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A Hairy Scheme for Electrostatic Stabilization

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ABSTRACT

To suppress oscillations in plasma potential in a stellarator one might use a system of thin dynamic probes, resembling hairs, which alternately sample the plasma potential and inject a charge to correct it. We find that if only the lowest azimuthal modes are dangerous, it may be possible to increase the confinement time an order of magnitude, but not more. The problem is that if the probes are made thin enough so that they do not drain the plasma, then they will burn up.

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1. INTRODUCTION

There are several schemes one can try at the present juncture to try to slow down pumpout and lengthen the confinement time τ in Model C. For instance one can increase the aperture, or increase the conductivity by catastrophic heating of the electrons, or make further changes in the magnetic topology. None of these schemes is likely to increase τ by more than an order of magnitude, if any improvement at all can be seen. The scheme proposed herein is also not likely to succeed, but we think its chances are no worse than those of other proposals. Its merit is that it is far easier and cheaper to try than most other methods.

Our proposed scheme is based on certain preconceived ideas about how pumpout works. We do not attempt to justify these preconceptions but only describe them, as follows. Low-frequency electrostatic oscillations arise from various instabilities. The fluctuating electric field $\mathbf{E}$ lies mainly in the $\underline{\theta}$ direction. E_θ causes ions and electrons to oscillate in the $\underline{r}$ direction because of the $\mathbf{E} \times \mathbf{B}$ drift. Those particles near the edge drift out of the aperture during part of the cycle and are lost to the limiter. The apparently constant amplitude of the oscillations is due to a balance between the scraping off of plasma and the growth of the oscillation, feeding on the density gradient in the interior. The lowest azimuthal modes (small m-numbers) have the largest amplitudes but the smallest growth rates. These small-m modes may attain their large nonlinear amplitude by growing all the way from the linear regime, or they may be fed from the fast-growing large-m modes by the coalesence of eddies (a phenomenon occurring in two-dimensional turbulence).

Since fluctuating electric fields play a much larger role in pumpout than the fluctuating magnetic fields, we seek to eliminate the electric fields directly. Clearly, it would be very difficult to short out oscillations of all wavelengths, but we may be able to short out the lowest-m modes. These modes are the most effective in causing pumpout because (a) they have large amplitudes; (b) they have lower frequency, so that the drift excursions are large; and (c) shear has a tendency to quench the high-m modes.* (These reasons are not necessarily independent of one another.) Therefore, a significant improvement in τ may be achieved even if the high-m modes are allowed to remain. There is the distinct possibility, however, that when the low-m modes have been eliminated, the lowest remaining mode will have an amplitude much larger than it had before.

To eliminate the lowest radial and the mth azimuthal mode and all lower ones, we envision an array of 4m wires pointing radially toward the axis. This array may be repeated at several ports. Each probe samples the floating potential at a rate much faster than the frequencies of the electrostatic oscillations. If the potential deviates from the average, the probe draws or emits

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*This is not obvious but has been shown experimentally by Hartman at Livermore and theoretically by the author in MATT-385.

electrons to bring the potential on its line of force back to the average. In Sec. 2 we consider the number of probes required. In Sec. 3 we compute the probe current required. In Sec. 4 we compute that available. In Sec. 5 we compute the loss of plasma to the probes. In Sec. 6 we consider the destruction of the probes by the plasma. In Sec. 7 we sketch the electronic circuitry.

2. SOME NUMBERS AND PICTURES

For the purposes of the numerical estimates in this paper we assume a hydrogen plasma of radius $R = 5$ cm, temperature $T_i \approx T_e = 100$ eV, density $n = 10^{13}$ cm^{-3} in a magnetic field $B = 35$ kG. The relevant parameters for this plasma are given below. The fundamental rotation frequency f_o of the plasma is computed from the diamagnetic drift $v_D = -(cKT_e/eB)(n'/n)$. If the plasma rotates as a solid body, then $n(r)$ has the shape $n = n_o \exp - (r/r_o)^2$. Let $n(R) \approx 0.135\ n_o$; then $r_o{}^2 = \frac{1}{2}R^2$, and $n'(R)/n(R) = -2R/r_o{}^2 = -4/R = -0.8$ cm^{-1}. This gives $v_D = 2.3 \times 10^5$ cm/sec and $f_o = V_D/2\pi R = 7.2$ kHz. A radial electric field may change f_o a factor of 2 one way or the other, but this is the order of magnitude of f_o and is in agreement with measurements.

