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Tool Positioning for Noncircular Cutting With Lathe[1]

This paper presents the design and implementation of a digital controller for a lathe to machine workpieces with noncircular cross sections. Noncircular cutting is accomplished by controlling the radial tool position in the direction normal to the surface of workpiece. A discrete time model for the tool carriage in the radial direction is obtained by a least squares method applied to input and output data. The model is used for designing digital feedback and feedforward controllers. The zero phase error tracking control algorithm is applied as a feedforward control law for positioning of the tool along desired time varying signals. The effectiveness of the proposed controller is demonstrated by experiment and simulation.

I Introduction

The traditional way of utilizing a lathe is to cut the workpiece in circular cross sectioned shapes: i.e., the radial tool position is fixed in the direction normal to the surface of workpiece and the work surface can be represented by a "surface of revolution." However, it is possible to produce the cross sections of the workpiece with arbitrary shapes, if the tool position is controlled in this radial direction and its motion synchronized with the spindle angular position. We call the production of such surfaces by a machining operation noncircular cutting (Higuchi et al., 1984).

Noncircular cutting offers a number of advantages in manufacturing. Traditional approaches to prepare workpieces with irregular shapes have been casting or forging, which require a final finishing process to attain necessary surface finish or dimensions. Noncircular cutting allows us either to machine the part completely or to finish a certain class of irregular-shaped workpieces on the lathe. This yields a significant reduction in time and cost to produce such workpieces as well as introduces new flexible manufacturing techniques to a variety of difficult to produce components.

In order to establish noncircular cutting as a reliable and useful machining technique, it must be studied carefully both from control viewpoint and metal-cutting viewpoint. From the viewpoint of control, the servo controller for fast tracking of desired time varying output signals is of fundamental importance for accurate finished dimensions. From the viewpoint of metal cutting, consideration to the design and control of cutting tool geometry is important since the relative angle between the tool and the workpiece will vary during the machining. For each group of workpiece materials (ferrous, nonferrous, nonmetallic), there is generally a set of optimal tool angles in turning (Kalpakjian, 1984).

This paper is concerned with the first problem: i.e., design and implementation of the tracking servo controller. Two kinds of tracking controllers are considered: one for tracking the desired signals which can be represented as combinations of sinusoidal components, and the other for tracking arbitrary shaped desired signals. In the latter case, the controller is based on the idea of the zero phase error tracking algorithm (Tomizuka, 1985).

The remainder of this paper is organized as follows. The next section describes the experimental setup for noncircular cutting. Section III presents modeling and identification of dynamics involved in noncircular cutting. The design of controller along with simulation results is presented in Section IV followed by experimental results in Section V. Conclusions are given in Section VI.

II Experimental Setup

Figure 1 shows the experimental system which consists of a PDP-11/23 microcomputer, interface hardware and TREE-

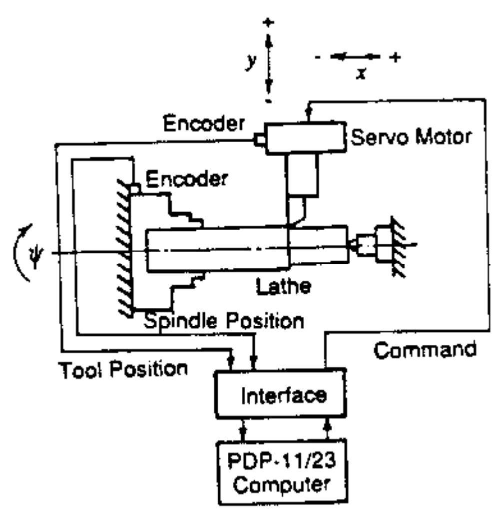

Fig. 1 Experimental setup

[1]This work was supported in part by NSF Grant No. MEA-8112843.

Contributed by the Dynamic Systems and Control Division for publication in the JOURNAL OF DYNAMIC SYSTEMS, MEASUREMENT AND CONTROL. Manuscript received by the Dynamic System and Control Division October 20, 1986.

