\[ \beta = \frac{4\pi \alpha_0 L}{\nu_{gs}} \]

\[ \frac{1}{\nu_{to}} = \frac{1}{\nu_{gs}} + \frac{1}{\nu_{to}} \]

\[ \nu_{to} = \frac{2}{\nu_{gs} - \nu_{to}} \]

\[ \nu_{to} = \frac{2}{\nu_{gs} - \nu_{to}} \]

\[ \nu_{0} = \frac{1}{\nu_{to}} = \frac{1}{\frac{2\pi \alpha_0 L}{(\nu_{gs} - \nu_{to})^2}} \]

Eqn - 1 mark

Plot - 1 mark

Total - 2 marks

Eqn - 1 mark

Plot - 1 mark

Total - 2 marks
\[ g_m = \frac{H \cos \alpha \omega}{L} (\omega g - \omega_0) \]

\[ g_{m0} = \frac{2}{L} (\omega g - \omega_0) \]

Assuming \( \omega \) is constant.

\[ y_0 = \frac{1}{\gamma J_0} + \frac{1}{\gamma} \frac{H \cos \alpha \omega (\omega g - \omega_0)}{L^2} \]

\[ g_{m0} = \sqrt{\frac{4 H \cos \alpha \omega}{L^2}} \cdot \frac{1}{\gamma J_0} \cdot \frac{1}{\gamma} \cdot \frac{1}{\gamma} \]

Assuming \( \omega \) is constant.

\[ E = n - 1 \]

\[ \text{plot} \rightarrow 1 \]
\[ g_m = \beta (V_{gs} - V_{th}) \]
\[ = \beta (V_{gs} - \left( V_{th0} + \gamma \left( \frac{a_F + V_{th0} - \sqrt{a_F}}{a} \right) \right)) \]

\[ \delta_0 = \frac{1}{\delta_0} \cdot \frac{1}{\frac{\beta (V_{gs} - V_{th})}{\alpha}} \]
\[ = \frac{1}{\frac{\beta}{\alpha} \left( V_{gs} - \left( V_{th0} + \gamma \left[ \frac{a_F + V_{th0} - \sqrt{a_F}}{a} \right] \right) \right)^2} \]

\[ g_m_{\infty} = \frac{2}{\delta_0 (V_{gs} - V_{th})} \]
\[ = \frac{2}{\delta_0 \left( V_{gs} - \left( V_{th0} + \gamma \left[ \frac{a_F + V_{th0} - \sqrt{a_F}}{a} \right] \right) \right)} \]

As the bulk potential \( V_{th} \) increases, the bulk potential \( V_{th} \) becomes negative and the device gain will go off.

\[ V_{th} = V_{gs} \]

\[ V_{bo} = \frac{V_{gs} - V_{th0} + \sqrt{a_F}}{\gamma} + a_F \mu + V_g \]

\[ 1 \text{  } 6 \]
\[ v_{in} = 0 \quad M_1 \text{ is off} \quad v_{out} = V_{CC} \quad M_2 \text{ is in linear region} \]

\[ v_{in} > v_{th} \text{ current through } M_1 \text{ decreases} \]

\[ M_2 \text{ stays in linear region} \quad v_{out} \text{ drops} \]

\[ \frac{R_P (v_{in} - v_{th})^2}{2} = \frac{R_P (V_{CC} - v_{th})(v_{o} - v_{th})}{2} \]

\[ + \frac{v_{CC} - v_{out}}{R_P} \]

\[ \text{when } v_{out} = v_{th} + v_{lamp} \quad M_2 \text{ enters saturation at this point. The corresponding } v_{in} \text{ is given by} \]

\[ \frac{R_P (v_{in} - v_{th})^2}{2} = \frac{R_P (V_{CC} - v_{th} - v_{lamp})^2}{2} + \frac{v_{CC} - v_{out}}{R_P} \]

\[ \text{for } v_{in} > v_{in1} \quad M_2 \text{ stays in saturation conducting constant current, } v_{out} \text{ drops as per equation} \]

\[ \frac{R_P (v_{in} - v_{th})^2}{2} = \frac{R_P (V_{CC} - v_{th} - v_{lamp})^2}{2} + \frac{v_{CC} - v_{out}}{R_P} \]

\[ \text{when } v_{out} \text{ drops to } v_{in} = v_{th} \quad M_1 \text{ will enter into linear region} \quad \text{corresponding } v_{in} = v_{in2} \text{ is given by} \]

\[ \frac{R_P (v_{in2} - v_{th})^2}{2} = \frac{R_P (V_{CC} - v_{th} - v_{lamp})^2}{2} + \frac{v_{CC} - (v_{in2} - v_{th})}{R_P} \]
As $\text{Vin}$ increases, $\text{Vout}$ will continue to drop asymptotically, approaching zero.