Cyclotron frequencies: $\omega_{ci} = 3.4 \times 10^8$, $\omega_{ce} = 6 \times 10^{11}$, $f_{ci} = 5.4 \times 10^7$,
$f_{ce} = 10^{11}$ Hz

Plasma frequencies: $\omega_{pi} = 4.2 \times 10^9$, $\omega_{pe} = 1.8 \times 10^{11}$, $f_{pi} = 6.7 \times 10^8$,
$f_{pe} = 2.9 \times 10^{10}$ Hz.

Larmor radii: $r_{Li} = 4 \times 10^{-2}$, $r_{Le} = 10^{-3}$ cm

Debye length: $\lambda_D = 2.3 \times 10^{-3}$ cm

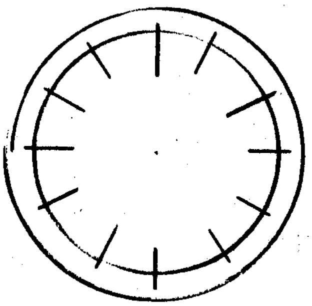
Thermal and sound speeds: $v_i \approx v_s = 10^7$, $v_e = 3 \times 10^8$ cm/sec

Drift speed: $V_D = 2.3 \times 10^5$ cm/sec

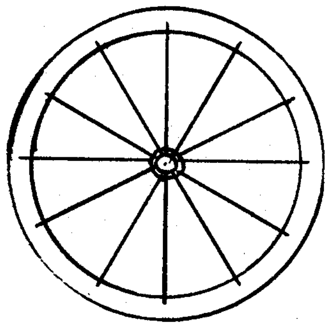
Rotation frequency: $f_o = 7.2 \times 10^3$, $\omega_o = 4.5 \times 10^4$ Hz

Probe radius and length: $r_p \approx 2.5 \times 10^{-3}$, $\ell_p \approx 2$ cm.

An array of probes at one port might look like this:



or this:



In the first arrangement the probes are self-supporting tungsten wires imbedded in a ceramic ring surrounding the plasma. In the second arrangement a small ceramic ring, perhaps 1/8" or 1/4" in diameter, holds the wires together at the center. Since the wires we shall be considering are of the order of 0.002" in diameter, the first method may not work if the wires curl up when heated by the plasma. On the other hand, accurate positioning of the wires is not necessary. The second method suffers from having a large foreign body on the axis and from having the probe potential applied near the axis, where it is neither needed nor desired.

A 12-wire array, as shown, will eliminate the $m < 6$ modes of a standing wave and will also attenuate higher-m modes of traveling waves if their correlation length is long enough. The question now is how many such arrays are needed. To determine this we note that drift waves are Landau damped if $\omega/k_{\parallel} < v_i$, so that $k_{\parallel max} = \omega/v_i \cong k_{\perp} v_D/v_i = m v_D/r v_i = 2\pi/\lambda_{\parallel min}$. This gives $\lambda_{\parallel min} \approx 1300/m$ cm with our parameters, or $\lambda_{\parallel min} \approx 200$ cm for $m = 6$. This is only a rough estimate because the density gradient and the Landau damping condition may vary by a factor of 2 or 3, probably in the direction that increases $\lambda_{\parallel min}$. Eliminating $m = 6$ would require an array every 50 cm or so. This is impractical because, as we shall see, the losses on the probes are too large. One or two arrays may be sufficient to do some good if our estimates have been too pessimistic. There is some experimental evidence that a single array will help: the oscillations outside the aperture in Model C are considerably smaller than inside when there is a single conducting aperture limiter.

