1. Vectors

- notation
- examples
- vector operations
- linear functions
- complex vectors
- operation counts
Vector

- a vector is an ordered, finite list of numbers
- we use two types of notation: vertical and horizontal arrays; for example
  \[
  \begin{bmatrix}
  -1.1 \\
  0.0 \\
  3.6 \\
  7.2
  \end{bmatrix}
  = (-1.1, 0.0, 3.6, 7.2)
  \]
- numbers in the list are the elements (components, entries, coefficients)
- number of elements is the length (size, dimension) of the vector
- a vector with length \( n \) is called an \( n \)-vector
- set of \( n \)-vectors with real elements is denoted \( \mathbb{R}^n \)
Conventions

• we usually denote vectors by lowercase letters

\[ a = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix} = (a_1, a_2, \ldots, a_n) \]

• \(i\)th element of vector \(a\) is denoted \(a_i\)
• \(i\) is the index of the \(i\)th element \(a_i\)

Note

• several other conventions exist
• we’ll make exceptions, e.g., \(a_i\) can refer to \(i\)th vector in a list of vectors
Block vectors, subvectors

vectors can be stacked (concatenated) to create larger vectors

Example

- stacking vectors $b$, $c$, $d$ of length $m$, $n$, $p$ gives an $(m + n + p)$-vector

$$a = \begin{bmatrix} b \\ c \\ d \end{bmatrix} = (b_1, \ldots, b_m, c_1, \ldots, c_n, d_1, \ldots, d_p)$$

- other notation: $a = (b, c, d)$
- $b$, $c$, $d$ are blocks or subvectors of $a$
Special vectors

Zero vector and vector of ones

\[ 0 = (0, 0, \ldots, 0), \quad 1 = (1, 1, \ldots, 1) \]

length follows from context (if not, we add a subscript and write \(0_n, 1_n\))

Unit vectors

- there are \(n\) unit vectors of length \(n\), written \(e_1, e_2, \ldots, e_n\)
- \(i\)th unit vector is zero except its \(i\)th element which is 1; for \(n = 3\),

\[
e_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad e_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad e_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}
\]

- length of \(e_i\) follows from context (or should be specified explicitly)
Outline

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Geometry

**Position:** coordinates of a point in a plane or three-dimensional space

\[ x = (x_1, x_2) \]

**Displacement:** shown as arrow in plane or 3-D space

other quantities that have direction and magnitude, *e.g.*, force vector
Signal or time series

values of some quantity at different (and finitely many) times

- $x_k$ is value at time $k$, or in period $k$
- $n$-vector $x$ can be displayed by plotting $x_k$ versus $k$

- $x$ can represent a sampled function $f(t)$ of a continuous time variable $t$

$$x = (f(t_1), f(t_2), \ldots, f(t_n))$$
Images

Monochrome (black-and-white) image
grayscale levels of $M \times N$ pixels stored as $MN$-vector (e.g., columnwise)

Color image
• three $MN$-vectors with R, G, B intensities
• or concatenated as one vector of length $3MN$

Video sequence
• $K$ frames of size $M \times N$ as $K$ vectors or length $MN$ (if B&W)
• or concatenated as one $KMN$-vector
Feature vectors

contain values of variables or attributes that describe members of a set

Examples

• age, weight, blood pressure, gender, . . . , of patients in a database
• square footage, #bedrooms, list price, . . . , of houses in an inventory

Note

• vector elements represent very different quantities, in different units
• can contain categorical features (e.g., 0/1 for male/female)
• ordering has no particular meaning
Vectors of counts, histograms, occurrence vectors

Word counts

- vector represents a document
- length of vector is number of words in a dictionary
- \(i\)th element is number of times word \(i\) occurs in document

Histogram

- vector represents a histogram or distribution
- \(i\)th element is the frequency in bin \(i\)

Occurrence, set membership

- vector represents an object that can belong to \(n\) different sets
- \(i\)th element is 1 if object is in set \(i\); zero otherwise
Probability

- random event with $n$ possible outcomes, numbered 1, 2, . . . , $n$
- probability vector $p$ has length $n$
- $i$th element is probability of outcome $i$
- elements of $p$ are nonnegative and add up to one
Economics and finance

