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RAPPORTEUR PAPER ON DRIFT WAVES AND DIFFUSION *

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Text of Rapporteur Presentation

on Alkali Plasmas

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I bring greetings from Professor Stix, who could not attend and asked that I take his place in giving this review paper on Alkali Plasmas. It befalls me to summarize 7 papers containing the results of 16 experiments and 3 theories.

It may seem strange that such low temperature plasmas as one encounters in Q-machines should be considered in a conference on thermonuclear fusion. But the philosophy of those who work with alkali plasmas is that careful tests of our understnading of the behavior of plasmas is of utmost importance to fusion research, and that these tests are best carried out in quiescent plasmas in uniform magnetic fields. The papers I shall review will illustrate the success of this approach. It has been said that before one can learn to run, first one must learn to walk; this, then, might be called the pedestrian approach to thermonuclear fusion.

A. Anomalous Transport.

Let me now proceed to the first topic, Anomalous Transport, the study of which has led to some rather fascinating results. The problem is as follows: In a so-called double-ended Q-machine, there are two end plates— made of tungsten, tantalum, or rhenium — which are heated to a temperature of 2000-3000° K. These emit electrons, which are then confined by a magnetic field $\mathbf{B}$. A neutral beam of Cs, K, or Ba atoms is directed at the end plastes by some sort of collimator (Fig. 1)— sometimes at one end and sometimes at both ends — and these atoms are ionized upon contact with the hot plates. A flux of ions is therefore injected along with the electrons to form a plasma with density between 10^8 and 10^{12} cm^{-3} and temperature $T \approx 0.2$ eV in equilibrium with the end plates. These experiments were carried out under electron-rich conditions; that is to say, the hot plates emit an excess of electrons, and the potential ϕ as a function of z has the behavior shown in Fig. 1.

Fig. 1

There is a negative potential drop in the sheath, so that plasma electrons are not confined along $\underset{\sim}{B}$ but stream out with the random flux $\frac{1}{4} n\bar{v}_e$. This flux is balanced by thermionic emission given by the Richardson current j_R, and the excess emission current is turned back by the Coulomb barrier exp $(e\phi/KT)$, ϕ being negative:

$$\tfrac{1}{4}n\bar{v}_e = j_R e^{e\phi/KT} \tag{1}$$

The plates and sheaths can be thought of as self-adjusting sources which supply just the right number of electrons to preserve charge neutrality along each line of force.

The ions, on the other hand, are trapped in the z direction by the potential well and ideally can be lost from the plasma in only two ways:

1) classical diffusion across $\underset{\sim}{B}$, and

2) end-plate recombination.

In the latter process, the ions in the tail of the Maxwellian distribution penetrate the sheath and recombine on the end plates with a small probability of order 10-20%. Note, first, that the diffusion is not ambipolar: only the diffusion of ions is under investigation. Second, one can hope to distinguish between (1) and

(2) because (1) is B-dependent, whereas (2) is not. Third, we have neglected volume recombination and diffusion by ion-ion collisions; these processes have been shown to be negligible in the experiments.

For both diffusion and end-plate recombination, one would expect that the equilibrium density would be proportional to the square root of the ion flux Φ_i entering from the ends or, equivalently, of the neutral flux density j_o impinging on the end plates; thus,

$$j_o \propto \Phi_i \propto n^2. \qquad (2)$$

For classical diffusion, the n^2 dependence is obvious: we have $j_r = - D_\perp \nabla n$, and $D_\perp = 2n\eta KT/B^2$. For end-plate recombination, one factor of n is obvious; the other factor of n comes from the Coulomb barrier, which, according to Eq. (1), is proportional to n.

Previous experiments to verify Eq. (2) have given the following result (Fig. 2). At high densities, the experimental measurements (solid line) indeed obey Eq. (2). However, below about $n = 2 \times 10^{11}\ cm^{-3}$, the density is lower than predicted and follows an $n \propto \Phi_i$ dependence, indicating the presence of an anomalous loss mechanism. One can postulate that the loss is due to 1) an enhanced $D_\perp$, 2) enhanced end-plate recombination, or 3) plasma convection across $\underset{\sim}{B}$.

