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The Harmonic-Rejection Mixer

The harmonic-rejection mixer (HRM) dates back to 2001 [1] but has been finding increasingly wider application in radio-frequency (RF) transceivers [2]–[4]. The popularity of HRMs stems from two trends in RF design: the incentive to minimize the number of high-frequency filters and the demand for wide-band radios. In this article, we study the HRM's operation principle and design issues.

Background

A mixer is a circuit that multiplies two inputs in the time domain [Figure 1(a)]. Suppose a narrow-band random signal $x_1(t)$ is multiplied by a periodic waveform $x_2(t) = A\cos\omega_2 t$, typically provided by a local oscillator (LO), to generate $x_{out}(t)$. In the frequency domain, this operation is equivalent to convolving the spectrum of $x_1(t)$ with that of $x_2(t)$, which contains only an impulse at ω_2. Thus, as illustrated in Figure 1(b), the output contains two copies of the spectrum of $x_1(t)$, shifted up and down by ω_2. This mixing scenario corresponds to the behavior of an ideal analog multiplier.

In practice, circuit implementations of analog multipliers suffer from a low gain and high noise. Consider, e.g., the differential pair shown in Figure 1(c), where M_1 converts $x_1(t)$ to a current, and M_2 and M_3 steer this current according to the value and polarity of $x_2(t)$. For ideal multiplication, M_2 and M_3 must operate in the small-signal regime so that their transconductance varies linearly with the tail current and with x_2. Consequently, most of the tail current is wasted as a common-mode component, leading to a low gain for the mixer. Moreover, because M_2 and M_3 are simultaneously on, they inject substantial noise to the output.

In view of these issues, we prefer that M_2 and M_3 in Figure 1(c) switch abruptly and completely. That is, the LO voltage swing and $(W/L)_{2,3}$ are chosen large enough to ensure that M_2 and M_3 rapidly steer the tail current from one side to the other. Now, $x_1(t)$ is multiplied by a square wave, $S_{\pm 1}(t)$, that toggles between −1 and +1. We note that $x_1(t)$ experiences a gain of $(2/\pi) g_{m1} R_D$ as its spectrum is shifted up and down in the frequency domain, emerging as a differential output voltage, x_{out}. The multiplication by a square wave occurs even if the LO waveform is a sinusoid so long as the LO swing and $(W/L)_{2,3}$ are sufficiently large. Also, M_2 and M_3 contribute minimal noise because only one is on for about half of an LO cycle, and it is heavily degenerated by M_1.

While improving the mixer's performance, hard switching introduces a new issue. Because $x_1(t)$ is now multiplied by a square wave, its spectrum is also translated up and down by the higher harmonics of the LO. Figure 2 depicts the resulting output spectrum, where it is assumed that the first harmonic of the LO lies at the center of the spectrum of $x_1(t)$, i.e., $\omega_2 = \omega_1$. We observe that copies of $X_1(f)$ are translated to $2\omega_2, 4\omega_2$, and so forth. These effects become problematic in both RF transmitters and RF receivers.

Problem of LO Harmonics in Transmitters

Suppose we wish to transmit a low-frequency (baseband) signal $x_1(t)$. A mixer must multiply $x_1(t)$ by an LO waveform having a fundamental frequency of ω_{LO} so as to upconvert the spectrum of $x_1(t)$ to ω_{LO} [Figure 3(a)]. With hard switching, however, the spectrum also appears at

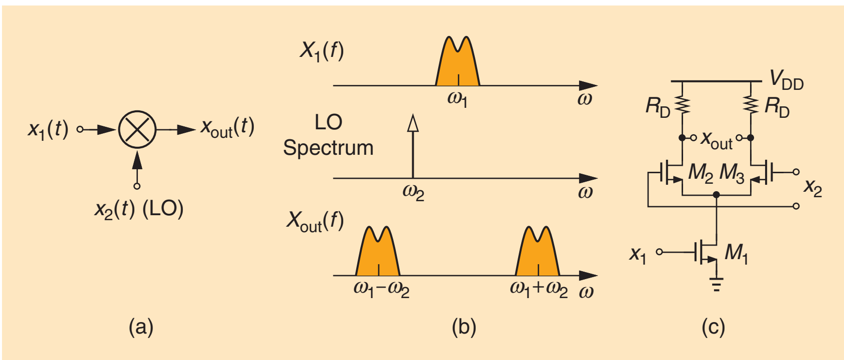

FIGURE 1: (a) The basic mixer, (b) its input and output spectra, and (c) a simple circuit realization.