Portfolio

- vector represents portfolio of investments in $n$ assets
- $i$th element is amount invested in asset $i$, or #shares of asset $i$ held

Cash flow

- vector represents cash flow for $n$ periods (e.g., quarters)
- $i$th element represents payment to us (if positive), by us (if negative)

Resource vector

- vector represents manufacturing process for a product
- process requires $n$ resources (energy, labor, material, . . . )
- $i$th element is amount of resource $i$ used
Polynomials and generalized polynomials

a polynomial of degree \( n - 1 \) or less

\[
f(t) = c_1 + c_2 t + c_3 t^2 + \cdots + c_n t^{n-1}
\]

can be represented by an \( n \)-vector \((c_1, c_2, \ldots, c_n)\)

**Extensions:** for example, a cosine polynomial

\[
f(t) = c_1 + c_2 \cos t + c_3 \cos(2t) + \cdots + c_n \cos((n - 1)t)
\]

can be represented by an \( n \)-vector \((c_1, c_2, \ldots, c_n)\)
Summary

• vectors are used in a wide variety of applications

• can represent very different types of information

• usefulness of vector representation for a particular application depends on the relevance of vector operations for the application
Outline

• notation
• examples
• **vector operations**
• linear functions
• complex vectors
• operation counts
Addition and subtraction

\[ a + b = \begin{bmatrix} a_1 + b_1 \\ a_2 + b_2 \\ \vdots \\ a_n + b_n \end{bmatrix}, \quad a - b = \begin{bmatrix} a_1 - b_1 \\ a_2 - b_2 \\ \vdots \\ a_n - b_n \end{bmatrix} \]

- **commutative**

\[ a + b = b + a \]

- **associative**

\[ a + (b + c) = (a + b) + c \]

Vectors 1-15
Scalar-vector and componentwise multiplication

Scalar-vector multiplication: for scalar $\beta$ and $n$-vector $a$,

$$
\begin{bmatrix}
a_1 \\
a_2 \\
\vdots \\
a_n
\end{bmatrix}
\begin{bmatrix}
\beta
\end{bmatrix}
= 
\begin{bmatrix}
\beta a_1 \\
\beta a_2 \\
\vdots \\
\beta a_n
\end{bmatrix}
$$

Componentwise multiplication: for $n$-vectors $a$, $b$

$$
a \circ b = 
\begin{bmatrix}
a_1 b_1 \\
a_2 b_2 \\
\vdots \\
a_n b_n
\end{bmatrix}
$$

(in MATLAB: $a \ .* \ b$)
Linear combination

A *linear combination* of vectors $a_1, \ldots, a_m$ is a sum of scalar products

$$\beta_1 a_1 + \beta_2 a_2 + \cdots + \beta_m a_m$$

The scalars $\beta_1, \ldots, \beta_m$ are the *coefficients* of the linear combination.

Vectors $a_1 = (4, 0)$ and $a_2 = (2, 2)$.

$$\frac{3}{4}a_1 + \frac{3}{2}a_2 = (6, 3)$$
Inner product

the inner product of two $n$-vectors $a$, $b$ is defined as

$$a^T b = a_1 b_1 + a_2 b_2 + \cdots + a_n b_n$$

• a scalar

• meaning of superscript $^T$ will be explained when we discuss matrices

• other notation: $\langle a, b \rangle$, $(a \mid b)$, $\ldots$

(in MATLAB: $a^\prime \ast b$)
Properties

for vectors $a, b, c$ of equal length, scalar $\gamma$

- $a^T a = a_1^2 + a_2^2 + \cdots + a_n^2 \geq 0$

- $a^T a = 0$ only if $a = 0$

- commutative:
  $$a^T b = b^T a$$

- associative with scalar multiplication:
  $$(\gamma a)^T b = \gamma (a^T b)$$

- distributive with vector addition:
  $$(a + b)^T c = a^T c + b^T c$$
Simple examples

Inner product with unit vector

\[ e_i^T a = a_i \]

Differencing

\[ (e_i - e_j)^T a = a_i - a_j \]

Sum and average

\[ 1^T a = a_1 + a_2 + \cdots + a_n \]

\[ \frac{1}{n} \mathbf{1}^T a = \frac{a_1 + a_2 + \cdots + a_n}{n} \]
Applications