Three experiments on this problem are reported here. Each experiment has its own gimmick; that is, a special feature which allows one to distinguish among these possibilities. I wish to emphasize that all these experiments were performed in the quiescent regime, in which the operating parameters have been adjusted so that the level of low-frequency oscillations is too low to account for the anomalous loss. The obvious mechanism of diffusion by fluctuations has been excluded.

1. Hashmi, Houven van Oordt, Wegrowe (Garching), paper E-7.

The gimmick in this experiment was the use of barium instead of cesium or potassium. Barium has a resonance line which can be observed spectroscopically. By this technique, the density of the neutral beam can be measured; also, the density of ions can be measured without using probes, which can be a source of plasma loss. Fig. 3 shows experimental results on the proportionality of n to j_o in the low-density regime.

Fig. 3 (Fig. 1 of paper E-7)

The significant point here is that the abscissa is j_o rather than the usual Φ_i, which is easily obtained by dropping a negatively biased collector plate in front of each hot plate. Thus the anomaly is not connected with the method of measurement of Φ_i. Note that the density is as much as an order of magnitude lower than expected classically.

Fig 4 shows that j_o is indeed proportional to Φ_i. Therefore, there is no peculiarity in the behavior of the ionization function.

Fig. 4 (Fig. 2 of paper E-7)

Fig. 5 gives a comparison of the profile of the plasma and the profile of the neutral atomic beam. There appears to be very little spreading of the plasma, indicating a small radial diffusion rate. This is made clear

Fig. 5 (Fig. 3 of paper E-7)

Fig. 6 (Fig. 4 of paper E-7)

in Fig. 6, in which the observed ratio of plasma to neutral beam radii is compared with that expected from Bohm diffusion (dashed line).

The absence of an anomaly in $D_\perp$ was further verified by ion flux measurements (Fig. 7). The input flux Φ_i was measured by plate 1.

Fig. 7

The flux Φ_p arriving at large cold plate 3 was measured with plate 1 removed. It was found that $\Phi = \Phi_p$, so that the ions did not spread past the radius of plate 3, at least in a single-ended configuration. When the neutral beam profile was changed, Φ_i/n remained constant, indicating a loss insensitive to ∇n. The ring 2 measured Φ_R, the flux escaping radially with plates 1 and 3 removed. It was found that $\Phi_R \approx 0.2 - 0.5\Phi_i$. This was found to be consistent with ionization occurring outside the inner radius of ring 2, so that the magnitude of Φ_R did not necessarily indicate a large radial flux. Harshmi et al. concluded that radial losses could not account for the density anomaly. This leaves enhanced end-plate recombination as the only alternative, but no mechanism was suggested for this unlikely process. The crucial point here is the interpretation of the Φ_R measurement; we shall return to this point later.

2. Motley and von Goeler (Princeton), paper E-1

The gimmick in this experiment, done on the Q-3 machine at Princeton, was the use of a special hot plate with a small central hole, through which the neutral atoms could be introduced (Fig. 8). Very narrow collimation of the neutral beam

Fig. 8 (Fig. 1a of paper E-1)

was therefore possible. The width of the density profile could then be observed to decrease with increasing B, as shown in Fig. 9. However, the

Fig. 9 (Fig. 1b of paper E-1)

plasma radius was always found to be nearly equal to the Larmor radius, as shown in Fig. 10. This small spread of the profile and the n and B dependences are consistent with

Fig. 10 (Fig. 2 of paper E-1)

classical diffusion. Nevertheless, the magnitude of n, the proportionality of Φ_i to n, the constancy of confinement time τ (as measured by a pulsed neutral beam), and the plate-temperature dependence indicate an anomalous loss process. The radial flux was measured directly with a ring collector; and, in agreement with the Garching results, the ratio Φ_R/Φ_i was found to be between 0.3 and 0.5, as shown in Fig. 11. However, the interpretation here was entirely

Fig. 11 (Fig. 3 of paper E-1)

different. Because of the narrow collimation, Φ_R could not be attributed to contact ionization of neutrals near the edge of the hot plate. Also, the radius of the ring collector could be varied. The results are consistent with the picture that all of the anomalous loss is from radial motion, probably fast dc convection in the region outside the beam.

3. Chen, Mosher, and Rogers (Princeton), paper E-2.

The gimmick in this experiment is the "hard-core", a current-carrying conductor along the axis which adds a B_θ to the uniform B_z and allows one to study the shear dependence of the anomalous loss. Fig. 12 shows the experimental arrangment at one end of the machine.

