COMMENTS ON PLASMA PHYSICS AND CONTROLLED FUSION

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A New Role for Infrared Lasers†

Aside from plasma diagnostics, the use of lasers in connection with controlled fusion has centered around two schemes: (1) ignition of a solid pellet by a Nd-glass or CO_2 laser,[1] and (2) heating of a gaseous plasma (produced by a θ-pinch or by the laser itself) by a CO_2 laser.[2] Development of a powerful far-infrared laser, such as the HCN laser, may make possible a third application: (3) plasma stabilization in conventional confinement devices. The wavelengths and critical densities (at which $\omega = \omega_p$) for these lasers are summarized below:

	λ (μm)	n_c (cm^{-3})
Nd-glass	1.06	10^{21}
CO_2	10.6	10^{19}
HCN	337	10^{16}

In pellet-fusion, no magnetic confinement is possible because of the large plasma density. The laser energy must be delivered in picosecond pulses, and this apparently causes insuperable problems with solid materials such as Nd-glass. A second problem is that the energy is deposited only in the skin, where $n = 10^{21}$ cm^{-3}; and some sort of rapid energy transfer must be presumed to take place to heat the core, where $n = 10^{23}$ cm^{-3}. Use of CO_2 lasers would greatly diminish the materials-damage problem, but the heat conduction problem becomes worse, since $n_c = 10^{19}$ cm^{-3}.

In CO_2-laser heating of gaseous plasmas, one is similarly caught between the horns of a dilemma. Because of the relatively low plasma density, magnetic confinement is needed to satisfy the Lawson criterion. If one operates at $n \approx n_c = 10^{19}$ cm^{-3}, multimegagauss fields would be required. If one presupposes reasonable fields (< 200 kG), the density is limited to $\sim 10^{17}$ cm^{-3}, and the absorption length is a sizable fraction of a kilometer. (A separate length criterion is imposed by the end losses of a linear system.) The best hope

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is to find a nonlinear absorption process that can occur for $n \ll n_c$. The large field strengths required for such nonlinear processes are much more easily achieved in a small focal spot than throughout the volume of plasma to be heated. (The self-focussing effect may help by dividing the beam into a number of small diameter, high-intensity beams, but there is the danger of convective plasma losses when the heating is done nonuniformly.)

In laser-feedback stabilization, a nonlinear effect is still involved, but the high intensity radiation is required only in a finite number of spots. Only the long-wavelength modes will be suppressed, but these are the dangerous ones for plasma confinement. In tokamaks, for instance, suppression of the lowest mode of the kink instability would allow operation at lower values of the quality factor q. Indeed, feedback stabilization by external windings is being implemented on new tokamaks and θ-pinches. Laser beams have the advantages of being easier to modulate at high frequencies and of interacting with the interior of a plasma.

The possibility of probeless coupling of electromagnetic radiation to a plasma was demonstrated experimentally by Wong, Baker, and Booth.[3] In this experiment, a beam of microwaves polarized in the extraordinary mode was directed into a plasmas column and modulated at the frequency ω_i of electrostatic ion cyclotron waves. A nonlinear coupling occurs when $\omega = \omega_h$, where ω is the microwave frequency and ω_h is the upper-hybrid resonance frequency. Since the plasma density was not uniform, the condition $\omega = \omega_h$ was met at only one radius; and ion cyclotron waves were locally excited there by modulation. Furthermore, since the modulation frequency was a natural oscillation frequency of the plasma, only a very small power (a fraction of a milliwatt) was needed for excitation. In stabilizing an unstable wave, such as a drift wave, even less power would be required, since it is only a matter of triggering an instability 180° out of phase with the one that is there. This coupling mechanism has been named the "double resonance effect". The high-frequency resonance, $\omega = \omega_h$, localizes the interaction, and the low-frequency resonance $\omega = \omega_i$, ensures that the coupling efficiency is high. The nonlinear mechanism for this coupling is not completely understood; in the following paragraphs the relevant literature will be reviewed.

In applying the double resonance effect to fusion plasmas, one would have to use far-infrared lasers. An HCN laser at 337-μm wavelength has $n_c = 10^{16}$ cm^{-3}. Pulsed power up to 450 W at this wavelength has been reported.[4] A laser resonating with a plasma of 10^{15} cm^{-3} density would have $\lambda \approx 1$ mm. Although lasers approaching this wavelength have been made to work, very little power has been achieved. (It is difficult to invert the population of two closely spaced levels.) Another possibility is to create the desired frequency $\omega \approx \omega_p$ or ω_h by down-conversion of 10-μm radiation from two CO_2 lasers operating on different lines. A loss of efficiency will necessarily result from the down-conversion, but the energy available with CO_2 lasers is so immense that this application of lasers may be the only one among those mentioned above which is feasible with current technology.