**Diagram:**

- $V_{in}$
- $V_{out}$
- $V_{in1}$
- $V_{in2}$
- $V_{out}$
- $V_{in1}$ Sat
- $V_{in2}$ Sat

**Note:**

Each region twice mark total.

Each region

- 1 for boundary condition
- 1/2 for relevant equation & trend

Equation:

$$1 = \frac{1}{2}$$
Starting from $v_{x0} = V_{CC}$ for convenience.

(i) $v_{x0} = V_{CC}$ until $v_{y} = V_{CC}$

This continues until $v_{x} = v_{b} - V_{TM}$ at which point

$M_{1}$ will turn on in saturation. Then

$I_{x} = \frac{v_{x}}{R_{S}}$.

(ii) $v_{x} < V_{b} - V_{TM}$ in saturation.

When $v_{x}$ reaches $V_{b} - V_{TM}$ at $v_{x0} = V_{CC}$

$V_{y} = V_{CC}$.

$v_{y}^{2} = \frac{R_{C}}{2} (V_{b} - v_{x} - V_{TM})^{2}$

$v_{x} = v_{b} - V_{TM} - \sqrt{V_{y}^{2} - \frac{R_{C}}{2} (V_{b} - v_{x} - V_{TM})^{2}}$.

When current $I_{x}$ is

$\frac{v_{x}}{R_{S}} = \frac{R_{C}}{2} (V_{b} - v_{x} - V_{TM})^{2}$.

(iii) $v_{x} < v_{x1}$ in linear region.

$I_{x} = \frac{v_{x}}{R_{S}} = \frac{R_{C}}{2} \left[ (V_{b} - v_{x} - V_{TM})(v_{y} - v_{x}) - \frac{(v_{y} - v_{x})^{2}}{2} \right]$. 

If $v_{y} = V_{CC}$ then $v_{y} = V_{CC} - \left( \frac{v_{x} - J_{x}}{R_{S}} \right) R_{C}$. 

$I_{x} = \frac{v_{x}}{R_{S}} = \frac{R_{C}}{2} \left[ (V_{b} - v_{x} - V_{TM})(v_{y} - v_{x}) - \frac{(v_{y} - v_{x})^{2}}{2} \right]$. 

$V_{y} = V_{CC} - \left( \frac{v_{x} - J_{x}}{R_{S}} \right) R_{C}$. 

$I_{x} = \frac{v_{x}}{R_{S}} = \frac{R_{C}}{2} \left[ (V_{b} - v_{x} - V_{TM})(v_{y} - v_{x}) - \frac{(v_{y} - v_{x})^{2}}{2} \right]$. 

$V_{y} = V_{CC} - \left( \frac{v_{x} - J_{x}}{R_{S}} \right) R_{C}$.
At some point in region 1 or 2, current change polarity when again current reaches \( \frac{V_o}{R_o} \)

Each region - 

Total 3
Since there is no current flow through $R_1$, $V_y = V_{in}$.

(i) At $V_{in} = 0$, $M_1$ is off and $M_2$ is on in saturation. This is the state till $M_1$ turns on at $V_{in} = V_{th1}$.

So for $0 \leq V_{in} \leq V_{th1}$,

$$I_x = -\frac{B_p}{2} \left( V_{in} - V_{th1} \right)^2$$

(ii) For $V_{th1} \leq V_{in} \leq V_{th2}$ both $M_1$ and $M_2$ are on in saturation

$$I_x + \frac{B_p}{2} \left( V_{in} - V_{th1} \right)^2 = \frac{B_n}{2} \left( V_{in} - V_{th2} \right)^2$$

(iii) $V_{in} = V_{th2}$, $M_2$ is off.

$$I_x = \frac{B_n}{2} \left( V_{in} - V_{th2} \right)^2$$

Each region: 1

Total: 3
Small signal mean - 1

Analysis - 2 3 1 mark if approach is correct & final answer is wrong.