Fig. 12 (Fig. 1 of paper E-2)

By keeping the input ion flux constant, as measured by removable collector J, one can investigate the behavior of $D_\perp$ with shear by observing the equilibrium plasma density. Fig. 13 shows the results of peak density n_p plotted against the current I_s in the hard-core. At low shear, there is a loss connected with oscillations ($D_\perp \approx 3\ D_{Bohm}$), and the experimental points lie below the theoretical curve. At high shear, where the curve n_{osc} lies above the experimental points, the oscillations have been stabilized;

Fig. 13 (Fig. 2 of paper E-2)

and the loss is entirely due to the mysterious mechanism under study. Here the experimental points agree with theory.. The theory referred to here postulates that the loss is caused by dc convection across axially asymmetic electric fields.

If the hot end plates are not perfectly isothermal, one would expect the equipotentials in the plasma to be asymmetric, as exemplified by the off-center circle in Fig. 14. When the shear is applied, the rotational transform of the lines of force twists he circle into a long spiral, as shown. The ions

Fig. 14 (Fig. 3 of paper E-2)

are contained longer, since they drift along the equipotentials and now have a longer distance to drift to make a given excursion in radius. The existence of such convective patterns has been verified experimentally.

Fig. 15 (Fig. 4 of paper E-2)

Fig. 15 shows the floating potential as a function of radius at two values of shear; the spatial oscillations indicate the existence of spiral patterns. These patterns have also been mapped out over the entire cross section , and they change with shear in the manner expected from theory.

4. Conclusion

We can now draw the folloiwng conclusions regarding the intriguing problem of anomalous losses in Q-machines. In the case of shear, it has been shown that dc convection can account for the anomalous loss in the absence of oscillations, and the behavior with shear is explained by the picture of twisting convective patterns. In the case of no shear, all processes have been excluded except enhanced end-plate recombination and dc convection. The Garching group favors the former, and the Princeton group the latter; the difference lies in the interpretation of the ring-collector results. However, preliminary measurements by Motley at Princeton with a hot plate with an axially symmetric temperature distribution have indicated that dc convection may not be the whole story. Further measurements by both groups will, we hope, clear up this mystery within the next year. If dc convection turns out to be the dominant mechanism, there will be agreement with the shear results. There may also be an additional, smaller, shear-insensitive loss mechanism, which has not been excluded by the shear experiment.

In addition to this work, there are also results on diffusion in the presence of oscillations. At Garching, evidence of large anomalous diffusion is found when the plasma is unstable, but the modes have not yet been identified. At Princeton, Chu et al. have shown that the enhanced diffusion due to drift waves is connected with the phase shift between density and potential fluctuations. On the other hand, Chen et al., working at lower densities, find an enhanced oscillatory loss even when the n-ϕ phase shift is zero.

B. Wave Identification

1. Hendel, Chu, Politzer, Perkins, and Coppi (Princeton), paper E-1. Fig. 16 shows the data of Hendel et al. on the instability threshold and wave frequency

Fig. 16 (Fig. 5 of paper E-1)

of resistive drift waves localized in the region of the density gradient. The threshold agrees well with that given by a theory which includes ion viscosity; this predicts the $n^{\frac{1}{2}}$ dependence shown on the bottom graph. These results are published and well known. This work is a landmark in the study of low-frequency instabilities because it is the first time that a drift wave has been positively indentified. The following two experiments show new effects which modify the original picture.

2. Enriques, Levine, and Righetti (Frascati), paper E-3. In this experiment the effect of a radial electric field was investigated. The electric field was created by dividing the hot plate into two concentric sections

and applying a voltage between them. Two modes have been found. Fig. 17 shows the potential distribution across the plasma when 0.65 volts is applied

Fig. 17 (Fig. 1 of paper E-3)

across the gap. A 2 kHz oscillation peaked in the region of large elctric field (near the gap) is found. The density is nearly constant across the gap. The wave propagates in the same direction as, but more slowly than, the maximum $E \times B$ drift, which is of the order of the ion thermal velocity. The threshold occurs at applied voltages between .05 and 0.5 V. The n-ϕ phase shift is nearly 180°, suggesting that this is a flute mode. It is conjectured that this is a Kelvin-Helmholtz instability caused by the shear in $E \times B$ drift velocity.