Three theoretical problems are involved: (A) amplification of the electromagnetic wave near the plasma cutoffs and resonances, (B) the nonlinear coupling mechanism that converts energy at $\omega \approx \omega_p$ to energy at ion frequencies (essentially dc), and (C) the nonlinear mechanism that converts the energy of two lasers at ω_1, $\omega_2 \gg \omega_p$ to energy at $\omega \approx \omega_p$.

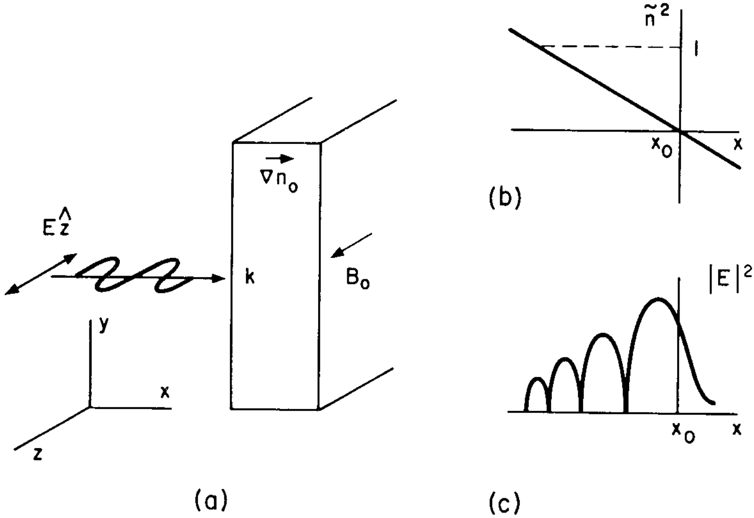

FIGURE 1.

A. Consider an electromagnetic wave incident on a plasma slab with density increasing monotonically in the x direction. The wave amplitude changes with x because the index of refraction $\tilde{n}$ changes. The qualitative behavior depends on the polarization of the wave and the angle of incidence. In Fig. 1(a) the wave is incident along ∇n_0, and $\mathbf{E}$ is along $\mathbf{B}_0$ (O-wave). The dc magnetic field $\mathbf{B}_0$ does not matter in this case, and the wave amplitude $|E|^2$ has the well-known behavior shown in Fig, 1(c). Far to the left of the cutoff at x_0 (where $\omega = \omega_p$), E varies as $\tilde{n}^{-1/2}$, as given by the WKB approximation. This approximation fails near x_0, where the wavelength becomes long; and the exact wave equation must be solved. For a linear density profile of slope n', this simple case[5-7] yields Bessel's equation of order 1/3, and the maximum amplitude is given by[8]

$$|E_m|^2 = 3.6\left(\frac{\omega/c}{n'/n_c}\right)^{1/3} E_0{}^2. \qquad (1)$$

The wave is completely reflected in the absence of collisions, and a standing wave exists to the left of x_0. Analytic solutions for other density profiles are also possible,[9] but Eq. (1) will always be obeyed approximately since the dependence on n' is weak, and any reasonable profile will behave linearly in the neighborhood of x_0. Because of the 1/3-power dependence in Eq. (1), it is difficult to achieve an enhancement factor much greater than about 15 in laboratory experiments. In the ionosphere, where the number of free-space wavelengths (at $\omega = \omega_p$) in a density scale distance is of the order of 10^4, $|E|^2$ may be enhanced by almost a factor 10^2. The same factor could occur in a fusion reactor of radius 10^2 cm, if 337-μm radiation is used.

We now turn to the case (Fig. 1a) of normal incidence with the E-vector perpendicular to B_0 (X-wave). The index of refraction (Fig. 1b) now has a zero (cutoff) at $x = x_1$ and an infinity (resonance) at $x = x_0$. The density at these points is given by

$$\omega = \omega_R \equiv \tfrac{1}{2}[\omega_c + (\omega_c{}^2 + 4\omega_p{}^2)^{1/2}] \quad \text{and} \quad \omega = \omega_h \equiv (\omega_c{}^2 + \omega_p{}^2)^{1/2},$$

respectively. The distance δ between these points is approximately

$$\delta \simeq (\omega_c/\omega)(n/n'). \tag{2}$$

The wave amplitude (Fig. 1c) now shows the typical cutoff behavior near x_1, a region of evanescence and tunneling between x_1 and x_0, and an infinity at x_0. This has been treated by a number of authors.[10-15] For a density profile which yields a linearly varying dielectric constant, E_x behaves like Whittaker's function $W_{k,m}$. Although the reflection and transmission coefficients are easily obtained, numerical computations for curves like Fig. 2(c) have only recently been accomplished.[16] The infinity at $x = x_0$, where $\omega = \omega_h \equiv (\omega_p{}^2 + \omega_c{}^2)^{1/2}$, is, of course, removed by collisions. In a collisionless plasma, finite electron Larmor radius effects limit the height of the resonance, and energy is coupled to the electrostatic Bernstein modes. This complicated problem has been extensively studied.[17-20]

