero phase error whether or not the estimated polynomial $\hat{B}(q^{-1}, k)$ converges to $B(q^{-1})$.

Let $y_n(k)$ denote the plant output under a tuned ZPET controller: i.e., the transfer function from $y_m(k)$ to $y_n(k)$ is

$$\frac{y_n(k+d)}{y_m(k+d)} = \frac{B(q^{-1})B(q)}{[B(1)]^2}\frac{B_0^-(q^{-1})B_0^-(q)}{[B_0^-(1)]^2} \quad (3.15)$$

It follows from equations (3.13) and (3.15) that $y_n(k)$ and $y(k)$ under the adaptive ZPET controller satisfy

$$\lim_{k\to\infty} \hat{A}(q^{-1},k)(y_n(k+d)-y(k+d))$$

$$= \lim_{k\to\infty} A_\infty(q^{-1})\left(\frac{B(q^{-1})B(q)}{[B(1)]^2}\right.$$

$$\left.-\frac{B_\infty(q^{-1})B_\infty(q)}{[B(1)]^2}\right)\frac{B_0^-(q^{-1})B_0^-(q)}{[B_0^-(1)]^2}y_m(k+d) \quad (3.16)$$

Notice that

$$\left(\frac{B(q^{-1})B(q)}{[B(1)]^2}-\frac{B_\infty(q^{-1})B_\infty(q)}{[B(1)]^2}\right)\frac{B_0^-(q^{-1})B_0^-(q)}{[B_0^-(1)]^2}y_m(k+d)$$

is close to zero at low frequencies whether or not $B_\infty(q^{-1})$ equals to $B(q^{-1})$.

In general, as long as A_∞ does not have roots on the unit circle, equations (3.13) implies

$$\lim_{k\to\infty} [y(k+d)-y_m'(k+d)] = 0, \quad (3.17)$$

and (3.16) implies

$$\lim_{k\to\infty} [y_n(k+d)-y(k+d)] = 0 \quad (3.18)$$

Note that even if there exist unstable roots in $A_\infty(q^{-1})$, equation (3.17) still holds, because if this were not the case, $y(k+d)$ would have to be unbounded which contradicts the already proven boundedness of $y(k)$.

To avoid the pathology of $A_\infty(q^{-1})$ having roots on the unit circle, the constrained least squares method (Goodwin and Sin, 1984) can be used to constrain the parameter space of $\hat{A}(q^{-1}, k)$ in a closed convex set which does not intersect any point having roots (as a polynomial) on the unit circle. Another way is to make $\phi(k)$ persistently exciting such that $A_\infty(q^{-1}) = A(q^{-1})$ is assured. For least squares type algorithms, $\phi(k)$ is persistently exciting if there exist some N, $\alpha, \beta > 0$ such that

$$\alpha I \ge \sum_{k+1}^{k+N} \phi(i)\phi^T(i) \ge \beta I, \quad k > 0,$$

(Goodwin and Sin 1984), or, equivalently, $\{r'(k)\}$ has a spectral distribution function which is nonzero at n points or more, where n is the number of parameters to be identified. It is possible that the time varying filter from y_m to u_m may suppress the spectral components of the signals. Therefore, the
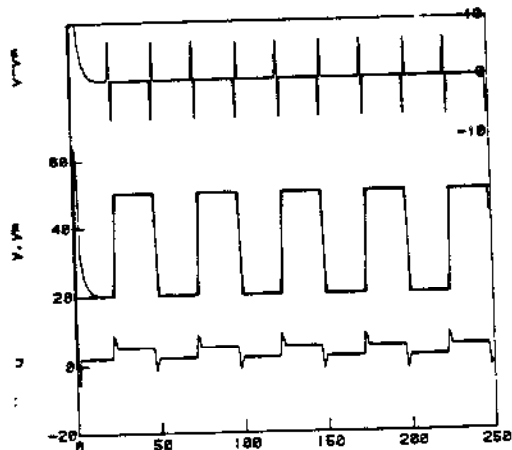
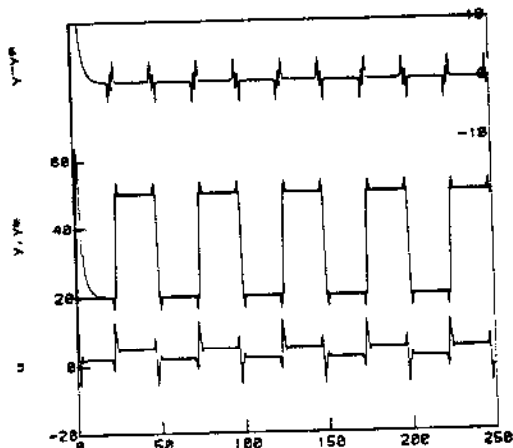Fig. 3 Simulation result of adaptive ZPETC