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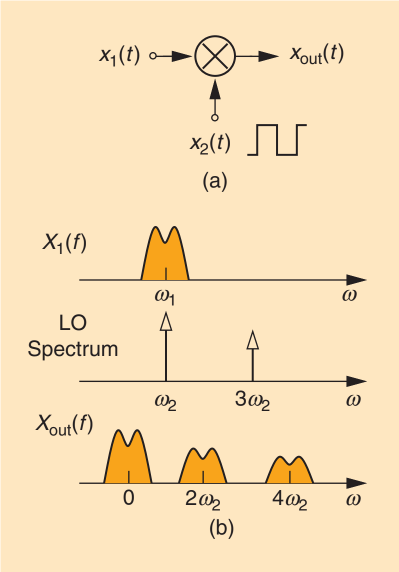

FIGURE 2: The mixer output components with a square-wave LO.

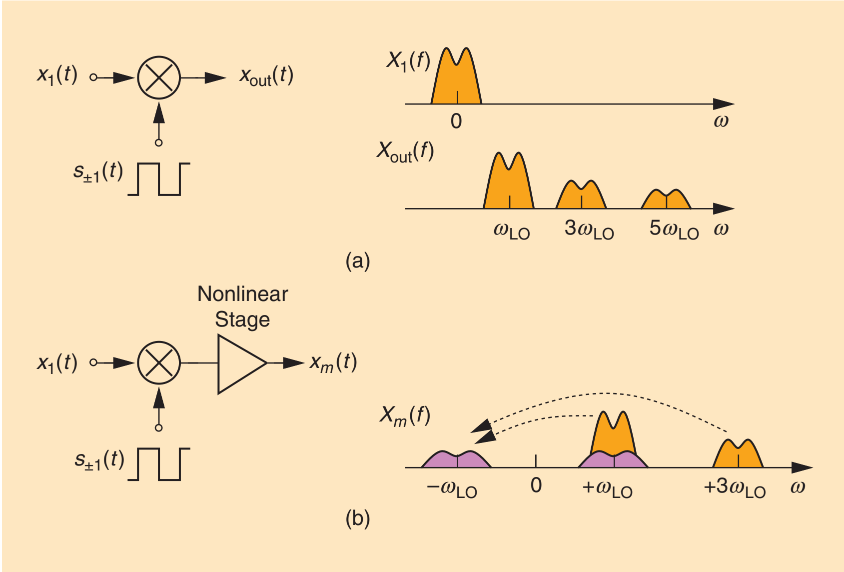

FIGURE 3: (a) The upconversion mixer in a transmitter and (b) intermodulation due to amplifier nonlinearity.

$3\omega_{LO}, 5\omega_{LO}$, and so on. The additional components appear benign because they lie far from the desired signal frequency and can be filtered as they travel through the transmitter. But if the stage following the mixer exhibits third-order nonlinearity, the signal may be severely corrupted. Specifically, let us model this nonlinearity by $y(t) = \alpha_1 x + \alpha_3 x^3(t)$, and note that a signal of the form $x(t) = A\cos\omega_a t + B\cos\omega_b t$ yields $y(t) = (3\alpha_3/4)A^2 B\cos(2\omega_a - \omega_b)t + (3\alpha_3/4)AB^2\cos(\omega_a - 2\omega_b)t + \cdots$, where these two terms are called *intermodulation products*. Thus, upon traveling through such nonlinearity, the spectral components at ω_{LO} and $3\omega_{LO}$ in Figure 3(a) produce a new one at $2\omega_{LO} - 3\omega_{LO} = -\omega_{LO}$ [Figure 3(b)], which also lands at $+\omega_{LO}$. It can be shown that this component occupies a wider bandwidth than does the desired signal, spilling into other users' channels.