\[
\begin{align*}
\text{gm}_2 V_a + \frac{V_{out}}{R_0} + \frac{V_a}{R_s} &= 0 \\
\Rightarrow \quad 1 + \frac{\text{gm}_2 R_s}{R_0} V_a &= -\frac{V_{out}}{R_0} \\
V_a &= -\frac{V_{out}}{R_0} \cdot \frac{R_0}{\text{gm}_2 R_s} - \frac{1}{1 + \frac{\text{gm}_2 R_s}{R_0}} \\
\text{Avo} \\
\text{gm}_1 (V_{in} - V_a) + \frac{V_{out} - V_a}{R_0} &= \frac{V_a}{R_s} \\
\text{gm}_1 \cdot V_{in} &= \text{gm}_2 V_a + \frac{V_a}{R_0} + \frac{V_a}{R_s} - \frac{V_{out}}{R_0} \\
&= \left[ \frac{1}{R_0} + \frac{R_0 + \text{gm}_1 R_0 (R_s)}{R_0 R_s} \right] \cdot \frac{R_s}{R_0} \cdot \frac{1}{\text{gm}_2 R_s} \\
&= \left[ \frac{(1 + \text{gm}_2 R_s) R_0 + R_0 + \text{gm}_1 R_0 (R_s)}{R_0 R_s (1 + \text{gm}_2 R_s)} \right] - \frac{V_{out}}{R_0} \\
\text{Vout} \div \text{Vin} &= \frac{\text{gm}_1 R_0 (R_0 + \text{gm}_1 R_0 (R_s))}{R_0 + (1 + \text{gm}_1 R_s) R_0 + (1 + \text{gm}_2 R_s) R_0}
\end{align*}
\]
Small Signal Model - 1

Analysis 2 3 (Mark for correct approach)

\[ g_m 2 (v_{a} - v_{out}) = \frac{v_{out}}{r_{02}} \]

\[ g_{m2} v_{a} = \left[ \frac{1 + g_m r_{02}}{r_{02}} \right] v_{out} \]

\[ v_{a} = \frac{1 + g_m r_{02}}{g_m r_{02}} v_{out} \]

\[ g_{m1} v_{in} + \frac{v_{a}}{r_{01}} + \frac{v_{a}}{r_{03}} + g_m v_{out} = 0 \]

\[ g_{m1} v_{in} + \left[ \frac{1}{r_{01}} + \frac{1}{r_{03}} \right] \frac{1 + g_m r_{02}}{g_m r_{02}} v_{out} = 0 \]

\[ \frac{v_{out}}{v_{in}} = \frac{g_m}{g_m + \left[ \frac{1 + g_m r_{02}}{g_m r_{02}} \right] \left[ \frac{1}{r_{01}} + \frac{1}{r_{03}} \right]} \]
\[ I_{in} = \frac{V_{in}}{R_1} \]

\[ V_{out} = \frac{1 + \frac{R_2}{R_1}}{1 + \frac{R_3}{R_2}} V_{in} \]

\[ I_{out} = \frac{V_{out}}{R_2} \]

\[ \text{Current mirror} \]

\[ I_{mirror} = I_{in} \]

\[ V_{mirror} = V_{out} \]
Current through $M_2 = 0 \implies V_A = 0$

1. $V_y(0^-) = 0 \implies M_1$ starts in saturation at $t = 0$

$$\frac{\beta}{2} (v_{bi} - V_{th})^2 + \frac{C_{dv_y}}{a} = 0$$

$$dV_y = -\frac{\beta}{2C} (v_{bi} - V_{th}) dt$$

$$V_y = V_{00} - \frac{\beta}{2C} (v_{bi} - V_{th})^2$$

2. When $V_y(t) = v_{bi} - V_{th} \implies M_1$ enters linear region

$$V_y = V_{00} - \frac{\beta}{2C} (v_{bi} - V_{th})^2$$

$$t_1 = \frac{(V_{00} - v_{bi} + V_{th})}{\frac{\beta}{2C} (v_{bi} - V_{th})^2}$$