3. REQUIRED PROBE CURRENT

The current that the probe must supply may be divided into three parts, aside from the displacement current, which we neglect. First is the current j_1 needed to cancel the continuously occurring charge separation that causes a drift wave to grow. Second is a current j_2 needed to drain off the space charge that exists initially if the ribbonlike tube of force intersecting the probe (T F I P) is at a potential different from that desired. Third is a polarization current j_3 (Spitzer, 2nd ed., p. 39) needed to set up the $E \times B$ drifts around the T F I P when it is given a potential different from that of the surrounding plasma.

(a) To compute j_1 we use the fact that for resistive drift waves the charge separation is caused primarily by the divergence of the ion current v_θ in the azimuthal direction [c.f. Phys. Fluids 8, 912 (1965) or MATT-306]. This charge accumulation is partly drawn off by electron flow along $\underset{\sim}{B}$, a flow depending on resistivity η; but we can get an upper limit to j_1 by neglecting the conductivity. Then we can get an answer independent of η and of the growth rate of the instability. We write the ion continuity equation as follows:

$$\partial n/\partial t = - \nabla \cdot (nv) = - i(m/r) n v_\theta + \text{ other terms,}$$

where

$$v_\theta = - i(\omega/\omega_{ci}) E_\theta / B = - (\omega/\omega_{ci})(m/r)\, \phi/B \quad .$$

Thus the relevant term in $\partial n/\partial t$ is

$$\partial n/\partial t = i\,(m/r)^2\, (\omega/\omega_{ci})\, n\, \phi/B \quad .$$

If we assume $\omega \approx m\omega_o = 2\pi m f_o$ and $\phi \lesssim KT_e/em$, where m is the mode number, we obtain

$$|\partial n/\partial t| \lesssim 10^8 (\omega_o/\omega_{ci}) (m/R)^2 (nKT_e/B) \quad ,$$

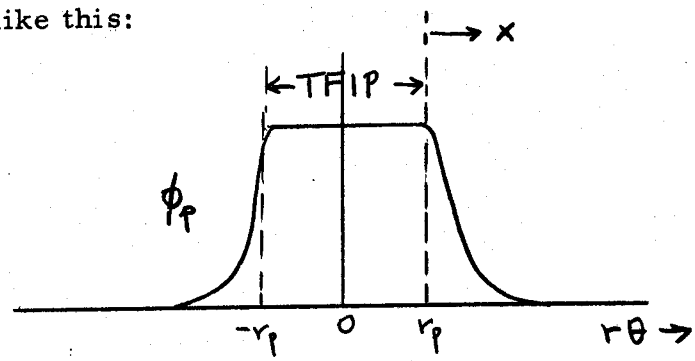
where KT_e is in eV. If L is the distance between arrays of probes, the flux into the T F I P of one probe is

$$j_1 = eL\, \partial n/\partial t \quad .$$

This is the current a probe has to provide per cm^2 of its projected surface (projected onto a plane perpendicular to $\underset{\sim}{B}$). For our numerical example, this works out to be $j_1 \lesssim 0.08$ amps/cm^2 if $m = 6$ and $L \approx 10^3$ cm . Finite Larmor radius and field curvature can increase this figure by some factor of order 2 or 3 .

This current of electrons is easily supplied by a probe. Since $r_p > r_{Le}$, these electrons will stay in the T F I P in the absence of diffusion. In practice, there will be some diffusion, classical or otherwise, and the probe must supply a somewhat larger current. In principle, to eliminate all modes below the mth , it is necessary to hold the potential only of the T F I P and not that of the whole region between probes. But if the potential between probes changes because of high-m modes, a potential drop will occur at the surface of the T F I P , and the currents j_2 and j_3 will be required.

(b) To compute j_2 we assume that the potential across the T F I P looks something like this:



The potential ϕ_p in the T F I P will fall off to the surrounding potential (taken here to be 0) in a sheath of thickness $= O(\lambda_D)$. If, for instance, ϕ_p is positive, there will be an excess of electrons in the sheath and an equal excess of ions in the T F I P. To bring ϕ up to ϕ_p, a negative charge must be removed from the T F I P ; we compute this by computing the negative charge in the sheath.

