

## The optimum plasma density for plasma wakefield excitation in the blowout regime

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**Abstract.** The optimum plasma density for achieving the largest wakefield accelerating gradient in a plasma wakefield accelerator (PWFA) for a given electron beam driver parameters (fixed charge, spot size and duration) is analyzed. It is found that the peak beam current  $I_p$  (charge per unit time) plays an important role in determining the optimum density. We show that for narrow beams of low peak current ( $I_p \ll I_A \approx 17$  kA and  $\sigma_r \ll \sigma_z$ ), the prediction from linear theory (Lu *et al* 2005 *Phys. Plasma* **12** 063101) that  $k_p \sigma_z = \sqrt{2}$  or  $n_p$  ( $\text{cm}^{-3}$ )  $\approx 5.6 \times 10^{19} / \sigma_z^2$  ( $\mu\text{m}$ ) for a bi-Gaussian bunch of length  $\sigma_z$  and spot size  $\sigma_r$ , works well for obtaining the maximum accelerating gradient. However, for narrow beams of high peak current ( $I_p \gtrsim I_A$  and  $\sigma_r \ll \sigma_z$ ), the optimum density can be an order of magnitude larger than that predicted by linear theory. In this regime, we show that a new condition  $n_p \sim n_{b0}$  should be used for  $1 \lesssim \sigma_z / \sigma_r \lesssim 10$ , where  $n_{b0}$  is the peak beam density. Theoretical arguments for this new condition are given and the predictions are confirmed by particle-in-cell (PIC) simulations.

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Recently, the field of plasma wakefield acceleration (PWFA) [1] has made amazing progress. In a series of PWFA experiments performed at SLAC, energy gains in the 1–50 GeV range have been demonstrated [2]–[5]. The most astounding result is the energy doubling of the 42 GeV electrons in less than 1 m. This is significant because it is the first time that ultrahigh gradient ( $\sim 50 \text{ GeV m}^{-1}$ ) acceleration was sustained over meter long distances, thereby providing energy gains of interests to high-energy physics [5]. In a typical PWFA experiment, a high gradient ( $\sim 10 \text{ GeV m}^{-1}$ ) multi-dimensional plasma wave wake with a phase velocity near the speed of light is excited by the passage of an intense ultra-short, ultra-relativistic, particle beam. A simple but fundamental question regarding this wake excitation process is as follows: given the available electron beam parameters (e.g. the total charge  $eN$ , the spot size  $\sigma_r$ , the pulse length  $\sigma_z$ , the electron energy  $\gamma$  and the normalized emittance  $\epsilon_n$ ), what is the optimum plasma density to obtain the largest wakefield and is there a simple expression for it? Due to the highly nonlinear and multi-dimensional nature of this problem, the answers are normally obtained empirically via laboratory experiments or particle-in-cell (PIC) simulations, both of which are expensive and time consuming. Therefore, a theory that takes into account all the key physical effects is needed, because it can help to identify the parameter space for carrying out experiments and simulations to precisely answer the question. In this paper, we will present a simple answer to this basic question by utilizing the physical insight obtained from a recently developed nonlinear theoretical framework of the blowout regime [6, 7]. The results work for parameters of recent and planned PWFA scenarios.

Linear theory provides a useful starting point for determining the optimum density. In linear theory, the density compressions and rarefactions of the wave are assumed to be small compared to the background density  $n_p$  and a rough condition for this is that the beam density  $n_b$  is less than  $n_p$ . The limitations and usefulness of the linear theory can be found in [8]. We start from an expression in [8] for the absolute wakefield amplitude  $E_{z0}$  (not normalized,  $E_{\text{norm}} = eE_{z0}/mc\omega_p$ ) for a bi-Gaussian drive bunch

$$E_{z0} = \sqrt{2\pi} \frac{mc^2}{e} \frac{n_{b0}}{n_p} k_p^2 \sigma_z e^{-k_p^2 \sigma_z^2 / 2} R(0), \quad (1)$$

where  $R(0) = (k_p^2 \sigma_r^2 / 2) e^{k_p^2 \sigma_r^2 / 2} \Gamma(k_p^2 \sigma_r^2 / 2)$ ,  $\Gamma(y) = \int_0^\infty t^{-1} e^{-t} dt$  and  $n_b = n_{b0} \exp[-(z^2 / 2\sigma_z^2) - (r^2 / 2\sigma_r^2)]$ .