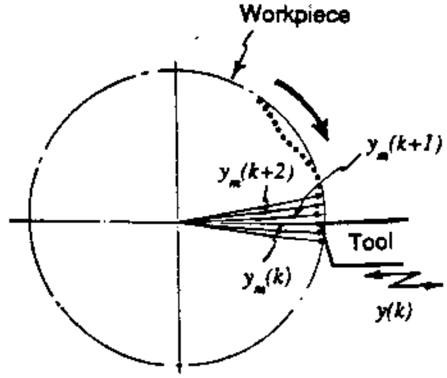

Fig. 2 Specification of noncircular cross-sectional shape

UP 1000 lathe. The tool carriage, i.e., the tool, is driven by a DC motor. Primary functions of the PDP-11/23 are digital control and data acquisition. As defined in the figure, y represents the radial direction normal to the workpiece surface and x the axial direction. ψ is the spindle angle. Experiments reported in this paper were conducted at the spindle speed of 93.7 rpm. The low spindle speed resulted from a limited bandwidth attainable from the existing DC servo motor for y-axis motion control.

If we specify the cross-sectional shape of workpiece by a series of points as shown in Fig. 2, the combinations of the spindle speed, n_s rpm, and a required number of points to describe the cross-sectional shape, N_p, determine the upper limit for the sampling period for digital control, T, by

$$T = 1/\{n_s/60)^* N_p\}\text{s}$$

In this paper, the sampling period and N_p are, respectively, 0.01 s and 64, which give a spindle speed of 93.7 rpm according to the above formula.[2]

III Modeling and Identification

In order to design the tracking controller for the tool carriage, i.e., tool, a dynamic model for carriage motion is required. Since the control algorithm will be implemented on computer, a discrete time modeling approach is considered. Furthermore, based on preliminary experiments, a third order discrete time transfer function model is judged to be appropriate for the sampling period of 0.01 s: i.e.,

$$\frac{y(k)}{u(k)} = \frac{q^{-1}B(q^{-1})}{A(q^{-1})} = \frac{q^{-1}(b_0 + b_1 q^{-1} + b_2 q^{-2})}{1 + a_1 q^{-1} + a_2 q^{-2} + a_3 q^{-3}} \qquad (1)$$

where the input, $u(k)$, is the D/A output of the computer, which becomes the reference input to the existing analog velocity servo controller for the y-axis motor, the output, $y(k)$, is the tool position, and q^{-1} is a one step backward shift operator. Notice that the input output relation given by equation (1) can also be represented as

$$y(k) = -\sum_{i=1}^{3} a_i y(k-i) + \sum_{j=0}^{2} b_j u(k-j) \qquad (2)$$

The parameters in this model, a_i's and b_j's, were estimated by a standard least squares method (e.g., Astrom and Wittenmark, 1984) applied to input and output data. To ensure the frequency richness requirement for the input signal for parameter convergence, the input was a pseudo random binary signal (Eveleigh, 1967).

Based on the identified parameters, the poles and zeros of the transfer function are:

$$\text{Poles} = 1.0,\ -0.32+0.45j,\ -0.32-0.45j,\ j=\sqrt{-1}$$

$$\text{Zeros} = -0.77,\ -2.43$$

[2]Sampling is initiated by the interrupt signal generated based on the spindle axis encoder. Actual values of T and N_p are subjected small variations.

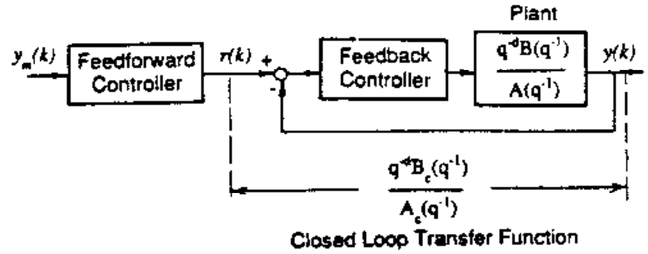

Fig. 3 Controlled plant under feedback/feedforward control

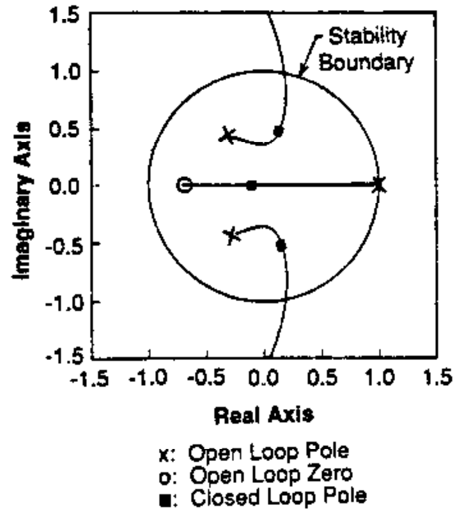

Fig. 4 Closed loop root locus for varying proportional feedback control gain K

The pole at 1.0 is due to integration from velocity to position in the open loop dynamics from $u(k)$ to $y(k)$. Notice that the system possesses one unstable zero, which must be taken into consideration in the design of tracking controllers (Tomizuka, 1985).