Simplifying to one dimension, we assume thermal equilibrium and write Poisson's equation for the sheath as follows:

$$d^2\phi/dx^2 = 4\pi\, e(n_e - n_i) = 8\pi\, n_o\, e \sinh\,(e\phi/KT)\ ,$$

where $n_{e,i} = \exp\,(\pm\, e\,\phi/KT)$. For $e\,\phi_p \ll KT$, we can replace the sinh by its argument and solve, obtaining

$$\phi = \phi_p \exp\,(-\sqrt{2}\ x/\lambda_D)\ .$$

The charge in the sheath is found by integrating $e(n_i - n_e)\,dx$ with the known form of $\phi(x)$. We find

$$q(esu/cm^2) = -(\sqrt{2}/4\pi)(\phi_p/\lambda_D) \ .$$

The surface area of the T F I P is approximately $2\ell_p L$, so that the total charge required is $Q = 2\ell_p L\,(\sqrt{2}/4\pi)(\phi_p/\lambda_D)$. This must be supplied by the probe at a frequency ω. Since the projected area of the probe is $2r_p\ell_p$, the required probe current is

$$j_2 = (\omega L/r_p)(\sqrt{2}/4\pi)(\phi_p/\lambda_D)/9 \times 10^{11} \ ,$$

where we have converted to volts and amperes. Taking $L \approx 10^3$ cm, $\phi_p \approx 10$ volts, $\omega \approx 6\,\omega_o$, we find $j_2 \approx 60$ amps/cm^2. This is a rather large current density, but it is probably greatly overestimated. Charge separations occur only because of currents of type j_1, which are 3 orders of magnitude smaller. Hence potentials of order $\phi_p = 10$ volts probably cannot be built up.

(c) To compute the polarization current j_3 we write, in emu,

$$Mn\,\partial \underset{\sim}{v}/\partial t = \underset{\sim}{j} \times \underset{\sim}{B} \ , \quad |\,j\,| = Mn\omega v/B \ , \quad v = E/B \ ,$$

the last being true because $\omega << \omega_{ci}$. This is the current density into or out of the T F I P needed to set up the $E \times B$ drift around the T F I P. Converting to practical units, we find for our numerical example

$$j = 1.4 \times 10^{-11}\,\omega E \ \text{amps/cm}^2 \ .$$

This has to be multiplied by $2\ell_p L$ and divided by $2\, r_p \ell_p$ to obtain the probe current density j_3. Taking $\omega \approx 6\,\omega_o$, $E \approx 10$ V/cm, we find $j_3 \approx 15$ amps/cm^2. This is somewhat less than j_2 and may be an over-estimate for the same reason.

4. AVAILABLE PROBE CURRENT

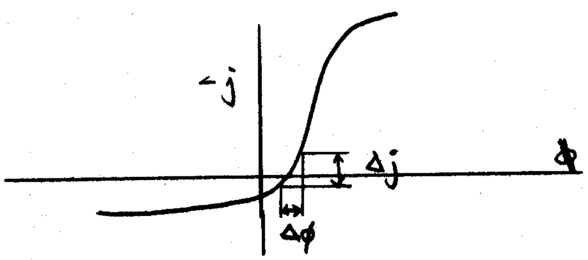
A floating probe draws equal numbers of ions and electrons. The ion flux is approximately

$$j_i = \frac{1}{2} ne\,(KT_e/M)^{1/2} \cong 8 \text{ amps/cm}^2 .$$

If the probe is pulsed several KT_e's negative, the electron current becomes negligible, and the probe can effectively inject 8 amps/cm^2 of electrons into the plasma. If the probe is pulsed several KT_e's positive, the saturation electron current can be drawn. This is

$$j_e = n\,(KT_e/2\pi m_e)^{1/2} \approx 220 \text{ amps/cm}^2 .$$

Thus the probe can effectively inject a very large positive charge into the plasma. If only a current of the order of j_1 (0.1 amps/cm^2) is needed, the probe can be operated in a narrow range around the floating potential, thus:



On the other hand, one can argue that if such small currents will suffice, a static array of probes will work as well as a dynamic system which actually injects the charge needed—perhaps a little more. If larger currents are needed because j_2 and j_3 are large, one can get up to 8 amps/cm^2 by pulsing to several hundred volts.