3. $t > t_1 \implies M_1$ is in linear region

$$\left[ 2(v_{bi} - V_{th})V_y - \frac{V_y^2}{a} \right] + \frac{C_{dv_y}}{a} = 0$$

$$dV_y = -\frac{\beta}{2C} dt$$

$$V_y \left[ \frac{1}{V_y} + \frac{1}{2(v_{bi} - V_{th})} \right] = -\frac{\beta}{2C}$$
\[ u_0(y) - u_1(y, t) = - \frac{\beta \epsilon (v_0 - v_1)}{c} \]

\[
\frac{\Delta v}{\Delta (v_0 - v_1)} = e^{\frac{-\beta \epsilon (v_0 - v_1)}{c}}
\]

\[ t = t_1 \quad v_1 = v_0 - v_{in} \]

\[
\frac{k (v_0 - v_{in})}{v_0 - v_{in}} - v_{in} = e^{\frac{-\beta \epsilon (v_0 - v_{in})}{c}}
\]

\[
\frac{v_{in}}{\Delta (v_0 - v_{in})} = e^{\frac{-\beta \epsilon (v_0 - v_{in})}{c}}
\]

\[ t_1 = e^{\frac{\beta \epsilon (v_0 - v_{in})}{c}} \]

\[ v_{in} = \frac{\Delta (v_0 - v_{in})}{1 + e^{\beta \epsilon (v_0 - v_{in})}} \]
To keep $M_1$ in saturation

\[ V_{by} = V_{gs1} - V_{th1} + V_{gs2} \]

Ignoring body effect,

\[ V_{by} = \sqrt{\frac{2I}{r_S}} + V_{th1} + \sqrt{\frac{2I}{r_P}} \]

\[ V_{by} \approx 0.754V \]

To keep $M_2$ in saturation

\[ V_{by1} = V_{DD} - \left( (V_{gs1} - V_{th1}) + V_{gs2} \right) \]

\[ V_{by1} = 3 - \left( V_{th1} + 2\sqrt{\frac{r_1}{r_P}} \right) \]

\[ V_{by1} \approx 2.0781V \]

\[ V_{out} = V_{DD} - 2xV_{by1} \]

\[ V_{out} \approx 2.5662 \]

\[ V_{out_{min}} = V_{DD} - 2xV_{by1} \]

\[ V_{out_{min}} \approx 0.28 \]

\[ V_{out_{DC}} = V_{DD} - V_{gs1} \]

\[ V_{out_{DC}} \approx 2.3951 \]
so even though the bias points are chosen such that the output DC point prevents the swing to

\[ V_{DD} = 3 \]

\[ V_{DD} = 0.562 = V_{MAX} \]

\[ V_{DC} = 0.295 \]

\[ \theta_{VDD} = 0.189 \]

\[ A = g_{m1} \times \left( g_{m2} + 0.11 \times g_{m3} \right) \]

\[ g_{m1} = \sqrt{AP_{1}} = 3.7 \times 2 = g_{m2} \]

\[ g_{m3} = \sqrt{AP_{2}} = 5.8 \times 2 = g_{m4} \]

\[ A = 3.7 \times \left[ \left( 3.7 \times \frac{1}{(15 \times 2)^2} \right) \times \left( 5.8 \times \frac{1}{(6 \times 2)^2} \right) \right] \]

\[ = 3.7 \times 1.58k \times 209k \]

\[ = 330 \]
Adjustment may be required to keep $m_{Bn M_4}$ in saturation due to body effects on $m_9 B M_3$.

\[ C_1 v_{IN} - v_a C_1 = (v_a - v_{OUT}) C_2 \]

\[ C_1 v_{IN} = v_a (C_1 + C_2) - v_{OUT} C_2 \]

\[ = -v_{OUT} \left( C_1 + C_2 - \frac{C_1 C_2}{A} \right) \]

\[ \frac{v_{OUT}}{v_{IN}} = \frac{C_1}{C_2 + \frac{C_1 C_2}{A}} \]

\[ \therefore \quad \frac{C_1}{C_2} = \frac{2}{1} \quad \text{(1)} \]

\[ \text{4 = 3) 6. = 3) Netstat/ present sera} \]

\[ \text{6 = 3) Ban currents & DC gain} \]

\[ A \text{ unreasonable, } 2 I \approx 50 \Omega \]

\[ 2 A \approx 50 \Omega \]

\[ \text{c) 1 = 3) mid Band gain = } 2 \pm 10\% \]