Another interesting phenomenon that can occur in the foregoing scenario was identified by [1] and, in recent years, has been called *counter intermodulation* (CIM) [5]. Suppose we wish to mix a baseband component, $x(t) = A\cos\omega_{BB}t$, with an LO. Simple multiplication, $A\cos\omega_{BB}t\cos\omega_{LO}t$, produces two upconverted sidebands, one at $\omega_{LO} - \omega_{BB}$ and another at $\omega_{LO} + \omega_{BB}$. If only one sideband is desired, we employ the single-sideband (SSB) modulator depicted in Figure 4(a), which consists of two mixers and a subtractor and operates based on the identity $A\cos\omega_{BB}t\cos\omega_{LO}t - A\sin\omega_{BB}t\sin\omega_{LO}t = A\cos(\omega_{BB} + \omega_{LO})t$. In this case, the upper sideband is retained.

Now, we consider the third harmonic of the LO. For the top and bottom mixers in Figure 4(a), the LO waveforms can be expressed as $\cos\omega_{LO}t - (1/3)\cos 3\omega_{LO}t$ and $\sin\omega_{LO}t + (1/3)\sin 3\omega_{LO}t$, respectively. It follows that $x_{out}(t)$ contains a sum of the form $-A\cos\omega_{BB}t\cos 3\omega_{LO}t - A\sin\omega_{BB}t\sin 3\omega_{LO}t = -A\cos(3\omega_{LO} - \omega_{BB})t$, i.e., this SSB mixing action translates the baseband component to the lower side of $3\omega_{LO}$ [Figure 4(b)].

Let us now subject the sidebands at $\omega_{LO} + \omega_{BB}$ and $3\omega_{LO} - \omega_{BB}$ to third-order nonlinearity. We obtain $2(\omega_{LO} + \omega_{BB}) - (3\omega_{LO} - \omega_{BB}) = -\omega_{LO} + 3\omega_{BB}$. As shown in Figure 4(c), this CIM product lands below ω_{LO}, potentially corrupting another user's channel.

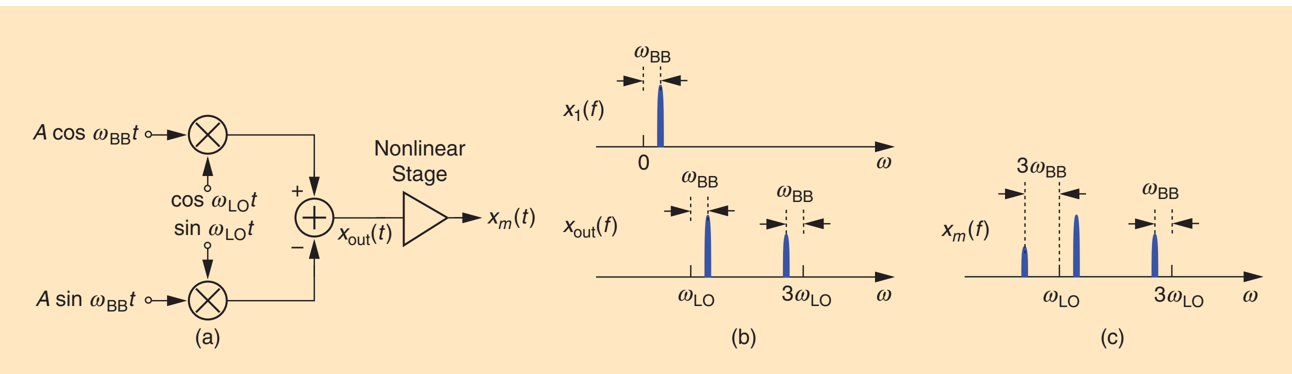

FIGURE 4: (a) A single-sideband mixer, (b) its output spectrum, and (c) counter intermodulation due to amplifier nonlinearity.

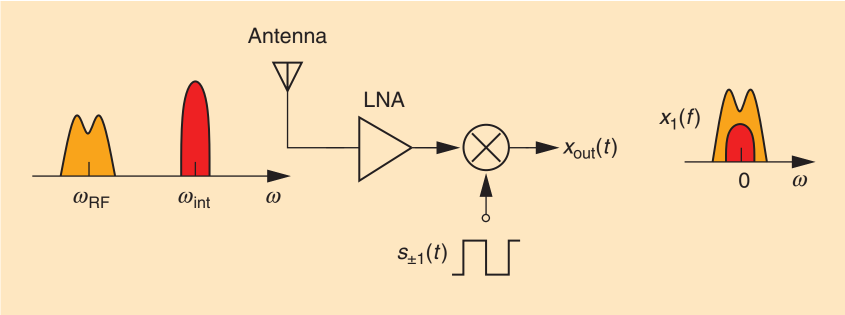

FIGURE 5: An RF receiver for illustrating the effect of LO harmonics.