Next, we find the plasma density that maximizes the absolute wakefield amplitude. At this point, it is important to be clear about what is being held fixed. For example, in the 1D or wide beam limit ( $k_p \sigma_r \gg 1$ ), if we assume that  $n_{b0}/n_p$  and  $\sigma_z$  are fixed, the absolute wakefield  $E_{z0}$  is maximized for  $k_p \sigma_z = \sqrt{2}$  or  $n_o \text{ (cm}^{-3}\text{)} = 5.6 \times 10^{19} / \sigma_z^2 \text{ (}\mu\text{m)}$ . On the other hand (as shown below), if we assume that the beam charge  $eN$  as well as  $\sigma_r$  and  $\sigma_z$  remain fixed, then for  $\sigma_r \gg \sigma_z$  the absolute wakefield is instead maximized for  $k_p \sqrt{\sigma_r \sigma_z} = \sqrt{2}$  or equivalently  $k_p \sigma_z = \sqrt{2} \sqrt{\sigma_z / \sigma_r} \ll \sqrt{2}$ .

For the rest of this paper, we will assume the beam parameters are fixed and only the plasma density varies. This is usually the situation of relevance when designing an accelerator. For this situation it is useful to explicitly write out  $n_{b0}$  as  $n_{b0} = N / [(2\pi)^{3/2} \sigma_r^2 \sigma_z]$ . Equation (1) now becomes

$$E_{z0} = e k_p^2 N e^{-(k_p^2 / 2) [\sigma_z^2 - \sigma_r^2]} \Gamma(k_p^2 \sigma_r^2 / 2). \quad (2)$$

In [8] it was shown that when optimizing  $E_{z0}$  as a function of  $k_p^2$ , i.e.  $n_p$ , the optimized wakefield amplitude  $E_{zM}$  can be written as

$$E_{zM} = \frac{eN}{\sigma_r \sigma_z} \Theta \left( \frac{\sigma_r}{\sigma_z} \right) \quad (3)$$

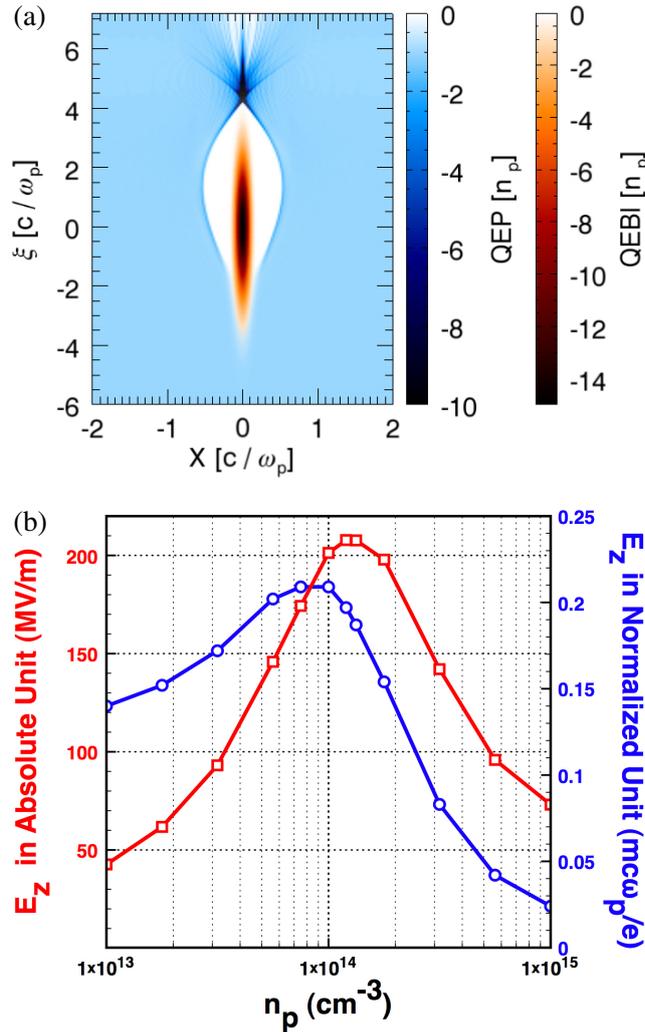
and the universal function  $\Theta$  was plotted in figure 2 of [8]. It is straightforward to obtain asymptotic expressions for  $k_p$  that optimizes  $E_{z0}$  in the  $\sigma_r/\sigma_z \ll 1$  and  $\sigma_r/\sigma_z \gg 1$  limits. When  $\sigma_r \ll \sigma_z$ , we call the bunch shape a cigar. In the opposite limit where  $\sigma_r \gg \sigma_z$  we call the bunch shape a pancake. In the cigar limit, one can obtain  $k_p \sigma_z \approx \sqrt{2} [1 - (1/\ln(2/k_p^2 \sigma_r^2))]^{1/2}$  which is essentially  $k_p \sigma_z = \sqrt{2}$  for  $k_p \sigma_r \ll 1$ . Interestingly although the limits are very different, for both the 1D limit ( $k_p \sigma_r \gg 1$ ) when  $n_{b0}/n_p$  and  $\sigma_z$  are held fixed and the narrow ( $\sigma_r \ll \sigma_z$ ) 3D limit where  $N$ ,  $\sigma_r$  and  $\sigma_z$  are held fixed, the optimum density is  $n_o$  ( $\text{cm}^{-3}$ ) =  $5.6 \times 10^{19}/\sigma_z^2$  ( $\mu\text{m}$ ). In the pancake limit, where  $k_p^2 \sigma_r^2/2 \gg 1$  and  $\Gamma(y) \sim ((1/y) + (1/y^2) + \dots)e^{-y}$ , one can easily show that the density which optimizes  $E_{z0}$  corresponds to  $k_p = \sqrt{2}/\sqrt{\sigma_r \sigma_z}$  or  $k_p \sigma_z = \sqrt{2} \sqrt{\sigma_z/\sigma_r} \ll \sqrt{2}$ . To determine the optimum density for beams with  $\sigma_r \simeq \sigma_z$ , one can refer to figure 2 in [8], where for a round beam  $k_p \sigma_z \simeq 0.9$ .