Fig. 4 Simulation result of adaptive ZPETC (controller order is over specified)
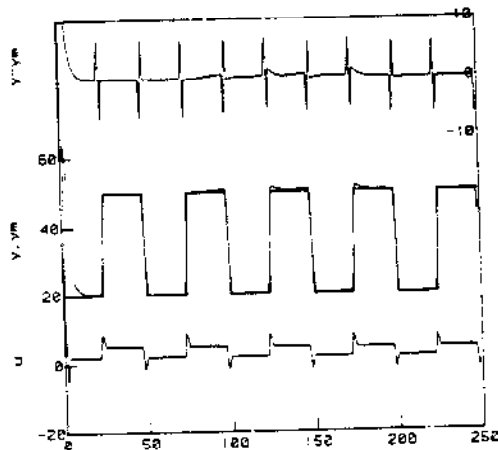

Fig. 5 Simulation result of adaptive ZPETC (plant zero varies from -2 at step 80 to -2.5 at step 180)

adaptive filter is kept fixed until sufficient excitation has been obtained (Bai and Sastry, 1986). The amount of excitation is measured by

$$\sum_{k}^{k+N} \phi(i)\phi^T(i).$$

In this case, the P.A.A. will update the parameters only at time instants k_i's, where $k_0 = 0$, $k_{i+1} = k_i + \delta_i$ and

$$\delta_i = \min_{\Delta \in S} \Delta \qquad (3.19)$$

$$S = \left\{ \Delta \,\Big|\, \sum_{k_i}^{k_i + \Delta} \phi\phi^T \geq \beta I \right\}$$
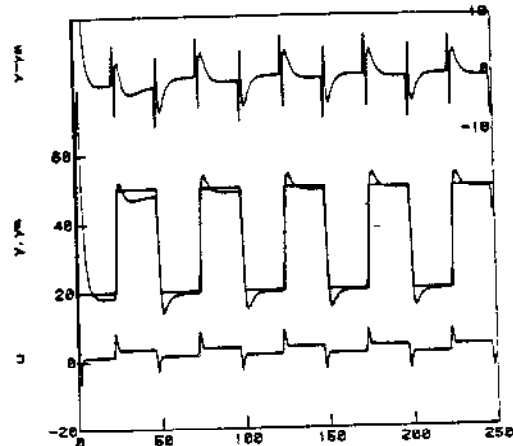
Fig. 6 Simulation result of adaptive ZPETC (unmodeled dynamics (l))

In the stationary case when the parameter estimations converge, the filter when y_m to r' can at most suppress $n_a + n_b + n_0$ points of the spectral distribution function of the signals where n_a, n_b, and n_0 are the orders of $A(q^{-1})$, $B(q^{-1})$, and $B_0^-(q^{-1})$, respectively. Hence, if y_m has at least $n + n_0 + n_a + n_b$ nonzero points in the spectral distribution function, (e.g., y_m is combinations of sine waves of $(n + n_0 + n_a + n_b)/2$ independent frequencies, where the factor of 2 is due to generation of two nonzero points by each frequency), r' has at least n nonzero points in the spectral distribution function. In this case, the parameter estimates converge to the true values:

$$\lim_{k \to \infty} \hat{\theta}(k) = \theta, \qquad (3.20)$$

and the tracking performance of adaptive feedforward controller is asymptotically same as that of nonadaptive one, namely:

$$\lim_{k \to \infty} \left[y(k+d) - \frac{B(q^{-1})B(q)}{[B(1)]^2} \frac{B_0^-(q^{-1})B_0^-(q)}{[B_0^-(1)]^2} y_m(k+d) \right] = 0 \qquad (3.21)$$

