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Turbulent Correlation Measurements of Drift Waves

D. Mosher and F. F. Chen

In a previous report,[1] we ~~have shown that~~ showed the effect of magnetic shear on low-frequency fluctuations and on plasma confinement in a hard-core Q-machine. This paper concerns more detailed measurements of the oscillations, particularly of phase velocity and correlated flux; the following paper will discuss the mechanism of anomalous transport.

Fig. 1 ~~shows~~ is a schematic of ~~one half of~~ the L-2Q device, which produces a thermally-ionized potassium plasma column, 2 inches in diameter by 326 cm long, with KT = 0.22 eV. An aluminum tube strung along the axis provides a shear field B_θ of up to 400 G at the periphery of the column; the axial field B_z is variable from 1 to 4 kG. A new addition to the apparatus is the θ-probe assembly at the midplane of the machine. Two small coaxially shielded probes, displaced 3.5 cm along $\hat{z}$, can be set at the same radius and moved independently in the azimuthal direction by the use of sliding vacuum seals. The axes of the two hot endplates and the axis of rotation of the θ-probe assembly can be aligned to within 1 mm by making use of the steep change in plasma potential at the edge of the plasma.

The amplitude and spatial distribution of the oscillations at zero shear can be seen in Fig. 2; the frequency spectrum is shown in Fig. 3. The large value of n_1/n_0 and the continuous spectrum indicate a highly turbulent, nonlinear situation. There are four reasons why we do not obtain the single-mode drift-wave oscillations observed in the Q-1 device: 1) we operate at low densities ($10^9 - 5\times10^{10}$ cm^{-3}), where viscous damping is unimportant; 2) the unusual length of the plasma minimizes the effect of endplate damping; ~~3) the hole in the center caused by the hard core~~ 3) the axial depression in density due to the presence of the hard core causes the density gradient to reverse sign, so that the diamagnetic drift frequency must vary with radius, and a single-mode oscillation of the whole plasma is not possible; and 4) the very small radial temperature gradient on the endplates, according to theory[2], decreases the azimuthal wave velocity and thus enhances the dephasing effects of radial propagation and wave growth.

To measure the propagation velocity of the turbulent fluctuations, we employed a PAR Model 100 Signal Correlator, which computes the crosscorrelation function $R(\tau)$ between two signals as a function of time delay τ applied to one of the signals. The function $R(\tau)$ displayed instantaneously on an oscilloscope is computed from information obtained and stored from the preceding several integrating periods; we used an integrating time of 5 seconds. For the two signals we took the saturation ion currents from two probes at the same radius and separated by a variable angle θ in the azimuthal direction. The ion currents were measured across a 1 kΩ resistor shunted by a cable capacitance of less than 500 pf, giving a frequency response from DC to beyond 500 kHz. The signals were amplified by Tektronix type 1A7 and 127 preamplifiers in series (DC to 500 kHz), and fed into the correlator (0.16 Hz to 250 kHz).

Displacement of the peak in $R(\tau)$ as the probes were displaced in θ gave the azimuthal propagation velocity for the perturbations. An example is shown in Fig. 4. To compare the wave velocity V_w with theory, we measured also the radial profiles of density, plasma potential, and oscillation amplitude, as shown in Fig. 5. The slope of the J_+ curve gave the diamagnetic drift velocity, and the slope of the V_f curve the E/B doppler shift. These slopes were taken at the position of the probes, usually the radius of maximum oscillation amplitude. Comparison of V_w with theory is shown in Fig. 6, in which the theoretical wave l.c.

velocity βV_{de} is plotted against the propagation velocity V_w measured with the correlator and shifted to the E=0 frame. Here $\beta \equiv I_0(b)e^{-b}/[2-I_0(b)e^{-b}]$, with $b \equiv k_\perp^2 r_L^2$, is a measure of the decrease in wave velocity when the wavelength becomes comparable to the ion Larmor radius. A further decrease due to the finiteness of $\mathrm{Im}(\omega)/\mathrm{Re}(\omega)$ is negligible at our low densities. Fig. 6 shows that the low-frequency fluctuations we have studied propagate with the phase velocity of linear drift waves, in spite of the fact that the fluctuations are of large amplitude.

The feasibility of a plot such as Fig. 6 is the result of a fortunate accident. When $\underline{b}$ is large, (low B_z), the waves are dispersive, each mode having a different phase velocity. But the condition for large $\underline{b}$ is just the condition for the existence of nearly coherent waves, for which $k_\perp$ can be measured and β calculated. On the other hand, when $\underline{b}$ is small, the fluctuations are turbulent; and many values of $k_\perp$ exist simultaneously. Fortunately, for $b \to 0$, β approaches unity; and the waves are not dispersive: it is then not necessary to know $k_\perp$ to be able to place a point on Fig. 6.