While linear theory provides a useful starting point, its assumptions fail for most current and planned PWFA experiments. As shown in [6]–[8], even if the assumptions fail the expressions for the wake amplitude can still be useful. The figure of merit to determine if the results from linear theory are valid is the normalized charge per unit length of the beam,  $\Lambda \equiv \int_0^\infty k_p r (n_{b0}/n_p) dk_p r = 2I_p/I_A$ , where  $I_p$  is the peak current of the beam and  $I_A \simeq 17$  kA is the Alfvén current. For beams with a bi-Gaussian profile,  $\Lambda = (n_{b0}/n_p) k_p^2 \sigma_r^2 = \sqrt{(2/\pi)} [r_e/\sigma_z] N$  ( $r_e = e^2/mc^2$  is the classical electron radius), or in physical units,  $\Lambda \approx 2.25(N/(2 \times 10^{10}))(20 \mu\text{m}/\sigma_z)$ . For  $k_p \sigma_r < \sqrt{\Lambda}$  i.e.  $n_{b0}/n_p > 1$ , the wakes are excited in a highly nonlinear multi-dimensional regime (the blowout regime) for which linear theory is invalid.

To illustrate the usefulness and the limitation of the linear prediction, we show two examples based on the parameters of PWFA experiments done at SLAC. In one of the examples, although  $n_b/n_p \gg 1$  so linear theory should not apply, the quantity  $\Lambda \ll 1$  and the prediction from linear theory remains useful. For the other example,  $\Lambda \gtrsim 1$  and the prediction for the optimum density from linear theory is actually off by a factor of ten.

At SLAC, the electron bunches typically have a total number of electrons  $N \approx 2 \times 10^{10}$ , or contain  $\sim 3$  nC of charge. The beams can be focused down to a few tens of microns by beam focusing optics. The pulse duration can be varied from ps down to 50 fs through a three-stage bunch compression method. The first example corresponds to the E157 experiment, where  $N = 1.9 \times 10^{10}$ ,  $\sigma_z = 700 \mu\text{m}$ ,  $\sigma_r = 30 \mu\text{m}$  and  $n_{b0} = 1.9 \times 10^{15} \text{cm}^{-3}$ . For these parameters  $\Lambda \approx 0.06$  and the linear theory predicts the optimum plasma density  $n_p \approx 1.15 \times 10^{14} \text{cm}^{-3}$ . For this density, the peak beam density  $n_{b0}$  is about 17 times higher than the plasma density, which suggests that the nonlinear blowout regime is indeed reached. This can be easily seen in figure 1(a) where the beam and plasma densities are plotted for  $n_p = 1.15 \times 10^{14} \text{cm}^{-3}$ .