To avoid parameter drifts due to disturbances from exogenous noise or time varying, nonlinear, and unmodeled dynamics effects, the parameter space should be constrained or a dead zone on the estimation error (Goodwin and Sin Sections 3.6 and 3.7) should be set.

4 Simulations of Adaptive Zero Phase Error Tracking Controller

Although it would be unfair to compare the algorithm with methods in Clarke (1984) which were aimed at self-tuning regulation, the Clarke's nonminimum phase open loop stable plant is used to find how the algorithm improves the tracking performance:

$$(1 - 0.7q^{-1})y(k) = (1 + 2q^{-1})u(k-1) \qquad (4.1)$$

The desired output signal y_m has a series of step changes between 20 and 50 taking place every 25 samples. The methods in Clarke do not cancel the plant zeroes. Therefore, the tracking error is

$$e(k) = y_m(k) - y(k)$$

$$= \left(1 - \frac{B(q^{-1})}{B(1)} \right) y_m(k) = \frac{2}{3} (y_m(k) - y_m(k-1)) \qquad (4.2)$$

For ZPETC, the tracking error is

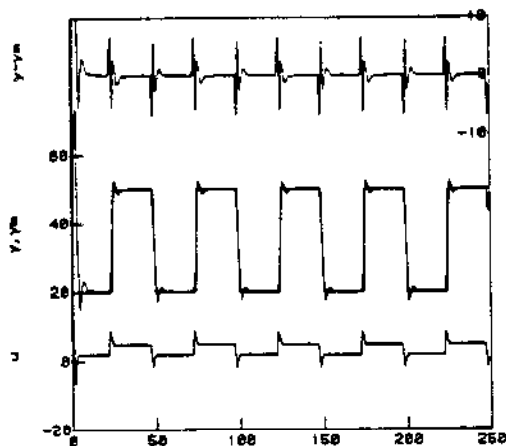$$e(k) = \left(1 - \frac{B(q^{-1})B(q)}{[B(1)]^2} \right) y_m(k)$$

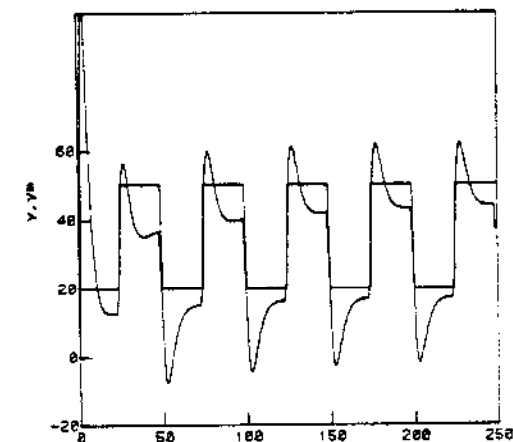Fig. 7 Simulation result of adaptive ZPETC (unmodeled dynamics (ii))

Fig. 9 Simulation result of adaptive ZPETC (square wave input for a servo system)

Fig. 8 Simulation result of adaptive ZPETC (unmodeled dynamics (iii))

Fig. 10 Parameter estimation errors

$$= \frac{2}{9} (y_m(k) - y_m(k-1) + y_m(k) - y_m(k+1)) \quad (4.3)$$

For the specified $y_m(k)$, the maximum errors from (4.2) and (4.3) are 20 and 6.7 respectively. Figure 3 shows the simulation result of the adaptive zero phase error tracking controller. The maximum tracking error is confirmed to be 6.7. In the simulation, the feedback controller was omitted since the open loop system is stable. The identifier was the one described by equations (2.8) and (2.9) with $\lambda(k) = 1$, and the method to avoid pathology of $\hat{A}(k)$ as described at the end of Section 3 was not used.