To minimize the intermodulation effects studied here, we must suppress the spectral copy produced by the mixers at $3\omega_{LO}$ before it experiences the subsequent nonlinearity. This can be accomplished by a passive on-chip filter, but transmit chains operating across multiple frequency bands (e.g., the 900-MHz band, the 2-GHz band) require a widely programmable filter response. Harmonic-rejection mixing provides an elegant solution here.

Problem of LO Harmonics in Receivers

The hard-switching action of mixers proves troublesome in receivers as well. As illustrated in Figure 5, suppose a receiver senses both a desired signal at ω_{RF} and an interferer at $\omega_{\text{int}} = 3\omega_{RF}$. Then, the third harmonic of the LO downconverts the interferer, corrupting the desired signal. We call this interferer a *harmonic blocker.*

Early receiver designs did not face the problem of harmonic blockers because they interposed a narrowband filter between the antenna and the low-noise amplifier (LNA). Modern receivers, however, support numerous frequency bands and should preferably avoid such filters.

Harmonic-Rejection Mixing

Our study thus far indicates that we prefer to utilize hard-switching mixers but we also wish to suppress the effect of the LO harmonics. Our first step toward resolving this dilemma is to recognize that the (baseband or RF) input signal can be multiplied by an approximation of a sinusoid that can still be realized by means of hard-switching mixers. Specifically, we consider multiplication by a quantized sinusoid, $x_q(t)$ [Figure 6(a)] [1], surmising that this waveform exhibits weaker harmonics than does a square wave. In other words, we wish to use a number of hard-switching mixers and combine their outputs such that the input signal is equivalently multiplied by a quantized sinusoid [Figure 6(b)]. This, in turn, requires that we express $x_q(t)$ in terms of square waves. Fortunately, this is possible. Three square waves having phase differences of 45 and 90°, satisfy this condition. Expressing three such waveforms as

$$x_{S1}(t) = \cos\omega_{LO}t - (1/3)\cos(3\omega_{LO}t) + (1/5)\cos(5\omega_{LO}t) - \cdots, \quad (1)$$

$$x_{S2}(t) = \cos(\omega_{LO}t + 45^\circ) - (1/3)\cos(3\omega_{LO}t + 135^\circ) + (1/5)\cos(5\omega_{LO}t + 225^\circ) - \cdots, \quad (2)$$

$$x_{S3}(t) = \cos(\omega_{LO}t + 90^\circ) - (1/3)\cos(3\omega_{LO}t + 270^\circ) + (1/5)\cos(5\omega_{LO}t + 450^\circ) - \cdots, \quad (3)$$

we observe that the harmonics exhibit the phasor relationships depicted in Figure 6(c). It is, therefore, possible to cancel the higher harmonics if the phasor magnitudes are properly weighted. In fact, $x_{S2}(t)$ simply needs to be scaled up by a factor of $\sqrt{2}$ with respect to $x_{S1}(t)$ and $x_{S3}(t)$ so as to suppress the third and fifth harmonics [1].

But we still have not reached a solution! Recall from our earlier discussion that hard-switching mixers multiply the input by +1 and −1, values that cannot be weighted. Nevertheless, because each mixer output can be written as $x_{\text{in}}(t)S_{\pm1}(t)$, we can apply the weighting factor to either its input or its output. This point leads to two different solutions. 1) For active mixers, e.g., that in Figure 1(c), the transconductance of the input voltage-to-current