IV Controller Design

Both feedback and feedforward controllers for carriage motion are required in the radial direction normal to the workpiece surface for successful noncircular cutting (see Fig. 3). The feedback controller is for regulation against disturbances and the feedforward controller for positioning the tool along the desired time varying output signal y_m.

As a feedback controller, a simple proportional controller was adopted. The proportional control gain was determined based on the root locus plot in Fig. 4. For a selected proportional control gain of 16.7, the closed loop poles are at -0.14, $0.15+0.15j$ and $0.15-0.15j$. Notice that the closed loop zeros are at the same places as the open loop zeros. Figure 4 shows the frequency response of the closed loop system. While the frequency gain plot is reasonably flat to approximately 100 rad/s, an appreciable amount of phase shift is apparent, which degrades the tracking performance. To improve the tracking performance, a feedforward controller is required for compensating the dynamic delay in the closed loop system.

If the desired signal is composed of sinusoidal components, the feedforward controller may simply scale the magnitude and lead the phase of each sinusoidal component based on the frequency response in Fig. 5 so that the frequency response of the overall system, i.e., the feedforward controller followed by the closed loop system, becomes unity at that frequency. This principle is illustrated in Fig. 6.

Another approach to the design of feedforward controllers is based on pole-zero cancellation and phase cancellation. If the feedforward controller can cancel all the closed loop poles and zeros, the overall transfer function from the desired signal to the tool position becomes 1 and the tool position can follow

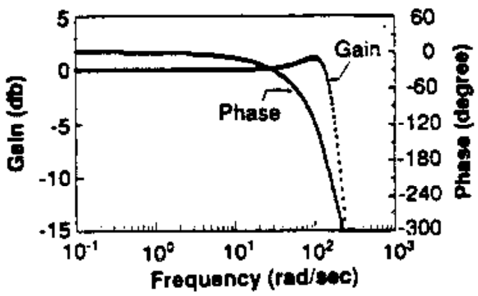

Fig. 5 Frequency response of closed loop system under proportional feedback control

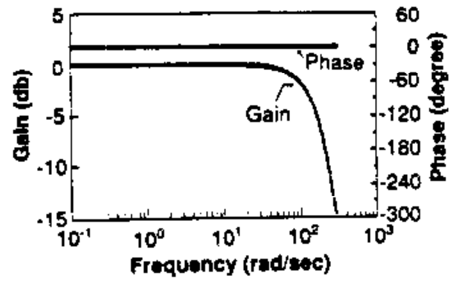

Fig. 8 Frequency response between desired output and actual output under zero phase error tracking control