The height of the resonance in Fig. 2(c) and, as we shall see, the slope of the curve are important for determining the strength of nonlinear interactions that occur there. It is clear that the height of the peak at x_0 depends not only on the dissipative mechanism but also on the thickness of the evanescence layer $\delta = x_0 - x_1$. In laboratory experiments such as Wong's,[3] this distance is comparable to the free space wavelength λ of the incident radiation; an appreciable fraction of this, therefore, tunnels through to the resonance point, where the field strength is enhanced. In a fusion reactor, however, δ/λ is of order 10^3, and essentially no radiation reaches x_0.

The physical reason for the peak in $|E_x|^2$ at $x = x_0$ can be understood as follows. Before the incident wave enters the plasma, $\mathbf{E}$ has only a transverse component in the y direction; this is entirely displacement current. After the

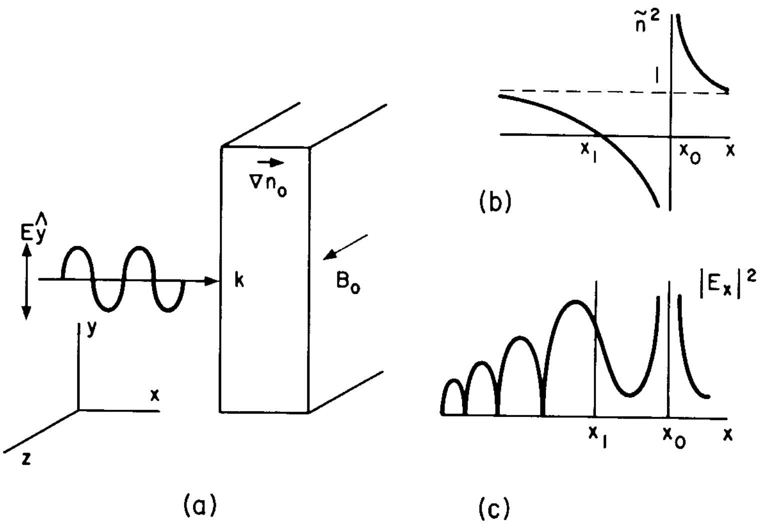

FIGURE 2.

wave enters the plasma, the plasma electrons, in responding to E_y, gyrate in circles because of $\mathbf{B}_0$. The x component of the electron motion is in the direction of $\mathbf{k}$, and the resulting oscillatory space charge causes the wave to develop a longitudinal component E_x. At the resonance point x_0, the wave is at the natural oscillation frequency of the electron fluid, and energy is continuously fed into the electrostatic oscillations. The oscillating space charge and E_x therefore grow to large amplitude. Note that it is the longitudinal component of $\mathbf{E}$ that becomes large; E_y is finite at x_0.

In the case of the O-wave (Fig. 1a), the electrons oscillate in the z direction, perpendicular to all gradients, and no space charge ever develops. The situation is entirely different if the wave enters the plasma *at a slight angle.* Consider now the case of Fig. 3, where an ordinary wave ($B_0 = 0$) enters the plasma at an angle θ_0 to the normal, with the E-vector *in the plane of incidence.* The electrons oscillate in the direction of $\mathbf{E}$, but since there is now a component of their motion in the ∇n_0 direction, a periodic space charge develops. This is shown in Fig. 3(b), where the thick arrows indicate an excess of electrons brought in from the zero-order density distribution. When the wave frequency is resonant with that of plasma waves, the electrostatic field builds up, and the initially transverse wave acquires a large longitudinal component. Because of this, the wave amplitude behavior, shown in Fig. 3(c), resembles that of normally incident X-waves for $B_0 \neq 0$. A cutoff occurs at $x = x_3$, where $\tilde{n} = \sin\theta$ or $\omega = \omega_p \sec\theta$; this is simply a consequence of Snell's law of refraction. There is an evanescent region, and then at $x = x_2$, where $\omega = \omega_p$, the resonance causes $|E_x|^2$ to rise to a value limited by collisions or Landau damping. This behavior was first pointed out by Ginzburg,[21] and numerical computations have been provided recently by Freidberg *et al.*[22]

In fusion applications the effect of nonnormal incidence is important because the tunneling distance can be controlled by adjusting θ, and the resonance can be made accessible. However, in reactors ω_c is of order ω_p, and it is interesting to see what the effect of $\mathbf{B}_0$ will be. In Fig. 4 an extraordinary wave is incident at an angle θ in the plane normal to $\mathbf{B}_0$. It can be