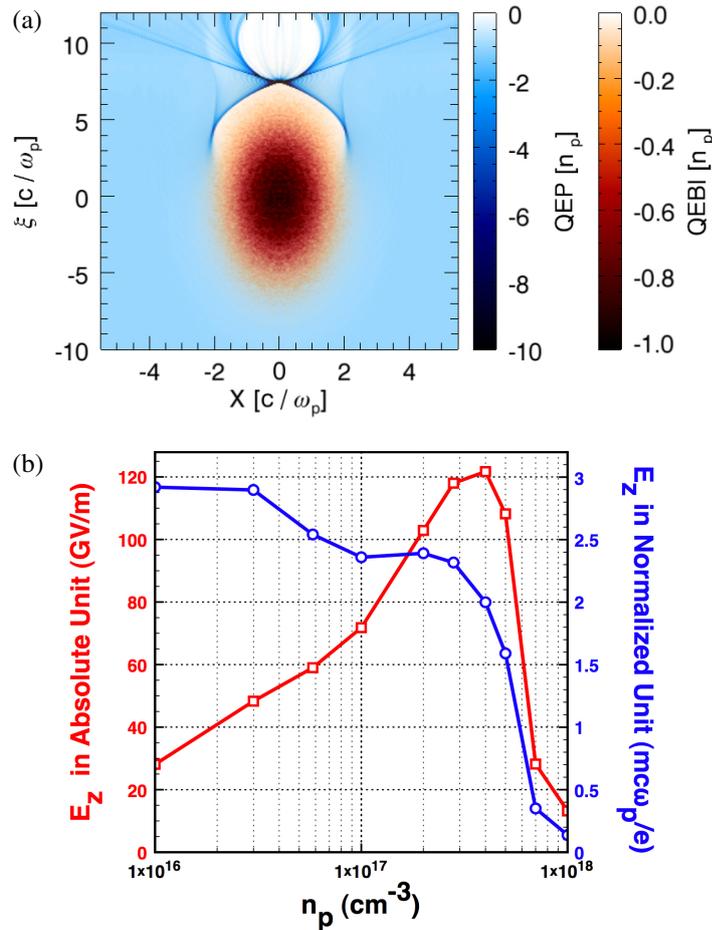
The second example is from the E164 experiment, where the bunch length was significantly reduced through compression. For typical E164 parameters,  $N = 1.8 \times 10^{10}$ ,  $\sigma_z = 32 \mu\text{m}$ ,  $\sigma_r = 10 \mu\text{m}$  and  $n_{b0} = 3.6 \times 10^{17} \text{cm}^{-3}$ . For these parameters,  $\Lambda \approx 1.27$  and the linear theory predicts the optimum plasma density  $n_p \approx 3.5 \times 10^{16} \text{cm}^{-3}$ . We note that for these parameters the asymptotic expression for the optimum density for  $\sigma_r/\sigma_z \ll 1$  is no longer applicable since  $\sigma_r/\sigma_z \sim 0.3$ . Therefore, we use a value from figure 2 in [8] where one sees that  $k_p \sigma_z \approx 1.13$  is less than  $k_p \sigma_z = \sqrt{2}$  consistent with the asymptotic formula.



**Figure 1.** (a) The beam and plasma density distributions at the optimum density  $n_o = 1.15 \times 10^{14} \text{ cm}^{-3}$  for a bi-Gaussian electron beam with  $N = 1.9 \times 10^{10}$ ,  $\sigma_z = 700 \mu\text{m}$  and  $\sigma_r = 30 \mu\text{m}$ . (b) The normalized and absolute useful wakefield amplitudes for different plasma densities.

To find out how well the linear theory predictions work, we performed full-scale 3D PIC simulations with code QuickPIC [9] by scanning the plasma density for the above two examples. The results are shown in figures 1(b) and 2(b) where we plot both the absolute ( $\text{GV m}^{-1}$ ) and the normalized ( $mc\omega_p/e$ ) useful wakefield amplitude against the plasma density. For the first example (figure 1), the density range scanned is from  $10^{13}$  to  $10^{15} \text{ cm}^{-3}$ . One can see clearly for this weakly nonlinear case the plasma density that maximizes the absolute wakefield amplitude is very close to the linear theory prediction,  $n_p = 1.15 \times 10^{14} \text{ cm}^{-3}$ .