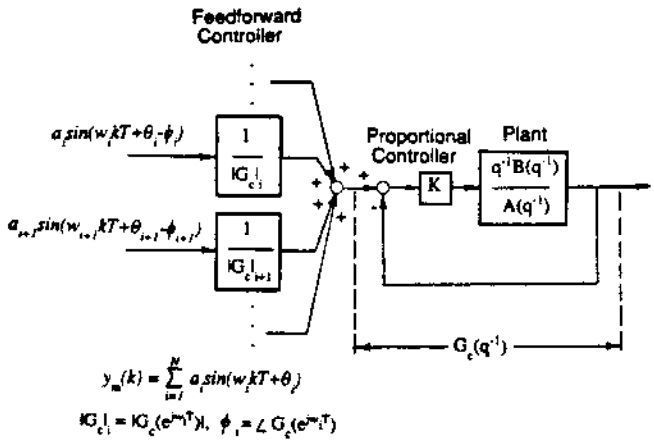

Fig. 6 Feedforward controller for tracking sinusoidal signals

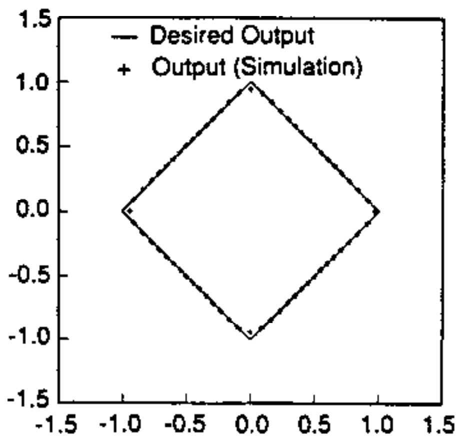

Fig. 9 Simulation result for square cross-section

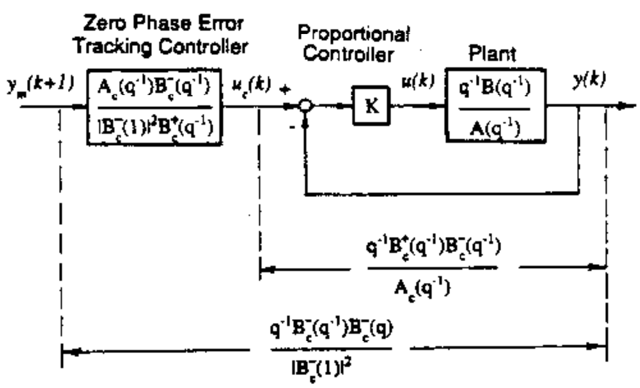

Fig. 7 Zero phase error tracking controller for tracking arbitrary signals

any desired signal perfectly. However, this simple approach cannot be utilized in the present problem because of the unstable zero at -2.43. As shown by Tomizuka (1985), the zero phase error tracking controller (ZPETC) ensures a superior performance of tracking arbitrary shaped signals under presence of unstable zeros.

Figure 7 shows the block diagram of the tool positioning system with ZPETC. In Fig. 7, the closed loop transfer function is represented by

$$G_c(q^{-1}) = \frac{q^{-1}B_c(q^{-1})}{A_c(q^{-1})} = \frac{q^{-1}B_c^+(q^{-1})B_c^-(q^{-1})}{A_c(q^{-1})} \tag{3}$$

where $B_c^+(q^{-1}) = k_c(1+0.77q^{-1})$ includes the stable zero at -0.77 and the scaling factor k_c, $B_c^-(q^{-1}) = 1+2.43q^{-1}$ includes the unstable zero at -2.43 and $A_c(q^{-1})$ is $(1+0.14q^{-1})\,[1+(-0.15-0.15j)q^{-1}]\,[1+(-0.15+0.15j)q^{-1}]$. As shown in the figure, ZPETC generates the reference input to the closed loop system, $u_c(k)$, from the desired output signal, $y_m(k)$, by

$$u_c(k) = \frac{A_c(q^{-1})B_c^-(q)}{|B_c^-(1)|^2 B_c^+(q^{-1})} y_m(k+1) \tag{4}$$

where $B_c^-(q) = 1+2.43q$. The overall transfer function from $y_m(k)$ to $y(k)$ then becomes

$$G(q^{-1}) = \frac{B_c^-(q^{-1})B_c^-(q)}{|B_c^-(1)|^2} = \frac{(1+2.43q^{-1})(1+2.43q)}{(3.43)^2} \tag{5}$$

From equation (4), ZPETC cancels all the closed loop poles and the zero at -0.77. For the uncancellable zero at -2.43, it provides a dynamic compensation so that the phase shift from the desired output y_m to the actual output y is zero for all frequencies, $0 \le \omega \le \pi/T$ where T is the sampling time, which can be checked by evaluating the frequency response of the overall transfer function given by equation (5). Figure 8 shows the frequency response. As shown in Fig. 8, the frequency response gain characteristics are flat in the low frequency region. Therefore, a superior tracking is expected for desired trajectories which do not contain high frequency components. For example, in an extreme case of cutting a square by lathe, which is not possible in actual cutting under any circumstances, a simulation study shows that ZPETC misses the corner points about 5.8 percent (see Fig. 9); otherwise tracking is perfect on the edges.

From equation (5), $y(k)$ is written as

$$y(k) = [2.43y_m(k+1) + 6.9y_m(k) + 2.43y_m(k-1)]/11.76 \tag{6}$$

Equation (6) can be utilized in several ways. For example, given a desired trajectory, it can be used for off-line analysis of the maximum error. It can be also utilized for the design of desired trajectories so that the error does not exceed a prespecified limit. Notice that from equation (6) the tracking error due to ZPETC at time k is

$$y(k) - y_m(k) = [2.43y_m(k+1) - 4.86y_m(k) + 2.43y_m(k-1)]/11.76 \tag{7}$$

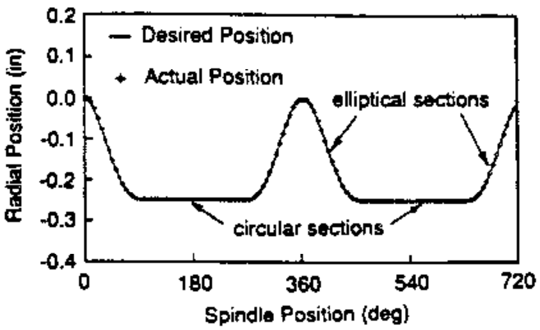

Fig. 10 Experimental result of noncircular cutting (cross-sectional shape = semicircle joined by semiellips)