A second type of instability is found with larger applied voltages of 1 - 3 volts. Fig. 18 shows that a large demsity gradient is created in the

Fig. 18 (Fig. 3 of paper E-3)

interior region in this case and that there is an oscillation peaked in the region of the gradient. The n-ϕ phase shift is nearly zero, indicating a drift mode. This is conjectured to be a drift wave ocurring at low densities such that ion viscosity is unimportant.

3. Chen and Rogers (Princeton), paper E-2.

This experiment is also concerned with the radial electric field, but in a double-ended rather than a single-ended Q-machine. The E field is changed by applying a voltage V_c to the aperture limiters. Fig 19 shows the oscillation amplitude variation with V_c.

Fig.19 (Fig. 6 of paper E-2)

There is a range of V_c for which the plasma is quiescent. For smaller V_c, turbulent fluctuations occur. For larger V_c, coherent drift waves are excited at a well-defined threshold. Fig. 20 shows this threshold voltage versus magnetic field for different densities.

Fig. 20 (Fig. 8 of paper E-2)

At large values of V_c, the curves approach asymptotically a critical magnetic field B_c, above which waves are excited. As V_c is lowered, B_c is increased, indicating that the electric field corresponding to the lower values of V_c has a stabilizing effect. Fig. 21 shows the radial variation of radial electric

Fig. 21 (Fig. 7 of paper E-2)

field E_r, as measured by a new technique involving synchronous detection of floating potential signals from an oscillating probe. There is some evidence that coherent oscillations can occur only when E (r) has a linear portion corresponding to solid-body rotation. The conclusion is that E(r) has a large effect not only on the frequency but also on the excitation threshold of resistive drift waves.

C. Critical Fluctuations

1. Hendel, Chu, and Politzer (Princeton), paper E-1.

The recent work of the Q-1 group at Princeton is concerned with critical fluctuations: the enhancement of thermal noise at the drift-wave frequency in the regime in which the waves are stable. Fig. 22 shows the noise amplitude at the drift-wave eigenfrequency as the magnetic field is varied through the

Fig. 22 (Fig. 6 of paper E-1)

instability threshold. Above threshold, the wave amplitude rises rapidly and reaches a nonlinear limit. Below threshold, one can measure an increase in the noise level as threshold is approached. Both regions are described by a single master equation:

$$\frac{\partial A}{\partial t} = \gamma A + CA_{noise} - \alpha A^3, \qquad (3)$$

in which A is the oscillation amplitude, A_{noise} is the thermal noise amplitude at the eigenfrequency, γ is the growth rate, and C and α are constants. This equation is quite general and has been given by such authors as Landau and Kadomtsev and, most recently, by Stix, who derived it from a mode-mode coupling calculation. We see that in steady state, in the limit of small A, the A^3 term can be dropped; and we have $A \propto \gamma^{-1}$. In the limit of large A, the middle term can be dropped; and we have $A \propto \gamma^{\frac{1}{2}}$. In Fig. 22 one sees that these two dependences agree well with experiment in the stable and unstable regions, respectively.

D. Drift-Wave Stabilization.

1. <u>Chen and Rogers</u> (Princeton), paper E-2. Shear stabilization. The effect of magnetic shear on drift waves has been studied in the hard-core Q-machine described earlier. Previously published results from this experiment were concerned with the suppression of a turbulent spectrum of drift waves. New results reported here are concerned with the stabilization of single modes, obtained by adjusting the radial electric field, as previously described. In this case, there are two aperture limiters, one on the outside of the plasma, and one on the inside, on the hard core. By varying the

voltages on the two aperture limiters, it is possible to make sinusoidal oscillations on the outer density gradient, as shown in Fig. 23

Fig. 23 (Fig. 10 of paper E-2)

The amplitude of this drift wave varies with hard-core current in the manner shown in Fig. 24. There is a mode shift as the shear changes the

Fig. 24 (Fig. 12 of paper E-2)

electric field distribution, and then there is a cut-off as the wave is stabilized. This stabilization point versus B_z is shown in Fig. 25, where it is compared

Fig. 25 (Fig. 13 of paper E-2)

with the theory of Krall and Rosenbluth. The agreement to within a factor of 2 can be considered good in view of the fact that the theory has uncertainties of this order or even larger.