One must, of course, inject zero net charge into the plasma, lest it begin to rotate furiously. Thus to make use of the large saturation electron current j_e one has to increase the ion saturation current correspondingly. This can be done with a hot probe which emits electrons and hence effectively collects more ions. However, space charge limitations arising from the disparity between T_p (probe temperature) and T_e limit the current available. In principle, if the probe is pulsed many KT_e's negative, the space charge can be overcome, and a current $2j_e$ can be emitted [c.f. Summer Institute Notes on Probe Techniques, p. 136]. However, one is emission limited at about 10 amps/cm^2; this already requires a probe temperature of 2950° K. Thus symmetrical currents up to ~ 20 amps/cm^2 of either sign can be obtained with hot probes and large voltages; currents up to ± 1 amps/cm^2 are easy to get with cold probes and small voltages. There is, by the way, no problem with heating the probes; as we shall see, the plasma does a nice job of it.

For future reference we compute the plasma-probe impedance at floating potential ϕ_f. The electron current density there is $j_f = j_e \exp(e\phi_f/KT_e)$. The condition $j_f = j_i$ determines ϕ_f. The effective resistance seen by the probe is $(dj/d\phi_f)^{-1} = KT_e/ej_i \approx 100/8 = 13\ \Omega\ cm^2$. A 2-mil dia probe 2 cm long has projected area $10^{-2}\ cm^2$ and hence sees an impedance $R_p = 1300\ \Omega$. We have assumed that the sheath has time to come to equilibrium; this is true if $\omega << \omega_{pi}$, as is the case here [c.f. MATT-269].

5. LOSS OF PLASMA TO PROBES

The rate of recombination on the probe surface is determined by the ion flux, given approximately by $j_i = \frac{1}{2}n(KT_e/M)^{1/2}$. For our example this amounts to $j_i = 5 \times 10^{19}\ cm^{-2}\ sec^{-1}$. The total loss rate on N probes is then $J_i = N2\pi r_p \ell_p j_i = 6 \times 10^{20}\ Nr_p\ sec^{-1}$ for $\ell_p = 2$ cm. We compare this to the present pumpout loss rate $J_\tau \approx nV/\tau$, where V is the C stellarator plasma volume and τ the observed confinement time. With $V \approx 10^5\ cm^3$ and $\tau \approx 10^{-3}$ sec, we obtain $J_\tau \approx 10^{21}\ sec^{-1}$. If we aim for a factor of 10 improvement in τ, we require $J_\tau \geq 10\ J_i$ or $J_i = 6 \times 10^{20}\ Nr_p \leq 10^{20}$, $Nr_p \leq 0.17$. A reasonable lower limit for N is 24, so that we read $r_p \leq 0.007$ cm = 2.8 mils. Thus the probe diameter must be 5 mils or smaller. Since there will still be pumpout due to high-m modes, and since 48 probes (4 sets of 12) may be necessary, we shall think in terms of 2-mil probes.

6. PROBE LIFE

To protect such small probes from melting, one must meticulously prevent them from ever being exposed to runaway electrons; we assume this can be done. To see whether a probe can last in a dc discharge, we assume that the power input from the plasma comes primarily from ion bombardment, each ion carrying an energy of $\sim KT_e$. This comes to 800 watts/cm^2, whereas it takes only ~ 200 watts/cm^2 to melt tungsten. Therefore, if thermal conductivity along the probe can be neglected, as we shall assume, the plasma must be pulsed, so that the heat capacity of a probe can keep it from melting during a pulse. The heat capacity C of tungsten is 2.7×10^7 ergs/cm^3/$^\circ$C. Equating the total heat capacity to twice the power input from ions, we obtain