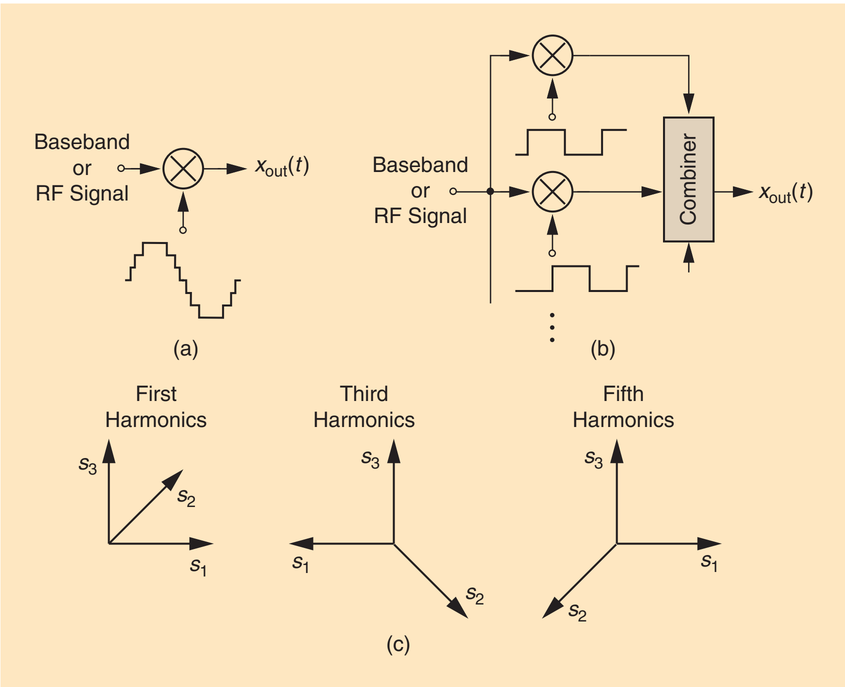

FIGURE 6: (a) A mixer driven by a quantized sinusoid, (b) a conceptual illustration of creating a quantized sinusoid by shifted copies of square-wave LO, and (c) phasor diagrams for LO harmonics.

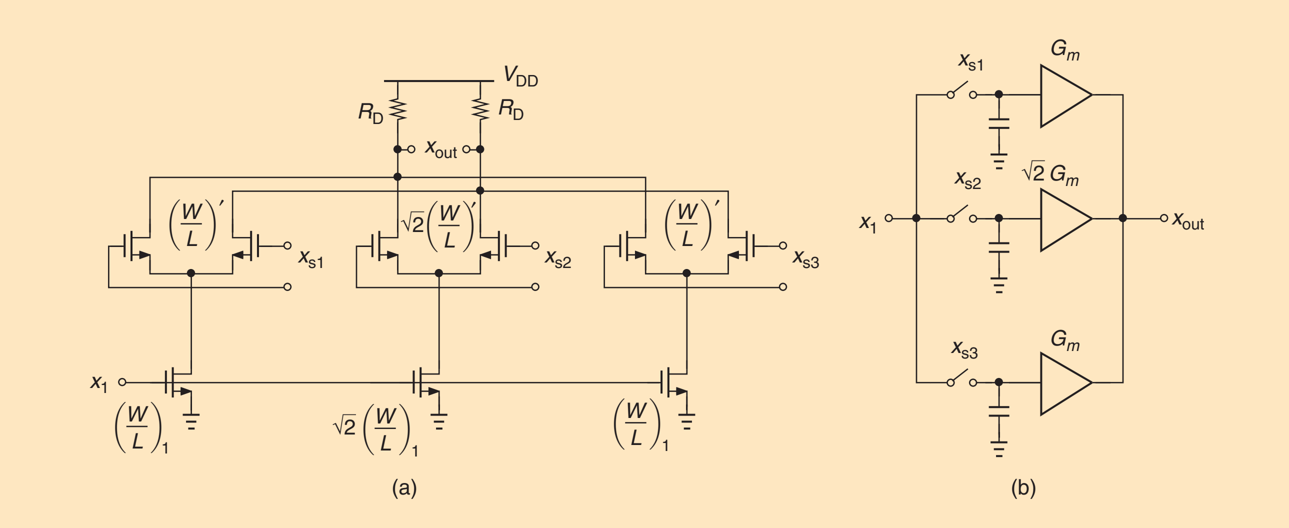

FIGURE 7: (a) The active and (b) passive implementations of HRMs.

converter is scaled, and, as shown in Figure 7(a), the outputs are summed in the current domain [1]. To obtain precise matching, the switching differential pairs are also scaled. [This topology is suited to the receive path; in the transmit path, each mixer would use a double-balanced (Gilbert cell) topology.] 2) For passive mixers, the weighting factor can be applied to the outputs, as exemplified by the voltage-sampling design shown in Figure 7(b). It is possible to employ both techniques to achieve a greater rejection [6].

