Distributed Control of Multi-Vehicle Systems

Richard M. Murray  Jason J. Hickey
Dong Eui Chang  Eric Klavins  Reza Olfati-Saber  Justin Smith

California Institute of Technology

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Outline
I.  Background and Motivation
II.  Graph rigidity and distributed control of multi-vehicle formations
III.  Computation and control (Hickey)
IV.  Caltech multi-vehicle wireless testbed status
Cooperative Control in Dynamic, Uncertain, Adversarial Environments

Team-based control for RoboFlag

- Theory for cooperative control of multi-vehicle systems in adversarial (competitive) environments
- Extend control approach to make use of tools from computer science (formal methods, protocol verification, etc)
Graph Rigidity and Distributed Control of Multi-Vehicle Formations

Reza Olfati-Saber and Richard M. Murray

References


- Distributed Cooperative Control of Multiple Vehicle Formations Using Structural Potential Functions, Reza Olfati-Saber and Richard M. Murray. To appear, 2002 IFAC World Congress.
Formation Operations

Control questions

• How do we split and rejoin teams of vehicles?
• How do we specify vehicle formations and control them?
• How do we reconfigure formations (shape and topology)

Initial approach: potential functions

• Provides natural mechanism for distributed control
• Can easily extend to optimization based control (with potential function as cost) ⇒ take into account constraints, nonlinearity
Definition A graph $G$ is called a *rigid graph* iff there exists a subgraph $H$ with $n$ nodes and $2n-3$ edges of $G$ such that

\[
(q_j - q_i)^T \cdot (p_j - p_i) = 0 \quad \forall e_{ij} \in E_H
\]

\[
\downarrow
\]

\[
(q_s - q_r)^T \cdot (p_s - p_r) = 0 \quad \forall r, s \in I, r \neq s
\]

$q_i$ = node position

$p_i$ = node velocity
Task Specification

Unique formation representations

- Distance constraints (rigidity)
- Area-based constraints (foldability)

\[ \Gamma = \left\{ q \in \mathbb{R}^{2n} : \begin{array}{l} \|q_j - q_i\| - d_{ij} = 0, \forall e_{ij} \in E, \\ (q_j - q_i) \otimes (q_k - q_i) - a_{ijk} = 0, \forall f_{ijk} \in F \end{array} \right\} \]

\( E = \) edges of \( G \)

\( F = \) faces of \( G \)

**Theorem** (CDC ’02): In \( \mathbb{R}^2 \), 2\( n \)-3 distance-based and \( n \)-2 area-based algebraic constraints associated with “properly placed” edges and faces are required to uniquely specify a formation of \( n \) agents.

**Formation Graph** \( G = (V, E, D, F, A) \)

Q: how do we stabilize a formation with graph \( G \)?
Formation Potentials

Constraint deviation variables → formation potential

\[ \eta_{ij} = \| q_j - q_i \| - d_{ij}, \]

\[ \delta_{ijk} = (q_j - q_i) \otimes (q_k - q_i) - a_{ijk}, \]

\[ V(q) = \sum_{e_{ij} \in E} \phi(\eta_{ij}) + \sum_{f_{ijk} \in F} \phi(\delta_{ijk}) \]

Potential function properties, example

\[ \begin{align*}
\phi(x) &> 0, \quad \forall x \neq 0 \\
\phi(0) & = 0 \\
\phi(x) &= \sqrt{1 + x^2} - 1 \rightarrow f(x) = \phi'(x) = \frac{x}{\sqrt{1 + x^2}}
\end{align*} \]

Example: Triangle

\[ V(q) = \phi(\eta_{12}) + \phi(\eta_{23}) + \phi(\eta_{32}) + \phi(\delta_{123}) \]
Control Results

**Problem statement:** Given \( H(q, p) = \frac{1}{2} \sum_{i \in I} \| p_i \|^2 + V(q) \) find \( u \) such that

\[
\frac{dH}{dt} \leq 0 \quad \text{and} \quad \lim_{t \to \infty} H(q, p) = 0
\]

**Distributed control (IFAC ‘02)**

\[
u_i = \sum_{j \in J_i} f(\|q_j - q_i\| - d_{ij}) \cdot \bar{n}_{ij} - c_d p_i
\]

**Theorem:** If each vehicle applies the control input in (*) , then the trajectory of the group of vehicles locally asymptotically converges to the desired formation in a collision-free manner.

**Optimization based control**

\[
u^* = \arg \min_{u} J = \int_{0}^{T} H(q, p) dt + H(q(T), p(T))
\]
Distributed Control of Multi-Vehicle Formations

![Graph showing control of multi-vehicle formations.](image-url)
Optimization-Based Tracking

Center of Mass
Computation and Control

Adam Granicz and Jason Hickey

References

• Formal Design Environments, Adam Granicz, Brian Aydemir, and Jason Hickey. Thereom Proving and Higher Order Logic (TPHOL), 2002.
• Adam Granicz and Jason Hickey. Phobos: An Approach to Domain Specific Compilers, HICSS Workshop on Domain Specific Languages
Caltech Multi-Vehicle Wireless Testbed

Richard M. Murray    Jason J. Hickey    Steven Low
Lars Cremean    William Dunbar    David van Gogh    Eric Klavins
Jason Meltzer    Abhishek Tiwari    Steve Waydo

References

• http://www.cds.caltech.edu/~mvwt
Multi-Vehicle Wireless Testbed for Integrated Control, Communications and Computation (DURIP)

Testbed features
- Distributed computation on vehicles + command and control console
- Point to point networking (bluetooth) + local area networking (802.11)
- Overhead vision system provides global position data (LPS)
Results to Date

- Manual Control (9 Nov 01)
- Classical Control (7 May 02)
- Trajectory Tracking (28 May 02)
- Leader Follower (28 May 02)
Motivation: Cooperative Control in Dynamic, Uncertain, Adversarial Environments

- RoboFlag as driver for theory of teams, cooperation, distributed control, etc
- Combine ideas from control and computer science \(\rightarrow\) higher levels of decision making

Current work

- Graph rigidity and distributed control: strong framework for future results
- Logical programming environments: basic infrastructure and theory
- Caltech multi-vehicle wireless testbed: experimental platform of testing ideas

Next steps

- Stronger theory for mixed logical/continuous operations (split/rejoin)
- Beat Cornell at RoboFlag through superior theory, strategy, and tactics