Figure 4 shows the simulation results for an overspecified order for the adaptive ZPETC. The reference model was overspecified to be third order. The 6 unknown parameters of the model do not converge to the true values. However, the control quality is still good. As shown in Fig. 4 the maximum error is even smaller than that in Fig. 3.

For slowly time varying system, the parameter adaptation gain with a constant trace is suited. Figure 5 shows the simulation results when the plant zero varied linearly from -2 at step 80 to -2.5 at step 180. The trace of $F(k)$ was set 1. The result shows that a good tracking performance is maintained even during the parameter variation period.

In order to examine the effect of unmodeled dynamics, the adaptive ZPETC designed for the plant (4.1) was tested for each of the following plants:

(i) $(1 - 0.3q^{-1})(1 - 0.7q^{-1})y(k)$

$$= (1 + 2q^{-1})u(k-1) \quad (4.4)$$
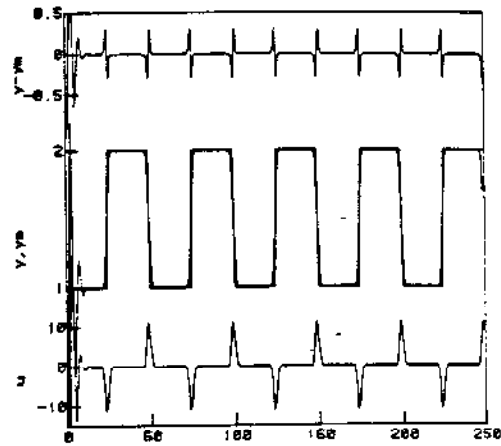
(ii) $(1 - 0.29q^{-1} + 0.25q^{-2})(1 - 0.7q^{-1})y(k)$

$$= (1 + 2q^{-1})u(k-1) \quad (4.5)$$

(iii) $(1 - 0.7q^{-1})(1 - 0.7q^{-1})y(k)$

$$= (1 + 2q^{-1})u(k-1) \quad (4.6)$$

Simulation results in Fig. 6 (case (i)), Fig. 7 (case (ii)) and Fig. 8 (case (iii)) show that the scheme can tolerate moderately fast unmodeled dynamics.

As an application to servo problems, a model of a machine tool carriage driven by a DC servo motor (Tomizuka et al. 1986) is used in the next set of simulations:

$(1 - 0.35240q^{-1} - 0.34426q^{-2} - 0.30397q^{-3})y(k)$

$$= (0.01093 + 0.03503q^{-1} + 0.02048q^{-2})u(k-1) \quad (4.7)$$

A proportional feedback controller with the feedback gain $k = 16.7$ results in a stable closed loop system:

$(1 - 0.16987q^{-1} + 0.24074q^{-2} + 0.03805q^{-3})y(k)$

$$= (0.18253 + 0.58500q^{-1} + 0.34202q^{-2})u(k-1) \quad (4.8)$$

Three types of reference inputs were tested: (i) a square wave, (ii) a single frequency sinusoidal wave, and (iii) a general wave. Figure 9 shows the results for case (i). The maximum error observed at discontinuities of the square wave is about 30 percent of the step change and coincides with the value calculated from equation (2.3). For case (ii), the identified parameters did not converge to true values due to the lack of persistency of excitation. As shown in Fig. 10, parameter estimation errors drift slowly from $\hat{\theta}^T(0) = (-0.330, -0.241, -0.038, 0.317, -0.585, -0.342)$ to $\hat{\theta}^T(10000) = (-0.332, -0.229, -0.012, 0.342, -0.572, -0.343)$. Figure 11 compares the tracking error under the adaptive scheme and that under the tuned ZPETC (with parameters known). The error magnitude of the tuned ZPETC is about 0.05 percent of the
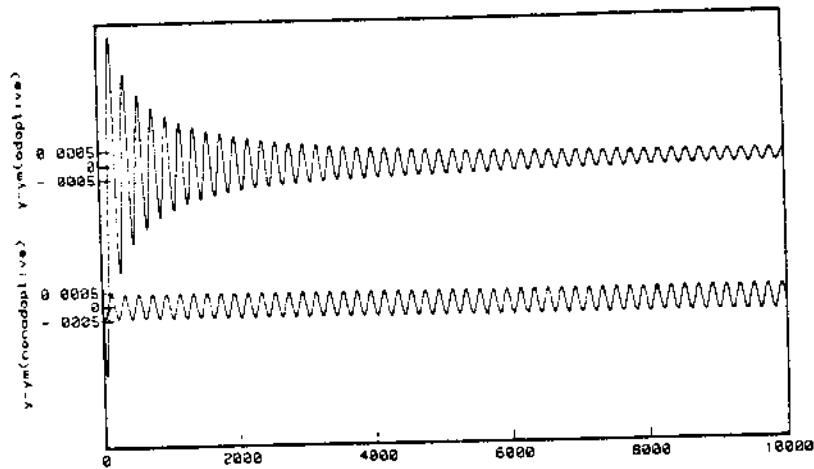
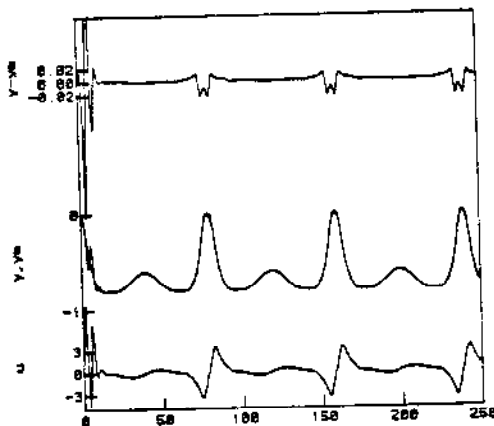Fig. 11 Tracking errors under adaptive ZPETC and tuned ZPETC

Fig. 12 Simulation result of adaptive ZPETC (general shape input for a servo system)

signal magnitude. The error of the adaptive scheme eventually becomes smaller than that of the tuned ZPETC. This is because the adaptive filter generates a more favorable magnitude scaling factor than the tuned ZPETC for this specific reference input. Figure 12 shows the results for case (iii). The parameters converge to the true values and the adaptive controller establishes the same tracking performance as that of tuned ZPETC. The error magnitude is about 2.5 percent of the signal magnitude.

5 Conclusions

This paper described the adaptive zero phase error tracking controller, which generates a reference input signal to let the output of a nonminimum phase plant follow time varying desired outputs with a small tracking error. The features of the proposed adaptive feedforward controller include: (i) the feedback loop is unperturbed by the adaptation algorithm, and hence, the adaptive feedforward controller can be easily switched on or off as a module; (ii) system stability is easily established because of the moving average characteristics of the feedforward controller and because the feedback loop is untouched; (iii) the preview action of the feedforward controller upgrades the tracking performance by canceling the phase lag caused by the closed loop dynamics.

Time varying or unknown aspect of the closed loop system addressed in this paper was primarily due to variations of the plant parameters. However, the simulation study included other uncertainties such as unmodeled dynamics. Although it was demonstrated by simulation that a superior tracking performance is maintained when unmodeled dynamics are stable and are moderately fast, this point must be further analyzed.

When adaptive regulation is necessary, the pole placement approach without zero cancellation can be used (Lozano and Goodwin, 1985). In this case, adding the adaptive feedforward filter $\hat{B}(q, k)/[B(1, k)]^2$ in front of the regulator loop, the zero phase error tracking can be achieved asymptotically.

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