The foregoing HRMs suppress the third and fifth LO harmonics but not the seventh and some others. We can assume that the transmit or receive path incorporates some filtering to attenuate higher harmonics. Alternatively, we can employ a greater number of LO phases to create a better approximation of the sinusoidal LO, but at the cost of higher complexity and power dissipation.

HRMs can also lower the noise of wide-band receivers. To see this point, we first observe that the single hard-switching mixer in Figure 8 downconverts to the baseband the noise produced by the antenna and the LNA at all of the LO harmonics. By contrast, an HRM removes this noise at $3\omega_{LO}$ and $5\omega_{LO}$.

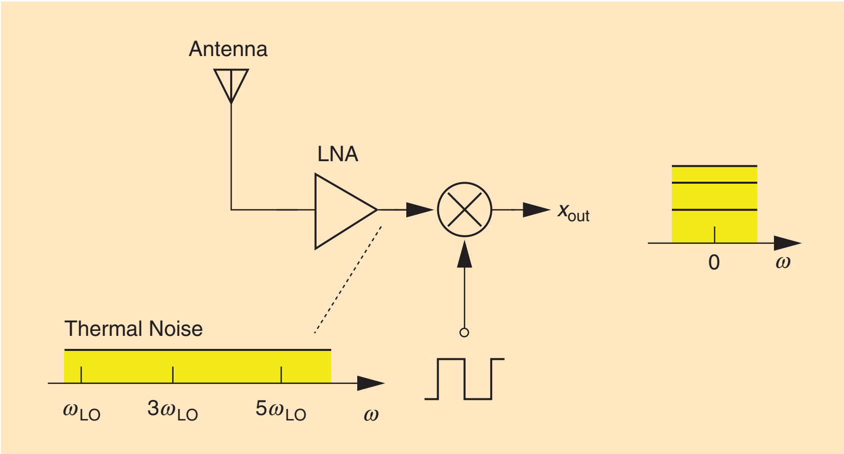

FIGURE 8: A white noise downconversion with a square-wave LO.

Design Considerations

The design of HRMs must deal with a number of issues. First, the amount of harmonic rejection depends on the matching among the three signal and LO paths in Figure 7(a) or (b). With typical random mismatches, the rejection is about 30–35 dB (excluding the amplitude factors of 1/3 and 1/5 for the third and fifth harmonics, respectively). Greater attenuations can be obtained through the use of calibration [7].

Second, the additional mixers raise both the power consumption and the capacitances presented at the inputs. From another perspective, if we design the topologies in Figures 1(c) and 7(a) for the same total bias current and transistor widths, the latter suffers from a lower gain and higher noise. This is because the first-harmonic phasors in Figure 6(c) do not add up in phase, i.e., some of the signal is wasted.

The third issue in HRM design is the need for LO phases with 45° separation. A common approach is to begin with a frequency four times the desired value and divide it by four using the network shown in Figure 9(a). Here, Latch 1 and Latch 2 form a master–slave flipflop, as do Latch 3 and Latch 4. Thus, the output of Latch 2 changes $T_{CK}/8$ seconds after the output of Latch 1 does [Figure 9(b)].

This approach can consume substantial power at high LO frequencies. The divider network of Figure 9(a), must operate with an input frequency of 20 GHz in a 5-GHz Wi-Fi transceiver. Although there are no significant blockers at the third LO harmonic (around 15 GHz), the fifth-generation (5-G) cellular radio standard allows communication around 25 GHz.