Using the same correlator, we have also measured the cross correlation between density and potential fluctuations at the same point in order to determine the radial flux due to the fluctuations. We employ the method of Bol and Ellis[3] and look for the slope of ~~as a function of probe separation~~ $\langle \tilde{n}_1 \tilde{V}_{f2}(\theta) \rangle$ at the point where $\langle \tilde{n}_1 \tilde{n}_2(\theta) \rangle$ is maximum. Measuring $\tilde{V}_f$, however, is a problem at our low densities and temperatures because the plasma presents a source impedance of $\sim 1\,\mathrm{M}\Omega$ at $n \simeq 10^9\ \mathrm{cm}^{-3}$, and the stray capacitance of the probe and cable causes appreciable phase shift even in the kilohertz range. We have overcome this difficulty by using capacitance neutralization with a shield driven by a unity-gain amplifier. The method is described elsewhere[4]. The frequency response and phase shift of the system is shown in Fig. 7. Since the 3° phase shift points are at 12 Hz and 100 kHz, a glance at the spectrum of Fig. 3 shows that our measurements of the n-V_f phase shift should be accurate to less than 3°.

A typical measurement of correlated flux is shown in Fig. 8. It is seen that the slope of the $\langle \tilde{n}\tilde{V} \rangle$ curve is very nearly zero at the point of maximum $\langle \tilde{n}\tilde{n} \rangle$ correlation. This means that the n and V fluctuations are very nearly in phase, resulting in very little transport due to the growth of the wave. The largest phase shift we have ever observed is shown in Fig. 9. Even though the oscillation amplitude was nearly 100% in this case, the measured n-V phase shift is a factor of 5 too small to account for the observed anomalous transport.

That the n-V phase shift is zero under a wide variety of conditions is verified without the tedium of the correlation measurements by merely looking at the similarity between oscilloscope traces from probes measuring density and potential on the same line of force. Fig. 10 shows such pairs of traces for different values of B_z, r, and n_1/n_0 (varied by applying shear). It is seen that the density and potential fluctuations are nearly in phase under all conditions.

We can explain the small phase shift observed if we assume that the theory of resistive drift instabilities[5] without ion viscosity is applicable to our oscillations at densities between 10^9 and 10^{10} cm^{-3}. The phase velocity has been shown to be consistent with that of drift waves, and the mean free path is always less than the machine length, so that the collisional destabilizing mechanism is dominant. In this case, the growth rate is given by[5]

$$\left| \frac{\mathrm{Im}(\omega)}{\mathrm{Re}(\omega)} \right| = r_L^2 \, \frac{k_\perp}{\Lambda} \, \frac{k_\perp^2}{k_\parallel^2} \left(\frac{n_0 e \eta}{B} \right),$$

$k_\parallel = \pi/300\ \mathrm{cm}^{-1}$,

where Λ is the density scale length and η the resistivity. Taking $B = 2kG$, $n = 2\times10^9\ \mathrm{cm}^{-3}$, $\Lambda = 1$ cm, and $k_\perp = m/r = 2/2$ cm^{-1}, we find $|\mathrm{Im}\,\omega/\mathrm{Re}\,\omega| \approx 4\times10^{-3}$. (C) Now the radial flux j_x can be expressed in terms of $\mathrm{Im}\,\omega$ as follows[6]:

$$j_x = \langle n_1 V_x^{(1)} \rangle = \tfrac{1}{2} n_0 \left(\frac{n_1}{n_0}\right)^2 \left(\frac{-n_0}{n_0'}\right) \mathrm{Im}\,\omega \approx 8 k_\perp \Lambda j_B \frac{\mathrm{Im}\,\omega}{\mathrm{Re}\,\omega} \left(\frac{n_1}{n_0}\right)^2 ,$$

where $j_B \equiv (KT/16eB)(n_0/\Lambda)$ is the Bohm flux. From this it is seen that if $\mathrm{Im}\,\omega/\mathrm{Re}\,\omega$ is small enough, j_x/j_B can be very small even for $n_1/n_0 = O(1)$. (A) The value of 4×10^{-3} for $|\mathrm{Im}\,\omega/\mathrm{Re}\,\omega|$ is entirely consistent with our observation of zero detectable n-V phase shift.