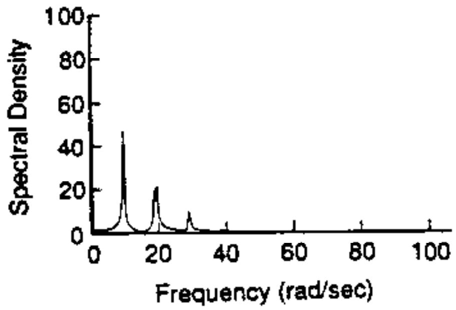

Fig. 11 Frequency spectrum of desired tool position (cross-sectional shape = semicircle joined by semiellips)

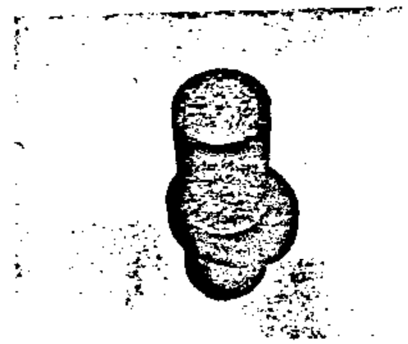

Fig. 12 Finished workpiece (Cross-sectional shape = semicircle joined by semiellips)

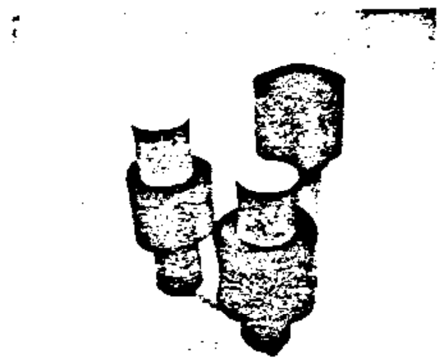

Fig. 13 Finished workpieces, $y_m(\psi) = c + a\sin(\psi n + kx)$ (left piece: $n = 3$ and $k = 0$; middle piece: $n = 3$ and $k \neq 0$; right piece: $n = 4$ and $k = 0$)

V Experimental Results

Because of a low spindle speed of 93.7 rpm, cutting experiments were performed with polyethylenen workpieces. Figure 10 shows the experimental result for a specified cross-section which is a semicircle jointed by a semiellipse. For this desired trajectory whose spectrum is concentrated in the low frequency region (see Fig. 11), the actual tool position remains close to the desired. Figure 12 shows the finished workpiece. The largest diameter of the specified shape was 1.75 in. (44.45 mm) and the maximum error in the experimental result was 0.025 in. (0.635 mm) which was 10 times larger than the error in simulation with the identified plant model given by equation (2). Since ZPETC is based on pole/zero cancellation and phase cancellation, it is sensitive to model/plant mismatches. The error amplification is attributed to model/plant mismatches. An adaptive version of ZPETC has been developed to overcome the problem mentioned above (Tsao and Tomizuka, 1986).

A number of other shapes including twists along the axial direction have been tried. Figure 13 shows the finished workpieces when cross-sectional shapes were described by

$$y_m(\psi) = c + a\sin(\psi n + kx) \tag{8}$$

where n corresponds to the number of lobes on the circumference of the part, and k is the rate of twisting along the x axis direction as defined in Fig. 1.

VI Conclusions

The paper described the design and implementation of the digital control algorithm for cutting workpieces with noncircular cross sections by a lathe. The feedforward controller is the most important element in the present digital control application. Two feedforward controllers, one based on magnitude and phase compensation for a particular frequency and the other based on ZPETC, have been proposed. ZPETC was demonstrated to be an effective method of noncircular machining.

Although the experiment supported the effectiveness of the proposed control approach, there still remains a number of problems which must be addressed from the viewpoint of control. One problem is associated with the bandwidth of the DC servo system for y-axis tool motion control. In order to cut metal, the spindle speed of 93.7 rpm is too slow. A hydraulic actuator will soon be installed on TREE-UP 1000 lathe to increase the bandwidth at least by 10 times. As stated in the previous section, the performance of the zero phase error tracking controller depends on the accuracy of model. Evaluation of the adaptive zero phase error tracking controller for noncircular machining is in progress.

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