Communication Complexity of Multi-Vehicle Systems

ERIC KLA VINS

Computer Science
California Institute of Technology

with: Jason Hickey and Richard Murray
A Theory of Decentralized Control Systems
(Gleaned from the LPE Reading Group)

Control + Computer Science

- Complexity and Scalability
- Combinatorics/Graph Theory
- Logic, Specification and Formal Methods
- Networking and Communications
Outline

• Specification of decentralized control systems
• Communication complexity
• Schemes with various complexities
• Related work: (Semi) Automatic Verification
Decentralized Control as Parallel Processing

DRL [Klavins&Hickey, Submitted to CDC 2002] is language for specifying and reasoning about parallel control systems. Based on UNITY [Chany&Misra, 1990].
# A Comparison of Formalisms

<table>
<thead>
<tr>
<th>Hybrid Automata Based Modeling</th>
<th>DRL</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Continuous/Discrete</td>
<td>• Discrete</td>
</tr>
<tr>
<td>• Simple Dynamics</td>
<td>• Arbitrary Dynamics</td>
</tr>
<tr>
<td>• State Based/Model Checking</td>
<td>• Symbolic Theorem Proving</td>
</tr>
<tr>
<td>• Small Systems</td>
<td>• Potentially large (homogenous) systems</td>
</tr>
<tr>
<td></td>
<td>• Domain Knowledge and Reuse</td>
</tr>
</tbody>
</table>
Modeling Dynamical Systems in DRL

Initial \((I)\): \( t = 0 \) \hspace{1cm} \text{Defines the initial conditions of the system}

Clauses \((C')\):

\[
\begin{align*}
    x_1 > 0 : & \quad u'_1 < 0 \\
    x_1 < 0 : & \quad u'_1 > 0 \\
    \text{true} : & \quad u'_2 = -k(x_2 - x_1)
\end{align*}
\]

Dynamics \((\Delta)\):

\[
\text{true} : \quad t' = t + \delta \quad \land \quad \forall i \quad ||x'_i - (x_i + \delta u_i)|| < \varepsilon
\]
Modeling Dynamical Systems in DRL

Initial (I): \( t = 0 \)

Clauses (C’):

\[
\begin{align*}
    x_1 > 0 & : u_1' < 0 \\
    x_1 < 0 & : u_1' > 0 \\
    \text{true} & : u_2' = -k(x_2 - x_1)
\end{align*}
\]

A set of clauses defines the program or controller. The rules may be nondeterministic.

Dynamics (\( \Delta \)):

\[
\text{true} : t' = t + \delta \land \forall i \|x_i' - (x_i + \delta u_i)\| < \varepsilon
\]
Modeling Dynamical Systems in DRL

Initial \((I)\): \( t = 0 \)

Clauses \((C')\):

\[
\begin{align*}
x_1 > 0 & : u'_1 < 0 \\
x_1 < 0 & : u'_1 > 0 \\
true & : u'_2 = -k(x_2 - x_1)
\end{align*}
\]

Dynamics \((\Delta)\):

\[
true : t' = t + \delta \land \forall i \ ||x'_i - (x_i + \delta u_i)|| < \varepsilon
\]
Modeling Dynamical Systems in DRL

Initial (I): \( t = 0 \)

Clauses (C):

\[
\begin{align*}
    x_1 > 0 & : u'_1 < 0 \\
    x_1 < 0 & : u'_1 > 0 \\
    true & : u'_2 = -k(x_2 - x_1)
\end{align*}
\]

Dynamics (\( \Delta \)):

\[
true : t' = t + \delta \land \forall i \ |x'_i - (x_i + \delta u_i)| < \varepsilon
\]
Modeling Dynamical Systems in DRL

Initial ($I$): $t = 0$

Clauses ($C'$):

\[
\begin{align*}
    x_1 > 0 : & \quad u'_1 < 0 \\
    x_1 < 0 : & \quad u'_1 > 0 \\
    \text{true} : & \quad u'_2 = -k(x_2 - x_1)
\end{align*}
\]

Dynamics ($\Delta$):

\[
\text{true} : t' = t + \delta \land \forall i \ |x'_i - (x_i + \delta u_i)| < \varepsilon
\]

This clause models the environment. Note the nondeterminism.
Scalability

- Scalability depends on computation and coordination

- “Coordination complexity” measures how much each agent relies on other agents

- Communication complexity is a surrogate for this

- Bad: $O(n^2)$. Good: $O(n)$

The drag about parallel computation:

But this isn’t the case for all tasks.

