Homework 9

Implement the Newton method on page 10–17 of the slides for the problem

$$\text{minimize} \quad f(x) = -\sum_{i=1}^{m} \log(1 - a_i^T x) - \sum_{j=1}^{n} \log(1 - x_j^2).$$

The variable x is an n-vector. The solution of this problem is the analytic center of the set of inequalities

$$Ax \preceq \mathbf{1}, \qquad -\mathbf{1} \preceq x \preceq \mathbf{1},$$

where A is an $m \times n$ matrix with rows a_i^T.

1. The starting point in Newton's method must satisfy $x^{(0)} \in \mathbf{dom}\, f$. An obvious choice for this problem is $x^{(0)} = 0$. Use $\epsilon = 10^{-8}$ in the stopping criterion on page 10–17, and $\alpha = 0.01$, $\beta = 1/2$ in the backtracking line search (page 10–6). In the exit condition for the line search on page 10–6, the left-hand side $f(x + t\Delta x)$ is interpreted as $+\infty$ if $x + t\Delta x \notin \mathbf{dom}\, f$.

   Test your code on randomly generated matrices A (for example, using the MATLAB command `A = randn(m,n)`), for different values of m and n (perhaps up to $m = 2000$, $n = 1000$).

   For a typical instance, plot the Newton decrement $\lambda(x^{(k)})$ and the step size t versus iteration number k.

2. Suppose $m \ll n$. Explain how to compute the Newton step Δx_{nt} in a number of operations that grows linearly in n (see page 10–30 of the slides). You do not need to implement this faster method.