**Weighted sum**
- \( f \) is vector of features; \( w \) is vector of nonnegative weights
- \( \mathbf{w}^T \mathbf{f} = w_1 f_1 + w_2 f_2 + \cdots + w_n f_n \) is total score

**Cost**
- \( \mathbf{p} \) is vector of prices of \( n \) goods; \( \mathbf{q} \) is vector of quantities purchased
- \( \mathbf{p}^T \mathbf{q} = p_1 q_1 + p_2 q_2 + \cdots + p_n q_n \) is total cost

**Expected value**
- \( \mathbf{p} \) is vector of probabilities of \( n \) outcomes
- \( f_i \) is the value of a random variable in the event of outcome \( i \)
- \( \mathbf{p}^T \mathbf{f} \) is the expected value of the random variable
Discounted value

• $c$ is a cash flow over $n - 1$ periods; $d$ is vector of discount factors

$$d = \left(1, \frac{1}{1 + r}, \frac{1}{(1 + r)^2}, \ldots, \frac{1}{(1 + r)^{n-1}}\right)$$

$r > 0$ is interest rate

• $d^T c$ is net present value of cash flow

$$d^T c = c_1 + \frac{c_2}{1 + r} + \frac{c_3}{(1 + r)^2} + \cdots + \frac{c_n}{(1 + r)^{n-1}}$$

Co-occurrence

• $a, b$ are 0-1 occurrence vectors; represent membership of A, B in $n$ sets

• $a^T b$ is number of sets that contain both A and B
Polynomial evaluation

- $c$ is vector of coefficients of $f(t) = c_1 + c_2 t + c_3 t^2 + \cdots + c_n t^{n-1}$
- $x = (1, u, u^2, \ldots, u^{n-1})$ is vector of powers of $u$
- $c^T x$ is value of polynomial at $u$

$$c^T x = c_1 + c_2 u + c_3 u^2 + \cdots + c_n u^{n-1} = f(u)$$
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Linear function

A function $f : \mathbb{R}^n \rightarrow \mathbb{R}$ is **linear** if superposition holds:

$$f(\alpha x + \beta y) = \alpha f(x) + \beta f(y)$$  \hspace{1cm} (1)

for all $n$-vectors $x, y$ and all scalars $\alpha, \beta$

**Extension:** If $f$ is linear, superposition holds for any linear combination:

$$f(\alpha_1 u_1 + \alpha_2 u_2 + \cdots + \alpha_m u_m) = \alpha_1 f(u_1) + \alpha_2 f(u_2) + \cdots + \alpha_m f(u_m)$$

for all scalars $\alpha_1, \ldots, \alpha_m$ and all $n$-vectors $u_1, \ldots, u_m$

(this follows by applying (1) repeatedly)
Inner product function

for fixed $a \in \mathbb{R}^n$, define a function $f : \mathbb{R}^n \rightarrow \mathbb{R}$ as

$$f(x) = a^T x = a_1 x_1 + a_2 x_2 + \cdots + a_n x_n$$

- any function of this type is linear:

$$a^T(\alpha x + \beta y) = \alpha(a^T x) + \beta(a^T y)$$

for all scalars $\alpha, \beta$ and all $n$-vectors $x, y$

- every linear function can be written as an inner-product function:

$$f(x) = f(x_1 e_1 + x_2 e_2 + \cdots + x_n e_n)$$

$$= x_1 f(e_1) + x_2 f(e_2) + \cdots + x_n f(e_n)$$

line 2 follows from superposition
Examples in $\mathbb{R}^3$

- $f(x) = \frac{1}{3}(x_1 + x_2 + x_3)$ is linear: $f(x) = a^T x$ with $a = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$

- $f(x) = -x_1$ is linear: $f(x) = a^T x$ with $a = (-1, 0, 0)$

- $f(x) = \max\{x_1, x_2, x_3\}$ is not linear: superposition does not hold for $x = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $y = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$, $\alpha = -1$, $\beta = 1$

so we have $f(x) = 1$, $f(y) = 0$, 

$$f(\alpha x + \beta y) = 0 \neq \alpha f(x) + \beta f(y) = -1$$
Exercise

- unit mass with zero initial position and velocity
- subject to piecewise-constant force $F(t)$ during interval $[0, 10)$:

  $F(t) = x_j$ for $t \in [j - 1, j)$, $j = 1, \ldots, 10$

- define $f(x)$ as position at $t = 10$, $g(x)$ as velocity at $t = 10$

are $f$ and $g$ linear functions of $x$?
Solution

• from Newton’s law \( s''(t) = F(t) \) where \( s(t) \) is the position at time \( t \)

• integrate twice to get final velocity and position

\[
s'(10) = \int_0^{10} F(t) \, dt = x_1 + x_2 + \cdots + x_{10}
\]

\[
s(10) = \int_0^{10} s'(t) \, dt = \frac{19}{2} x_1 + \frac{17}{2} x_2 + \frac{15}{2} x_3 + \cdots + \frac{1}{2} x_{10}
\]

the two functions are linear: \( f(x) = a^T x \) and \( g(x) = b^T x \) with

\[
a = \left( \frac{19}{2}, \frac{17}{2}, \ldots, \frac{3}{2}, \frac{1}{2} \right), \quad b = (1, 1, \ldots, 1)
\]
Affine function

A function $f : \mathbb{R}^n \to \mathbb{R}$ is \textbf{affine} if it satisfies

$$f(\alpha x + \beta y) = \alpha f(x) + \beta f(y)$$

for all $n$-vectors $x$, $y$ and all scalars $\alpha$, $\beta$ with $\alpha + \beta = 1$

\textbf{Extension:} if $f$ is affine, then

$$f(\alpha_1 u_1 + \alpha_2 u_2 + \cdots + \alpha_m u_m) = \alpha_1 f(u_1) + \alpha_2 f(u_2) + \cdots + \alpha_m f(u_m)$$

for all $n$-vectors $u_1$, $\ldots$, $u_m$ and all scalars $\alpha_1$, $\ldots$, $\alpha_m$ with

$$\alpha_1 + \alpha_2 + \cdots + \alpha_m = 1$$
Affine functions and inner products

for fixed $a \in \mathbb{R}^n$, $b \in \mathbb{R}$, define a function $f : \mathbb{R}^n \rightarrow \mathbb{R}$ by

$$f(x) = a^T x + b = a_1 x_1 + a_2 x_2 + \cdots + a_n x_n + b$$

i.e., an inner-product function plus a constant (offset)

• any function of this type is affine: if $\alpha + \beta = 1$ then

$$a^T (\alpha x + \beta y) + b = \alpha (a^T x + b) + \beta (a^T x + b)$$

• every affine function can be written as $f(x) = a^T x + b$ with:

$$a = (f(e_1) - f(0), f(e_2) - f(0), \ldots, f(e_n) - f(0)), \quad b = f(0)$$
Affine approximation

first-order Taylor approximation of differentiable $f : \mathbb{R}^n \to \mathbb{R}$ around $z$:

$$\hat{f}(x) = f(z) + \frac{\partial f}{\partial x_1}(z)(x_1 - z_1) + \cdots + \frac{\partial f}{\partial x_n}(z)(x_n - z_n)$$

- generalizes first-order Taylor approximation of function of one variable

$$\hat{f}(x) = f(z) + f'(z)(x - z)$$

- $\hat{f}$ is a local affine approximation of $f$ around $z$

- in vector notation: $\hat{f}(x) = f(z) + \nabla f(z)^T(x - z)$ where

$$\nabla f(z) = \left(\frac{\partial f}{\partial x_1}(z), \frac{\partial f}{\partial x_2}(z), \ldots, \frac{\partial f}{\partial x_n}(z)\right)$$

the $n$-vector $\nabla f(z)$ is called the gradient of $f$ at $z$
Example

\[ f(x_1, x_2) = x_1 - 3x_2 + e^{2x_1 + x_2 - 1} \]

Gradient

\[ \nabla f(x) = \begin{bmatrix} 1 + 2e^{2x_1 + x_2 - 1} \\ -3 + e^{2x_1 + x_2 - 1} \end{bmatrix} \]

First-order Taylor approximation around \( z = 0 \)

\[ \hat{f}(x) = f(0) + \nabla f(0)^T(x - 0) \]
\[ = e^{-1} + (1 + 2e^{-1})x_1 + (-3 + e^{-1})x_2 \]
Regression model

\[ \hat{y} = x^T \beta + \beta_0 \]

\[ = \beta_0 + \beta_1 x_1 + \cdots + \beta_p x_p \]