What is the analogous way to think about decentralized control systems?
Communication Complexity

Fix $\Pi = (I, C, \Delta)$. Suppose each clause $c \in C$ has cost $\gamma(c) \in \mathbb{N}$.

The communication complexity of a state $s$ is

$$cc(s) = \sum_{c \in C \land c.g(s)} \gamma(c).$$

The communication complexity of an execution $\{s_k\}$ of $\Pi$ is

$$cc(\{s_k\}) = \lim_{T \to \infty} \frac{1}{T} \sum_{k=1}^{T} cc(s_k).$$

The communication complexity of $\Pi$ is given by

$$cc(\Pi) = \max_{\{s_k\} \in E(\Pi)} cc(\{s_k\}).$$
Task 1: Full Communication

Define $\Pi_{dyn}$ to have the clause

$$\text{true} : t' = t + \delta \land \forall i \ ( ||x'_i - x_i|| < \delta v_{\text{max}} )$$

Suppose the task is to maintain the property

$$||e_{i,j} - x_j|| < \varepsilon$$

Initial (I):

$$t = 0 \land \forall i, j \ ( e_{i,j} = x_j \land l_{i,j} = 0 )$$

Clauses (C): For all $i \neq j$, include the clause $c_{i,j}$

$$(t + \delta - l_{i,j})v_{\text{max}} \geq \varepsilon : \ e'_{i,j} = x_j \land l'_{i,j} = t \land \gamma(c_{i,j}) = 1$$

**Thm 1:** Each clause is applied every $\tau \triangleq \frac{1}{\delta} \lfloor \frac{\varepsilon}{v_{\text{max}}} - \delta \rfloor$ steps. Thus, the $cc$ is $\frac{1}{\tau} n(n - 1) = O(n^2)$. 
Task 2: Communication with Neighbors

New task. Maintain

\[(i, j) \in E \Rightarrow \|e_{i,j} - x_j\| < \varepsilon\]

For all \(i, j\) such that \((i, j) \in E\), let \(c_{i,j}\) be

\[(t + \delta - l_{i,j})v_{\text{max}} \geq \varepsilon : e'_{i,j} = x_j \land l'_{i,j} = t\]

**Thm 2**: If the maximal degree of \((V, E')\) is constant for any \(n\), then \(cc(\Pi) = O(n)\).
Task 3: Distance Modulated Communication

New task. Maintain the property

\[ ||e_{i,j} - x_j|| < k||x_i - x_j|| \]

Initial (I):

\[ \forall i, j \ ( e_{i,j} = x_j \land l_{i,j} = 0 \land t = 0 ) \]

Clauses (C): For each \( i \neq j \) add the clauses \( c_{i,j} \)

\[ (t - l_{i,j})v_{\text{max}} \geq k\left(||e_{i,j} - x_i|| - (t - l_{i,j})v_{\text{max}}\right) : \]

\[ e'_{i,j} = x_j \land l'_{i,j} = t \]
How to Use DMC

Using $e_{i,j}$ and $l_{i,j}$ and knowing $v_{max}$

1) Define $O_{i,j}$ to be the set reachable by agent $j$ before the next communication event.

2) Plan around these sets to your goal.
Complexity of DMC

The communication complexity of this algorithm depends on how the vehicles are arranged.

a) ~1

b) Positions uniformly distributed in a square of area proportional to \( n \).

**Thm. 3:** If the vehicles are regulated to be (a) equispaced on a straight line, then \( cc(\Pi_{DMC}) = O(n \log n) \). If they are regulated to be (b) uniformly distributed in a square of area \( n/\rho \) then \( cc(\Pi_{DMC}) = O(n^{1.5}) \).
Task 4: Wanderer Communication Scheme

The task is still to maintain $||e_{i,j} - x_j|| < \varepsilon$, but only for a constant number of “wandering” vehicles.

• “Wanderers” must have accurate estimates of other’s positions.