For the second example (figure 2), the density range scanned is from  $10^{16}$  to  $10^{18} \text{ cm}^{-3}$ . For this more strongly nonlinear example, the absolute maximum amplitude occurs at a density near  $4 \times 10^{17} \text{ cm}^{-3}$ , which is surprisingly more than ten times larger than the linear theory prediction  $3.5 \times 10^{16} \text{ cm}^{-3}$ .



**Figure 2.** (a) The beam and plasma density distributions at the optimum density  $n_o = 4 \times 10^{17} \text{ cm}^{-3}$  for a bi-Gaussian electron beam with  $N = 1.8 \times 10^{10}$ ,  $\sigma_z = 32 \mu\text{m}$  and  $\sigma_r = 10 \mu\text{m}$ . (b) The normalized and absolute useful wakefield amplitudes for different plasma densities.

The above two examples clearly demonstrate the usefulness and limitation of the linear theory predictions regarding to the optimum plasma density for wakefield amplitude. The question then is how can we understand these results? It turns out that a clear understanding and a simple estimation can be obtained based on physical intuition from the nonlinear wakefield theory in [6, 7].

In the nonlinear framework of the blowout regime [6, 7], the plasma response to the electron beam driver can be divided into three distinct regions, namely the ion channel, the narrow electron sheath and the linear response region beyond the sheath. The key parameter for identifying the different regimes is the normalized blowout radius  $k_p R_b$ . In the non-relativistic blowout regime where  $k_p R_b \ll 1$ , the contribution from the linear response region dominates the wakefield structure; therefore, a formula similar to the linear theory expression can be used even though blowout occurs. In the regime where  $k_p R_b \sim 1$ , the contribution from both the ion channel and the linear response region is important. In the relativistic blowout regime where  $k_p R_b \gtrsim 2$ , the ion channel dominates the contribution. The normalized blowout radius  $k_p R_b$  is

mainly determined by the beam peak current  $I_p$  (charge per unit time) and an approximate formula for a narrow drive beam ( $\sigma_r < R_b$ ) is [7]

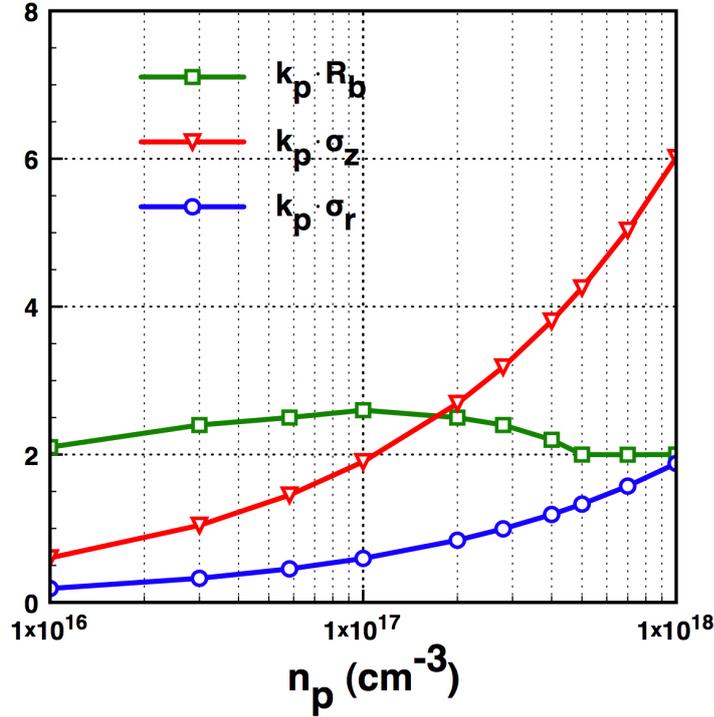
$$k_p R_b = 2\sqrt{\Lambda} = 2\sqrt{2I_p/I_A}. \quad (4)$$