- \( x \) is feature vector; \( x_i \)'s are regressors, independent variables, or inputs
- \( \beta = (\beta_1, \ldots, \beta_p) \) is vector of weights; \( \beta_0 \) is offset or intercept
- \( \hat{y} \) is (predicted) outcome or dependent variable

- regression model expresses \( \hat{y} \) as an affine function of \( x \)
- model parameters are coefficients \( \beta_i \)

Example

- \( \hat{y} \) is selling price of a house in some neighborhood
- regressors \((x_1, x_2, x_3, x_4) = (\text{lot size, area, \#bedrooms, \#bathrooms})\)
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Complex numbers

**Complex number**: \( x = \alpha + j\beta \) with \( \alpha, \beta \) real scalars

- \( j = \sqrt{-1} \) (more common notation is \( i \) or \( j \))
- \( \alpha \) is the *real part* of \( x \), denoted \( \text{Re} \, x \)
- \( \beta \) is the *imaginary* part, denoted \( \text{Im} \, x \)

set of complex numbers is denoted \( \mathbb{C} \)

**Modulus and conjugate**

- modulus (absolute value, magnitude): \( |x| = \sqrt{(\text{Re} \, x)^2 + (\text{Im} \, x)^2} \)
- conjugate: \( \bar{x} = \text{Re} \, x - j \text{Im} \, x \)
- useful formulas:
  \[
  \text{Re} \, x = \frac{x + \bar{x}}{2}, \quad \text{Im} \, x = \frac{x - \bar{x}}{2j}, \quad |x|^2 = \bar{x}x
  \]
Polar representation

nonzero complex $x = \Re x + j \Im x$ can be written as

$$x = |x| (\cos \theta + j \sin \theta) = |x|e^{j\theta}$$

- $\theta \in [0, 2\pi)$ is the argument (phase angle) of $x$ (notation: $\arg x$)
- $e^{j\theta}$ is complex exponential: $e^{j\theta} = \cos \theta + j \sin \theta$
Complex vector

- vector with complex elements: \( a = \alpha + j\beta \) with \( \alpha, \beta \) real vectors
- real and imaginary part, conjugate are defined componentwise:
  \[
  \begin{align*}
  \text{Re} \ a &= (\text{Re} \ a_1, \text{Re} \ a_2, \ldots, \text{Re} \ a_n) \\
  \text{Im} \ a &= (\text{Im} \ a_1, \text{Im} \ a_2, \ldots, \text{Im} \ a_n) \\
  \bar{a} &= \text{Re} \ a - j \text{Im} \ a
  \end{align*}
  \]
- set of complex \( n \)-vectors is denoted \( \mathbb{C}^n \)
- addition, scalar/componentwise multiplication defined as in \( \mathbb{R}^n \):
  \[
  \begin{align*}
  a + b &= \begin{bmatrix}
  a_1 + b_1 \\
  a_2 + b_2 \\
  \vdots \\
  a_n + b_n
  \end{bmatrix}, \\
  \gamma a &= \begin{bmatrix}
  \gamma a_1 \\
  \gamma a_2 \\
  \vdots \\
  \gamma a_n
  \end{bmatrix}, \\
  a \circ b &= \begin{bmatrix}
  a_1 b_1 \\
  a_2 b_2 \\
  \vdots \\
  a_n b_n
  \end{bmatrix}
  \end{align*}
  \]
Complex inner product

the inner product of complex $n$-vectors $a$, $b$ is defined as

$$b^H a = \overline{b_1}a_1 + \overline{b_2}a_2 + \cdots + \overline{b_n}a_n$$

- a complex scalar

- meaning of superscript $^H$ will be explained when we discuss matrices

- other notation: $\langle a, b \rangle$, $(a \mid b)$, $\ldots$

- for real vectors, reduces to real inner product $b^T a$

(in MATLAB: $b' * a$)
Properties

for complex $n$-vectors $a$, $b$, $c$, complex scalars $\gamma$

- $a^H a \geq 0$: follows from
  $$ a^H a = \bar{a}_1 a_1 + \bar{a}_2 a_2 + \cdots + \bar{a}_n a_n $$
  $$ = |a_1|^2 + |a_2|^2 + \cdots + |a_n|^2 $$