• Fixed vehicles can be ignorant.

• Wanderers may hand their right to move to a fixed vehicle.
WCS Algorithm

Initial (I):

\[ \forall i[(i \leq \kappa \rightarrow q_i = 1) \land (i > \kappa \rightarrow q_i = 3)] \land r_i = \bot \land \forall j(e_{i,j} = x_j) \]

Clauses (C): For each \( i \in \{1, \ldots, n\} \) define three clauses

\[
q_i = 1 : \forall j(q_j \neq 3 \rightarrow e'_{i,j} = x_j)
\]

\[
q_i = 1 \land \text{coin}(p, t) : q'_i = 2
\]

\[
q_i = 2 : q'_i = 3 \land r'_i = \bot \land \exists j[q_j = 3 \land q'_j = 1 \land r'_j = i \land \forall k(e'_{j,k} = e_{i,k})]
\]

\[
\begin{array}{c|c|c}
\text{cost} & \kappa - 1 \hfill & \\
\hline
q_i = 1 : \text{wandering} & 0 \\
q_i = 2 : \text{uploading} & n \\
q_i = 3 : \text{fixed} & \\
\end{array}
\]
WCS Algorithm

Dynamics ($\Pi_{dyn}$):

\[
\text{true} : t' = t + \delta \land \forall i[(q_i = 1 \rightarrow ||x'_i - x_i|| \leq \delta v_{max} \\
\land (q_i \neq 1 \rightarrow x'_i = x_i)]
\]

**Thm. 4:** If the value of $p$ in $\Pi_{WCS}(n)$ is constant, then

\[
E[cc(\Pi_{WCS}(n))] = O(n)
\]

and if $p \triangleq 1/n$ then

\[
E[cc(\Pi_{WCS}(n))] = O(1)
\]
Other Questions

• Define a “power aware” cost $p(c) = \|x_i - x_j\|^2 \gamma(c)$.

  • What are the efficient communication schemes?
  • Hops are better than direct (unlike with normal CC).

• Sensing costs and tradeoffs

• Incorporating control: $\Pi_{\text{dyn}} \circ \Pi_{\text{com}} \circ \Pi_{\text{control}}$
Current Work: (Semi) Automatic Verification

The RoboFlag Drill

\[ \Pi_{opp} = (I, C') \text{ where} \]

\[ I \equiv \forall i \in \mathbb{N} \ (b(i).y \geq 0 \land b(i+1).y > b(i).y + \delta v \land b(i).x \in [0, \text{max}]) \]

\[ C' = \{ \text{true} : b' = \lambda i. (q(i).x, q(i).y - \delta v) \} \]

and

\[ q = \text{if } b(0).y - \delta v < 0 \text{ then } \text{rest}(b) \text{ else } b \]

States that each opponent is above the line and in the playing field [0,max]. Also states that opponents are separated vertically (a convenience).

The opponents are modeled as a sequences of points. The new value of b is obtained from the old value by decreasing each y coordinate and throwing out the first element if it has crossed the line.
\( \Pi_{\text{def}}(k) = (I, C) \) where

\[
I \equiv x_k \in [0, \text{max}] \land y_k = 0
\]

\[
C = \{ \text{true} : \ x'_k = x_k + \delta u_k \}
\]

A system with \( n \) defenders is given by

\[
\Pi(n) = \Pi_{\text{opp}} \circ \Pi_{\text{def}}(1) \circ ... \circ \Pi_{\text{def}}(n)
\]

The goal is to define control specifications such that

\[
\Pi(n) \circ \Pi_{\text{control}}(1) \circ ... \circ \Pi_{\text{control}}(n)
\]

has

\[
b(i).y \in B_{\varepsilon_1}(0) \Rightarrow \exists k(||x_k - b(i).x|| < \varepsilon_2)
\]

as an invariant.
Toward a DRL Verification Assistant using Isabelle [Paulson et al., 1994]

Base DRL Theory (borrows heavily from Paulson’s UNITY implementation)

Real Number Theory

Special Purpose Theories (e.g. Theory of Opponent Sequences)

External Provers or Simplifiers? (e.g. Mathematica)

Specification

Desired Properties

Theorem Prover

Guarantees