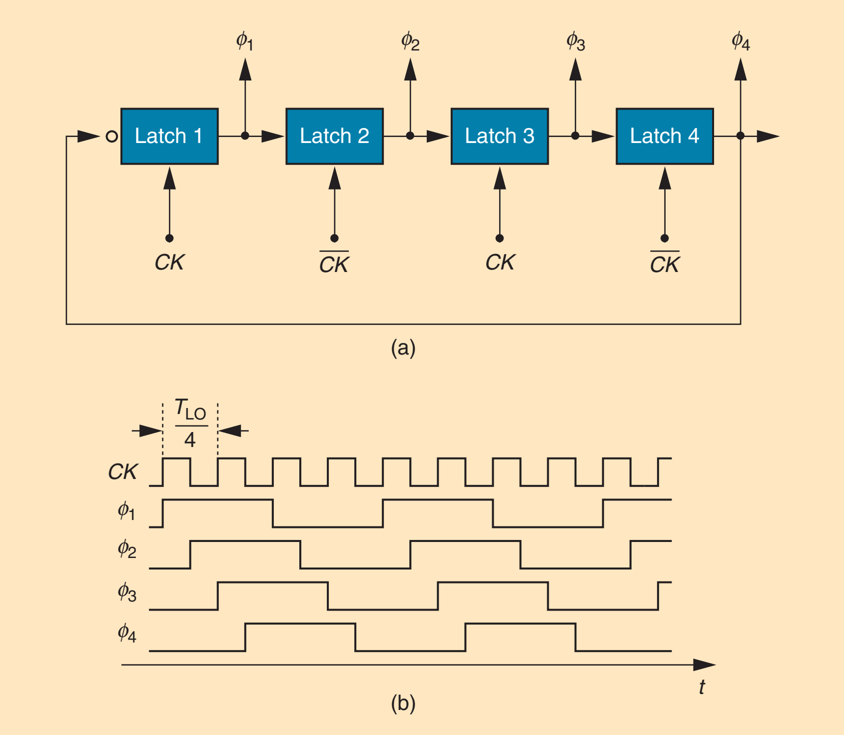

FIGURE 9: (a) A divider network generating 45° phases and (b) the circuit's waveforms.

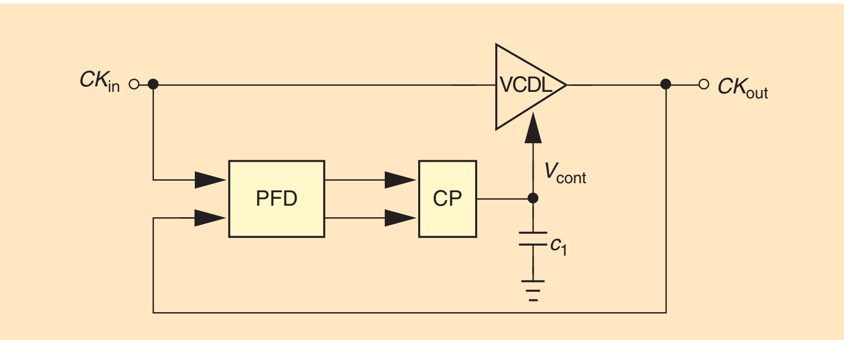

FIGURE 10: The basic DLL.

Thus, the rejection of the fifth LO harmonic becomes critical in a Wi-Fi radio that must coexist with 5-G cell phones.

Another interesting issue is that ϕ_1–ϕ_4 see different load capacitances and hence suffer from skews. This arises because, as depicted in Figure 7(a), the switching transistors driven by $x_{S2}(\equiv \phi_2)$ are wider than those driven by $x_{S1}(\equiv \phi_1)$ and by $x_{S3}(\equiv \phi_3)$. This disparity can be alleviated through the use of dummy transistors to equalize the load capacitances.

Questions for the Reader

1) How does the CIM result in Figure 4(c) change if the lower mixer's output is added to the top mixer's?
2) Calculate the gain of the mixer shown in Figure 7(a) with the aid of the first harmonic phasors shown in Figure 6(c).

Answers to Last Issue's Questions

1) Suppose the up and down currents in the charge pump of Figure 10 have a mismatch of ΔI. How does the delay-locked loop (DLL) react to this mismatch?

   If the current difference continues to flow through C_1, V_{cont} tends to infinity. The DLL therefore creates a constant phase offset between CK_{in} and CK_{out} so that the charge pump delivers a zero net charge in each phase comparison instant.

2) The CP imperfections in Figure 10 create a periodic ripple in V_{cont}. What is the effect of this ripple on the output waveform?

   Because the ripple occurs at the same rate as the output frequency, it does not produce any spurious components. Nevertheless, the ripple momentarily changes the delay, causing a small phase offset between CK_{in} and CK_{out}.

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