This formula has been verified in self-consistent simulations and it can be argued by equating the wakefield forces on an electron moving backward in the sheath with the peak space charge force of the beam [7]. As shown in [7], the normalized wakefield amplitude scales as  $\Lambda \ln(1/\Lambda)$  in the non-relativistic blowout regime ( $\Lambda \ll 1$ ), which is similar to the prediction of linear theory. While in the relativistic blowout regime ( $\Lambda \gtrsim 1$ ) the normalized wakefield amplitude scales as  $\sqrt{\Lambda}$ . For the given total beam charge  $eN$ ,  $\Lambda$  is independent of plasma density  $n_p$  and the spot size  $\sigma_r$  and only depends on the pulse length  $\sigma_z$ . However, for equation (4) to be meaningful in the blowout regime, there are indeed two implicit conditions: the first one is that the beam should not be much shorter than a plasma skin depth  $k_p^{-1}$ , otherwise the plasma blowout will not occur within the bunch and the plasma electrons simply receive an impulse from beam, in which case the blowout radius will be determined by the total charge in the bunch other than the peak current [7]; the other condition is that the beam density  $n_{b0}$  should be comparable or larger than the ambient plasma density  $n_p$ , such that the condition for trajectory crossing and blowout is satisfied. The first condition implies a lower limit on the plasma density, e.g.  $k_p\sigma_z \gtrsim 0.2$  and the second implies an upper limit on the plasma density, e.g.  $n_p \lesssim n_{b0}$  or  $k_p\sigma_r \lesssim \sqrt{\Lambda}$ .

We are now in a position to determine the optimum density in the blowout regime for either the non-relativistic ( $\Lambda \ll 1$ ) and relativistic ( $\Lambda \gtrsim 1$ ) regimes for cigar-shaped beams ( $\sigma_r \ll \sigma_z$ ). We fix the beam parameters ( $N$ ,  $\sigma_r$ ,  $\sigma_z$ ) and then gradually increase the density. We start at a sufficiently low density, such that the first condition in the previous paragraph is not met (e.g.  $k_p\sigma_z \lesssim 0.2$ ). In this regime, the meaningful parameter for determining the blowout radius and wakefield amplitude is the normalized total charge  $Q \equiv \Lambda k_p\sigma_z$  because only the total impulse from the beam matters ( $k_p R_b \propto \sqrt{Q}$ ) [7].  $Q$  increases as the plasma density increases from zero to the lower limit; therefore, for both the non-relativistic and relativistic blowout regimes in this density range, both the normalized and the absolute wakefield amplitudes increase as the density increases.

As the density is increased further into an intermediate range (where  $k_p\sigma_z \gtrsim 0.2$  and  $k_p\sigma_r \lesssim \sqrt{\Lambda}$ , assuming  $\sigma_r < \sigma_z$ ), we need to treat the non-relativistic and relativistic blowout regimes differently. In the non-relativistic blowout regime, the wake amplitude is roughly determined from the linear theory so the optimum density is about  $k_p\sigma_z = \sqrt{2}$ . On the other hand, for the relativistic blowout regime, the normalized wakefield amplitude is determined by the normalized blowout radius  $k_p R_b$ , which for this density range is approximately given by  $k_p R_b \approx 2\sqrt{\Lambda}$ . Since  $\Lambda$  does not depend on the plasma density, the normalized wakefield amplitude is insensitive to the density. Therefore, the absolute wakefield amplitude will increase with the plasma density in this density range, implying we should continue to increase the density.

For the relativistic blowout regime ( $\Lambda \gtrsim 1$ ), as the density is increased further it eventually exceeds the upper limit (e.g.  $n_{b0}/n_p \lesssim 1$  or  $k_p\sigma_r \gtrsim \sqrt{\Lambda}$ ). At this point the wake is now marginally excited in the blowout regime and the linear expression becomes valid again. Therefore, if  $k_p\sigma_z < \sqrt{2}$ , then the density can be increased further to increase the amplitude until  $k_p\sigma_z = \sqrt{2}$ . However, for cigar-shaped beams if  $k_p\sigma_r \gtrsim \sqrt{\Lambda}$ , then  $k_p\sigma_z \gg \sqrt{2}$ . As a result the wake amplitude will decrease as the density is increased further. We therefore conclude that the maximum wakefield amplitude is reached near  $n_{b0}/n_p \sim 1$ . For  $\Lambda \gtrsim 1$  and  $n_{b0} \sim n_p$ , the



**Figure 3.** The normalized beam parameters  $k_p \sigma_r$ ,  $k_p \sigma_z$  and the normalized blowout radius  $k_p R_b$  for a bi-Gaussian beam with  $N = 1.8 \times 10^{10}$ ,  $\sigma_z = 32 \mu\text{m}$  and  $\sigma_r = 10 \mu\text{m}$ .

normalized beam spot size is  $k_p \sigma_r = \sqrt{\Lambda} \sim k_p R_b / 2$ . Therefore, the absolute wake is maximized when the spot size is roughly matched to the blowout radius (for example, one can see this from the beam and plasma density plot in figure 2(a)).