- $a^H a = 0$ only if $a = 0$

- $b^H a = \overline{a^H b}$

- $b^H (\gamma a) = \gamma (b^H a)$

- $(\gamma b)^H a = \bar{\gamma} (b^H a)$

- $(b + c)^H a = b^H a + c^H a$

- $b^H (a + c) = b^H a + b^H c$
Example: power in electric networks

- $\tilde{v}(t)$ is voltage across circuit element at time $t$
- $\tilde{i}(t)$ is current through element

- $p(t) = \tilde{v}(t)\tilde{i}(t)$ is instantaneous power absorbed by element at time $t$

- for $n$ elements, with voltages $\tilde{v}_k(t)$, $\tilde{i}_k(t)$, $k = 1, \ldots, n$, total power is

\[ p(t) = \sum_{k=1}^{n} \tilde{v}_k(t)\tilde{i}_k(t), \]

the (real) inner product of two $n$-vectors of voltages and currents
Sinusoidal voltage and current

• assume voltage and current are sinusoids with the same frequency

\[\tilde{v}(t) = V \cos(\omega t + \alpha), \quad \tilde{i}(t) = I \cos(\omega t + \beta) \quad \text{(with } V, I \geq 0)\]

• can be represented by complex numbers (phasors)

\[v = \frac{V}{\sqrt{2}} e^{j\alpha}, \quad i = \frac{I}{\sqrt{2}} e^{j\beta}\]

instantaneous power

\[p(t) = VI \cos(\omega t + \alpha) \cos(\omega t + \beta)\]
\[= \frac{VI}{2} (\cos(\alpha - \beta)(1 + \cos 2(\omega t + \alpha)) + \sin(\alpha - \beta) \sin 2(\omega t + \alpha))\]
\[= \text{Re}(\bar{v}i) (1 + \cos 2(\omega t + \alpha)) + \text{Im}(\bar{v}i) \sin 2(\omega t + \alpha)\]
Complex power

\[ p(t) = \text{Re}(\bar{iv}) (1 + \cos 2(\omega t + \alpha)) + \text{Im}(\bar{iv}) \sin 2(\omega t + \alpha) \]

- \( P = \text{Re}(\bar{iv}) \) is called average (or real, active) power
- \( Q = \text{Im}(\bar{iv}) \) is reactive power
- \( P + jQ = \bar{iv} \) is complex power

for \( n \) elements: \( n \)-vectors of phasors \( v = (v_1, \ldots, v_n) \) and \( i = (i_1, \ldots, i_n) \)

\[ i^H v = \text{Re}(i^H v) + j \text{Im}(i^H v) \]

- \( \text{Re}(i^H v) \) is total average power
- \( \text{Im}(i^H v) \) is sum of reactive powers
- \( i^H v \) is sum of complex powers
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Floating-point operation

Floating-point operation (flop)

- the unit of complexity when comparing vector and matrix algorithms
- 1 flop = one basic arithmetic operation (+, *, √, . . . ) in \( \mathbb{R} \) or \( \mathbb{C} \)

Comments: this is a very simplified model of complexity of algorithms

- we don’t distinguish between the different operations
- we don’t distinguish between real and complex arithmetic
- we ignore integer operations (indexing, loop counters, . . . )
- we ignore cost of memory access
**Operation count**

**Operation count (flop count)**

- total number of operations in an algorithm
- in linear algebra, typically a polynomial of the dimensions in the problem
- a crude predictor of run time of the algorithm:

\[
\text{run time} \approx \frac{\text{number of operations (flops)}}{\text{computer speed (flops per second)}}
\]

**Dominant term:** the highest-order term in the flop count

\[
\frac{1}{3}n^3 + 100n^2 + 10n + 5 \approx \frac{1}{3}n^3
\]

**Order:** the power in the dominant term

\[
\frac{1}{3}n^3 + 10n^2 + 100 = \text{order } n^3
\]
Examples

flop counts of vector operations in this lecture (for vectors of length $n$)

- addition, subtraction: $n$ flops
- scalar multiplication: $n$ flops
- componentwise multiplication: $n$ flops
- inner product: $2n - 1 \approx 2n$ flops

these operations are all order $n$