2. Chen and Rogers (Princeton), paper E-2. Dawson rings. In this experiment one tries to suppress drift waves by chopping the plasma into short lengths and thus increasing the minimum value of $k_{\parallel}$. This is accomplished by inserting Dawson rings (Fig. 26), which carry a current opposite to that in the main B_z coils and create local regions of favorable curvature, which force the wave amplitude to vanish there. Fig. 27 shows an

Fig. 26 (Fig. 14 of paper E-2)
Fig. 27 (Fig. 17 " " ")

oscillogram of a drift wave with the Dawson ring off and on. There is complete suppression, but only part of the effect is due to the change in "connection length." Unfortunately, the plasma lifetime and the density gradient are also changed when the Dawson ring is turned on, so that a detailed check with theory is impossible.

3. Itatani, Obiki, and Takahashi (Nagoya), paper E-4.

In this experiment the possibility of suppressing drift waves with an AC signal between 1 and 500 kHz is tested. The signal is applied to a grid which cuts across the midplane of a single-ended Cs plasma. Fig. 28 shows the frequency spectrum of spontaneous low-frequency oscillations in

Fig. 28 (Fig. 2b of paper E-4)

the plasma before and after the application of a high-frequency signal. The observed suppression is independent of frequency as long as it is much higher than the drift-wave frequency. The grid, however, has a rectifying effect which causes a dc current to flow when an ac signal is applied. Fig. 29 shows the effect on oscillation frequency and amplitude of applying an ac voltage on the grid. The effect of the dc component has been carefully taken into account. It is seen that there is suppression of the drift waves for sufficiently large ac voltages. It is conjectured that the effect comes from a shortening of the effective length of the machine when the grid is energized. These results are sensitive to the sheath conditions and to the length of the plasma: there is no suppression for $L > 24$ cm.

Fig. 29 (Fig. 3b of paper E-4)

In addition to these results, the Japanese group has results on the interaction of drift waves with acoustic waves generated by the grid. There is a linear effect and a nonlinear effect. In the first case, an $m = o$ acoustic wave is generated at the aame frequency as the $m = 1$ drift wave. It can then be seen that the waves interfere constructively on one side of the column and destructively on the other side, where the oscillation amplitude can be made quite small. In the second case, linear combinations of the drift-wave and applied frequencies can be generated by nonlinear coupling.

E. Ion Waves

1. Doucet and Gresillon (Ecole Polytechnique), paper E-5.

This paper contains the results of three experiments on ion waves performed on a single-ended Cs plasma. The special feature of this experiment is that the ion temperature is varied by introducing varying amounts of neutral He into the plasma.

a. Collisional damping. Curves for the phase velocity and damping length vs. frequency have been calculated taking into account the variation of T_i due to collisions with the He atoms, and the unidirectional drift of the plasma from the hot plate to the cold plate. For example, Fig. 30 shows

Fig. 30 (Fig. 1a of paper E-5)

the theoretical variation of phase velocity with frequency for various values of the drift velocity for the mode propagating downstream. Fig. 31 shows the

Fig. 31 (Fig. 3a of paper E-5)

corresponding experimental data. By fitting the theory to the experimental points and using the calculated collision frequency, one can determine the drift velocity and the ion temperature. Fig. 32 shows the comparison of T_i measured this way and also directly by the use of an electrostatic analyzer, for various pressures of neutral He. There is very impressive agreement between the experimental points taken in these two completely different ways.

Fig. 32 (Fig. 5 of paper E-5)

b. Landau damping. By working at low densities, it is possible to measure the Landau-damping length.. However, in order to reduce T_i by collisions with neutrals, it is necessary to remove the neutrals in the region of measurement by differential pumping. In the one case measured so far, there was good agreement between theory and experiment.

c. Nonlinear Landau damping. For electron Langmuir oscillations, nonlinear damping of large amplitude waves shows up as a spatial modulation of the wave amplitude. The same sort of modulation has been seen in this experiment for ion acoustic waves. However, there are two points of disagreement with theory: the position of the first minimum does not vary as expected with amplitude, and the critical density for the transition from collisional to Landau damping is lower than expected. By use of correlation techniques, Doucet has found that only part of the observed spatial modulation is due to nonlinear damping; the rest is due to interference between the ion waves and a fast wave, whose identity is still unknown.