We note that for the above analysis to be valid, an additional condition,  $\sigma_z \lesssim 5 R_b \approx 10 \sigma_r$ , should be imposed, otherwise the beam plasma interaction will be in the adiabatic blowout regime where the ion channel is balanced by the electric and magnetic force of the beam and the wake amplitude scales as  $\sim \sqrt{\Lambda} / k_p \sigma_z$ . A detailed analysis will be presented in another publication.

To see the validity of the above analysis and reasoning, we revisit the two examples given earlier in this paper. In the first example, the electron pulse is relatively long ( $\sigma_z = 700 \mu\text{m}$ ) and it has  $\Lambda = 0.06$ ; therefore, it is within the non-relativistic blowout regime and the optimum plasma density should be close to the linear theory prediction. This was confirmed in figure 1(b) as mentioned earlier. For the second example, the electron pulse is much shorter ( $\sigma_z = 32 \mu\text{m}$ ) and it has  $\Lambda = 1.27$  and  $n_{b0} = 3.6 \times 10^{17} \text{cm}^{-3}$ ; therefore, it is in the relativistic blowout regime and an optimum plasma density close to the beam peak density  $n_{b0}$  should be expected. The PIC simulation results shown in figure 2(b) confirmed this: the trend of the absolute wake amplitude increasing with density and the optimum density,  $n_p \approx 4 \times 10^{17} \text{cm}^{-3}$ , is close to the prediction of  $3.6 \times 10^{17} \text{cm}^{-3}$ . In light of the factor of ten difference of the plasma density between the linear and nonlinear predictions, this agreement clearly shows the usefulness of the above reasoning. In the above we argued that  $k_p R_b$  is very insensitive to the density for fixed  $\Lambda$ . To show this we also plot the normalized blowout radius against plasma density for the simulations in figure 2(b). In figure 3, it is evident that the normalized blowout radius  $k_p R_b$

changes very little for plasma densities spanning two order of magnitude. The average  $k_p R_b$  is around 2.2 and is close to the theoretical estimate of  $2\sqrt{\Lambda} \approx 2.25$ . We note that for  $n_p > n_{b0}$  in figure 3, the blowout radius is not clearly defined and it is roughly deduced from the density perturbation in the simulations.

In conclusion, we have analyzed the optimum plasma density for achieving the largest accelerating gradient in an electron beam driven PWFA. It was found that the peak beam current  $I_p$  (charge per unit time) plays an important role in determining the optimum density. A single parameter  $\Lambda$  ( $\Lambda \equiv \int_0^\infty k_p r (n_b/n_p) dk_p r = 2I_p/I_A$ ) can be used to identify the different regimes, namely the non-relativistic blowout regime ( $\Lambda \ll 1$ ) and the relativistic blowout regime ( $\Lambda \gtrsim 1$ ). For low current beams ( $I_p \ll I_A \approx 17$  kA or  $\Lambda \ll 1$ ), the linear theory prediction for a cigar-shaped beam ( $k_p \sigma_z = \sqrt{2}$  for  $\sigma_r \ll \sigma_z$  [8]) provides a good estimation. But for high peak current cigar-shaped beams, such as  $I_p \gtrsim I_A/2$  or  $\Lambda \gtrsim 1$ , the optimum plasma density can be order of magnitude larger than the linear theory prediction. The major reason for this difference is that the wakefield amplitude in the relativistic blowout regime has a very different scaling than that of the linear regime and the non-relativistic blowout regime. The nonlinear analysis in this paper suggests that the optimum plasma density for this case is roughly equal to the peak beam density  $n_{b0}$  so long as  $1 \gtrsim \sigma_r/\sigma_z \gtrsim .1$  and this prediction was confirmed by full 3D PIC simulations.

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