$$\pi r_p^2 \ell_p C \, \Delta T = 2 j_i KT_e 2\pi r_p \ell_p \tau \quad ,$$

$$r_p = 4 j_i KT_e / C\Delta T = 0.4 \, \tau$$

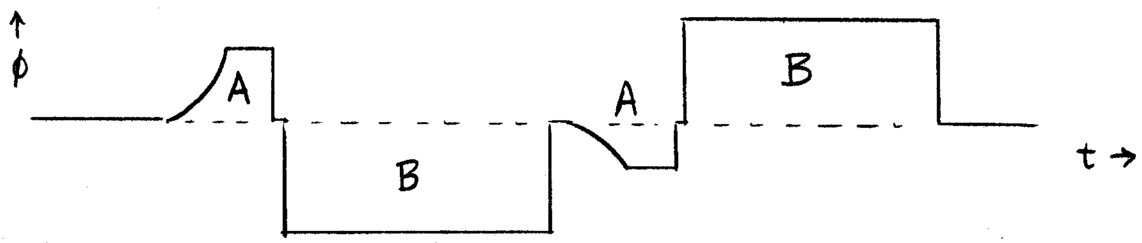
for $\Delta T = 3000^\circ$ C. Thus a 2-mil dia wire will in principle last for 6 msec. This is barely long enough for a significant increase in confinement to be measurable. To get a safety factor on probe life the best bet is to operate at densities lower than 10^{13} cm^{-3}.

The effectiveness of this method, therefore, depends on a delicate balance between short probe life, if the probes are too thin, and short confinement time, if the probes are too thick. The profile life will be even shorter if the probes must be pulsed to several hundred volts to obtain the large currents j_2 and j_3. Therefore, we must hope that only currents of the order of j_1 will be needed. If so, the probe power supplies considered *in the next section need provide only small voltages* $<< KT_e$.

7. TIMING AND CIRCUITS

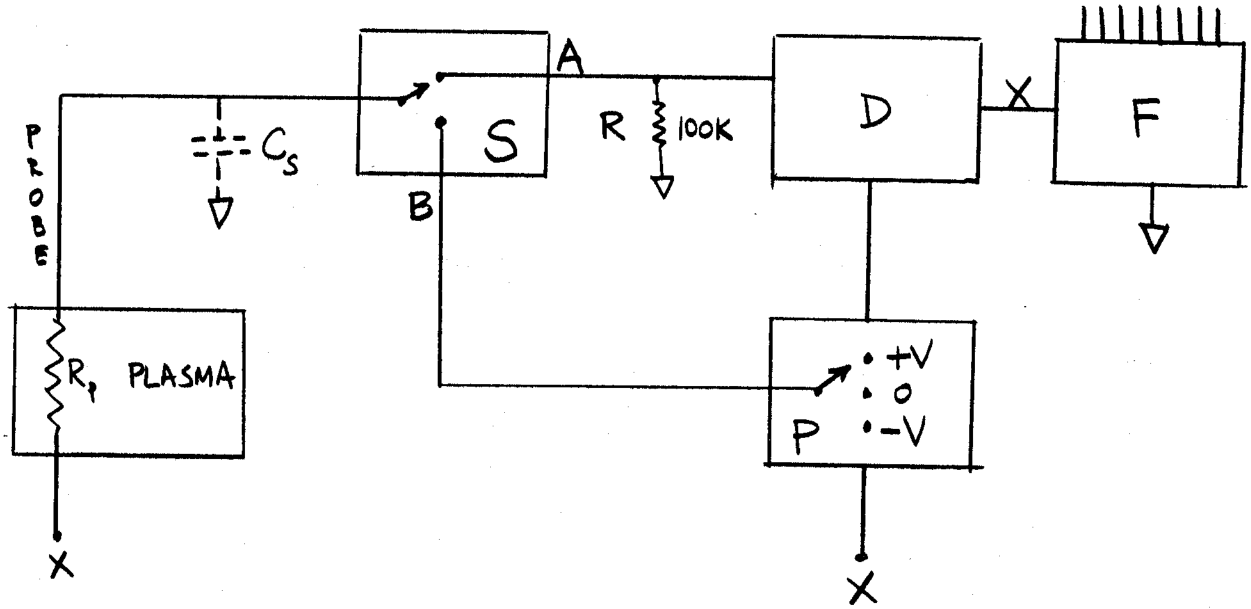
We propose to sample the plasma potential at a frequency ω_s such that $m\omega_o << \omega << \omega_{pi}$, or, from Sec. 2, $4.3 \times 10^4 << f_s << 6.7 \times 10^8$ Hz for $m = 6$. This will ensure that the probe sheath comes to equilibrium during each pulse. We also require time for the injected electrons to flow along the lines of force between probe arrays. Since $v_e \approx 3 \times 10^8$ cm/sec and $L \approx 300 - 600$ cm depending on the number of arrays, a frequency $f_s = 0.5\text{-}1$ MHz seems reasonable. This requires frequency response to about 10 MHz in the circuitry.