F. Theory

Finally, we come to the theoretical contributions. Two calculations are reported. Both concern the nonlinear behavior of drift waves; and, in order to keep the complexity within bounds, both make use of particularly simple approximations regarding the x-dependence of the perturbations. Because of this, neither can give a reliable estimate of anomalous transport.

1. Oraevskii, Tasso, and Wobig (Garching), paper E-6

In this calculation, the x-dependence (x being the direction of the density gradient) is neglected altogether, as is the ion temperature, the resistivity, and cylindrical geometry effects. Ion inertia and ∇T_e are retained to give a "universal" instability. In this case the nonlinear dispersion equation is found to be of the form

$$\frac{\partial\phi}{\partial y}(a + b\phi) = c\left(\frac{\partial^3\phi}{\partial y^3} - \frac{e}{KT_e}\frac{\partial\phi}{\partial y}\frac{\partial^2\phi}{\partial y^2}\right). \tag{4}$$

This has an explicit solution of the form

$$\left(\frac{\partial\phi}{\partial y}\right)^2 = \alpha\frac{KT_e}{e}e^{e\phi/KT_e} + \beta\phi + \frac{\gamma}{2}\phi^2 + \delta\,. \tag{5}$$

In various limits this equation can be shown to lead solitary waves, general periodic waves, or pure sine waves. When finite resistivity is included, Eq. (4) is replaced by two simultaneous equations, from which it can be shown that periodic solutions of finite amplitude are impossible.

2. Stix (Princeton), paper E-1.

A long and detailed theory by Stix takes into account resistivity, ion viscosity, and finite Larmor radius but neglects cylindrical geometry effects, zero-order electric fields, and variations in the density gradient. Here the density perturbation is assumed to have the same x dependence as the zero-order density profile. This complicated system is reduced to a nonlinear wave equation in a single variable. To solve this equation, Stix takes a trial function consisting of a sine wave and its first harmonic. If higher terms than the quadratic are neglected, it can be shown that a limit cycle is produced; that is, a steady oscillation of finite amplitude is reached, in which the nonlinear damping mechanism is the loss of energy to the harmonic via mode-mode coupling. However, the limiting amplitude is found to be much larger than is experimentally observed by Chu et al.

You will remember that the previous paper by the Garching group had the result that no limit cycle was possible in the resistive case. This discrepancy is apparently due to a difference in ordering. By adding a term to the equation of Stix it is possible to recover the equation of Oraevskii et al. Then it can be shown that periodic solutions are impossible. However, the added term is of higher order in the ordering scheme of Stix and is quite properly neglected. Then periodic solutions are possible. Thus it appears that the non-existence theorem exists only by virtue of a small effect. I am grateful to Professor Stix for pointing this out.

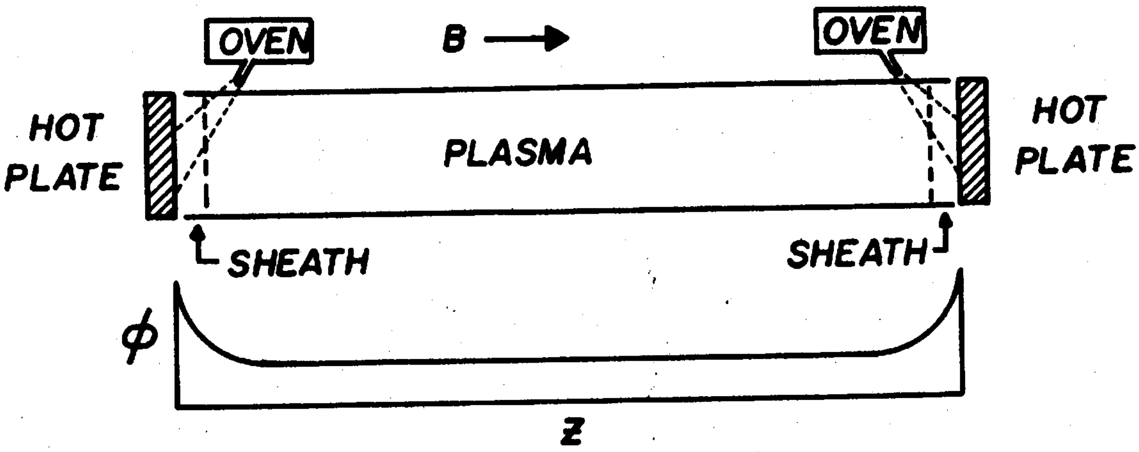
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Fig. 1