The absolute growth rate under the above conditions is very small: $\mathrm{Im}\,\omega \approx 20\ \mathrm{sec}^{-1}$. Waves can grow to large amplitude only if sufficient time is available. If end plate recombination is the limiting factor in the ion lifetime, τ, we estimate that with $T = 2550°K$, $n = 2\times10^9\ \mathrm{cm}^{-3}$, and a reionization probability of 80%, τ is about 5 sec. This long time is a consequence of the large sheath drop confining ions at low density. Thus there is time for 100 e-foldings even with the small growth rate. Apparently, the waves grow very slowly, accounting for the unmeasurable n-V phase shift, but have time to reach a large amplitude. Then the growth is halted by two mechanisms: the distortion of the zero-order density profile, and the loss of ions by scrape-off rather than correlated motion, at the aperture limiter. (B) This scrape-off mechanism, apparently accounts for the anomalous loss which we see associated with the appearance of oscillations.

¶ It can be shown that for drift waves the radial excursion Δx of an ion moving in the wave is given by $\Delta x/\Lambda \approx eV_1/KT \approx n_1/n_0$. In our case, with $n_1/n_0 = O(1)$, scrape-off is an important loss mechanism.

(A) Since the measured flux is of order j_B, this means that correlated flux can ~~be~~ contribute only a small fraction of the total loss.

(B) The nonlinear process determining the final wave amplitude is not yet understood in detail; but if the linear growth rate γ_L is small enough to give negligible phase shift, ~~then~~ it is reasonable to assume that the nonlinear ~~of~~ damping rate $\gamma_{NL} = -\gamma_L$ would also cause a negligible phase shift. At higher densities, $\mathrm{Im}\,\omega/\mathrm{Re}\,\omega$ can be as large as $\frac{1}{3}$, ~~giving rise to m~~ and one would expect to see an appreciable phase shift.

(C) Here we have ~~taken~~ given $k_\parallel$ the smallest possible value and have ~~taken~~ chosen a low m-number, since only ~~these~~ low-m modes have a large amplitude.

References

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Figure Captions

1. Scale drawing of one half of the L-2Q device as of July, 1967.
2. Radial profile of plasma density and oscillation amplitude, as given by the ~~ac~~ dc-coupled saturation ion current to a probe.
3. Typical spectrum of low-frequency oscillations at 4 kG, zero shear.
4. Cross/correlagrams of ion current fluctuations on two probes on the same line of force ($\theta=0$) and displaced in θ. Also shown are oscilloscope traces from the two probes at $\theta=0$, the position of maximum correlation.
5. Radial profiles of mean square oscillation amplitude ~~in ion current~~, probe floating potential, and ion saturation current for the case of Fig. 4, in which the oscillations have been made somewhat coherent by a small amount of shear current I_s in the hard core. Note that the floating potential curve follows the density curve according to $eV_f/kT \propto \ln n$, indicating a very uniform endplate temperature distribution (Ref. 2).
6. Comparison of ~~the~~ calculated and measured azimuthal wave velocities in the $E=0$ frame for various combinations of the parameters B, n, r, and I_s. Solid points were taken on the exterior density gradient; open circles were taken on the interior gradient near the hard core. The error bar is typical of all the points. The three points off the curve at the right represent cases in which a large asymmetry in endplate temperature caused a large radial component in the propagation vector.
7. Calibration of frequency response and phase shift ϕ of the actual probe-cable-~~capacitance neutralizing~~ preamplifier-driven shield system with a 1 MΩ resistor between the oscillator and the probe tip. In practice the gain is optimized for each plasma density by increasing it to a point just short of oscillation, with the probe inserted into the plasma.

8. A typical measurement ~~The normalized~~ of normalized cross correlation $R(0)$ between signals from probe 1, which is fixed and measures density fluctuations, and probe 2, which is moved through an angle θ and measures either density or potential fluctuations. Repeatibility errors are shown wherever they exceed the size of the points. The amplitude n_1/n_0 was of order unity in this case.

9. An atypical measurement showing a small but measurable n–V phase shift.

10. Oscillations on two probes 3 cm apart on the same line of force, one probe measuring ion saturation current J_+, and the other, floating potential V_f. Sweep speed: 1ms/cm. Frequency response: 6 Hz to 150 kHz.
(a) 2 kG, $I_s = 0$, probe at density peak. (b) 2 kG, $I_s = 0$, probe on outside density gradient. (c) 2 kG, $I_s = 150$ A, probe on outside gradient. (d) 2 kG, $I_s = 150$ A, inside gradient. (e) 4 kG, $I_s = 0$, outside gradient. (f) 4 kG, $I_s = 300$ A, outside gradient. The J_+ traces are dc coupled, and the baseline is shown in the margin.