The timing sequence might be as follows:



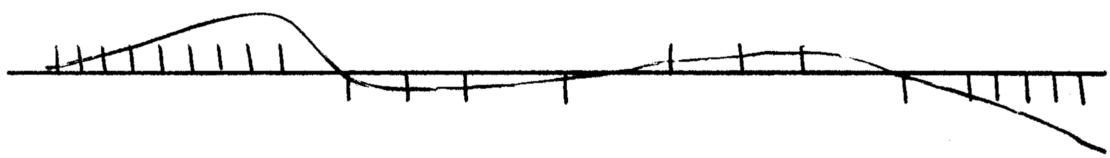
During the interval A in each cycle the probe is connected to a high impedance and rises to the floating potential. If this is found to be positive (negative) relative to a certain reference level, then in interval B the probe is connected to a low-impedance source providing a negative (positive) potential, thus reducing (increasing) the electron flux collected by the probe and thus pulsing the plasma potential back toward the reference level.

We propose to accomplish this by means of modern solid-state time-switching techniques. Each probe would be connected to a circuit like the one indicated on the following box diagram.



The plasma is represented by a resistance $R_p \approx 1300\ \Omega$, as found in Sec. 4. The effective plasma capacitance was discussed in Sec. 3 and because of its complexity is omitted here. The stray capacitance C_s represents the cable between the probe and the electronics. Circuit S is an electric switch which connects the probe alternately to A and B at 10^6 cps. In position A the probe is terminated in a high impedance $R \approx 100$ K and measures the floating potential. Circuit D is a discriminator which tells whether the probe is positive, negative, or $O \pm \epsilon$ with respect to a reference voltage X. According to the signal from D, the power supply P applies to the probe during interval B a dc bias equal to 0 or $\pm V$ by means of another electronic switch. V could be 12 or 24 volts, a convenient range for transistors. The reference level X is provided by circuit F, which averages the floating potentials from all the probes. One can add a constant potential to X by means of a knob, thus controlling the radial electric field in the plasma and hence its rotation frequency.

If we plot the current pulses B on a long time scale, they might look like this:



The plasma cannot respond to the high frequency of the pulses and therefore integrates over the pulses, thus receiving a current somewhat as shown by the curved line. Amplitude modulation of the probe current is accomplished by frequency modulation of constant amplitude pulses. There is a possibility that ion acoustic waves at $\omega = \omega_s$ will be excited, with $\lambda_{\parallel} = v_s/f_s \approx 10$ cm. If the ions are at all hot, these waves will be damped; at any rate, such waves are probably harmless to confinement.

There should be no problem with frequency response for the power pulse. The supply P should be made to provide up to 1 amp at 24 volts and hence must have an output impedance less than 25 Ω. To achieve 10 MHz response the RC_s time constant has to be less than 10^{-7} sec, so that C_s can be as large as 0.004 μf. However, during interval A the probe will rise to the floating potential in a time given by R_pC_s. For $R_pC_s = 10^{-7}$, $R_p = 1300\ \Omega$, C_s must be $\leq$ 80 pf. This is possible to achieve if the cables are kept short. Transistors must be chosen which are not affected by the fringing field of the stellarator. A 50 Ω terminated line unfortunately cannot be used because each probe must float during part of each cycle.

This work was performed under the auspices of the U. S. Atomic Energy Commission, Contract No. AT(30-1)-1238.