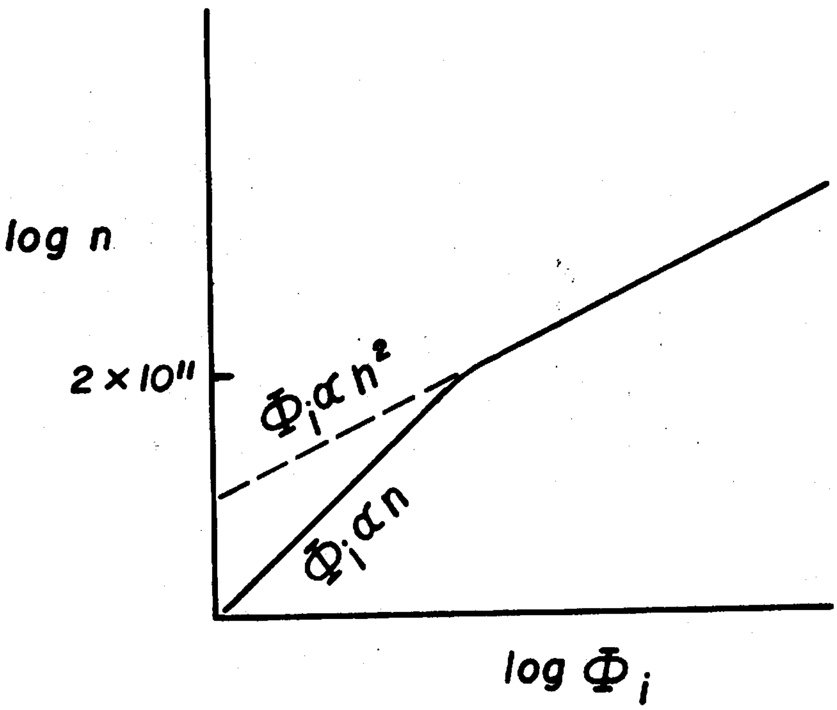
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Fig. 2

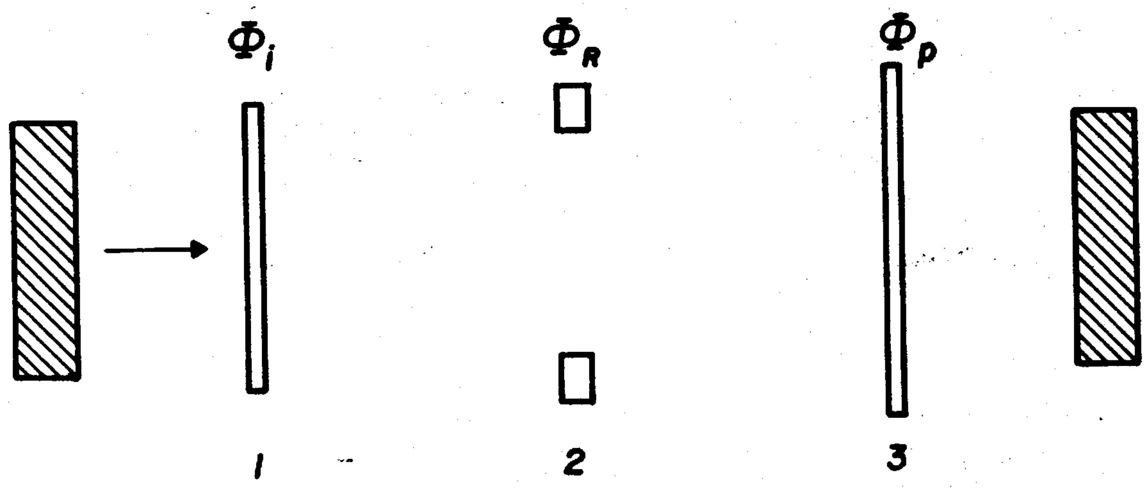

Fig. 7