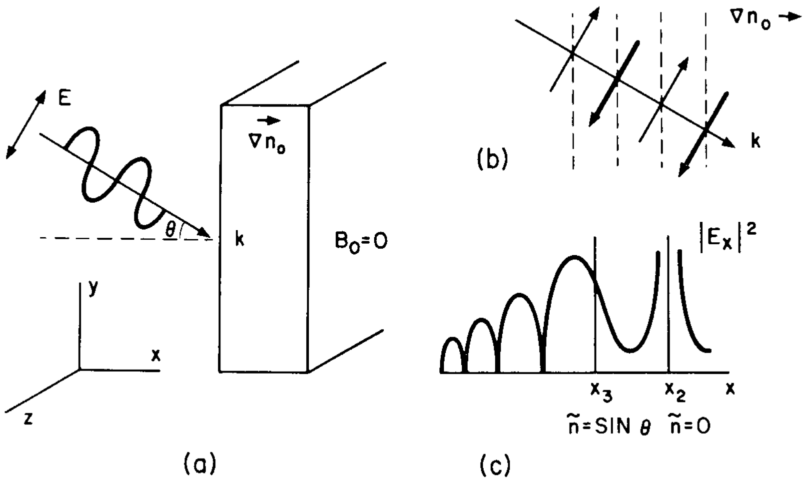

FIGURE 3.

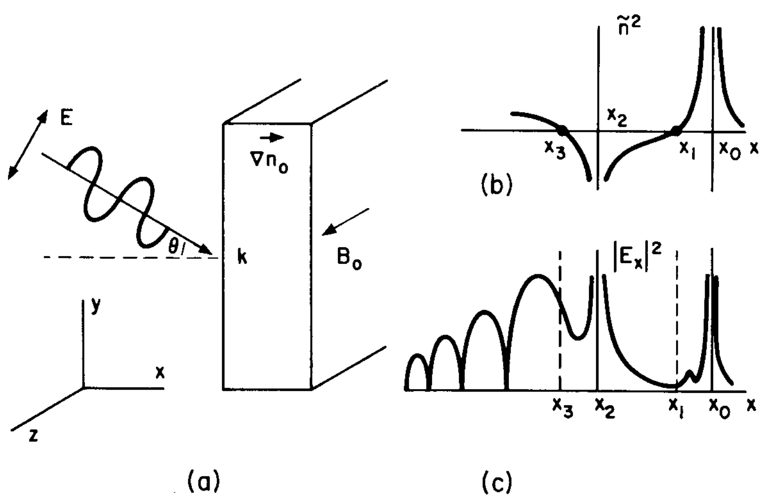

FIGURE 4.

shown[16] that the effective dielectric constant now has two resonances and two cutoffs (Fig. 4b). At $x = x_0$, there is the usual hybrid resonance $\omega = \omega_h$. At $x = x_2$, a second resonance occurs where $\omega^2 = \omega_c^2 + \omega_p^2 \sec^2 \theta$. The index of refraction vanishes at x_1 and at x_3, where there is reflection according to Snell's Law. The wave amplitude would vary as depicted in Fig. 4(c). As θ goes to 0, x_2 approaches x_0, x_1 disappears, and x_3 becomes the cutoff point x_1 of Fig. 2(c). As θ increases, the distance between x_3 and x_2 decreases, and that between x_2 and x_0 increases. To achieve high field strengths the angle θ would have to be chosen so as to optimize the tunneling between x_3 and x_2; the resonance at x_0 would then lie deep inside the plasma and would not be accessible.[16]

Finally, the angle of incidence can be adjusted so that **k** has a component k_z along $\mathbf{B}_0$. The *O*- and *X*-waves are then coupled, and the analysis is more complicated. Pearlstein and Bhadra[23] have treated this problem, but there are no new effects relevant to the problem of enhancement of the wave field.

B. We now turn to the question of how high-frequency electromagnetic energy is coupled to low-frequency plasma motions. First, consider the quasilinear drifts in a uniform, cold plasma describable by fluid equations. The second-order electron equation of motion will contain the terms $m\mathbf{v}^{(1)} \cdot \nabla \mathbf{v}^{(1)}$ and $-e\mathbf{v}^{(1)} \times \mathbf{B}^{(1)}$ representing second-order forces resulting from first-order motions. The $\mathbf{v}^{(1)} \times \mathbf{B}^{(1)}$ term gives a Lorentz force arising from the first-order motion in the wave's magnetic field. The $\mathbf{v}^{(1)} \cdot \nabla \mathbf{v}^{(1)}$ term gives a viscous force due to first-order velocity gradients. If the electromagnetic wave is an ordinary wave, $\mathbf{v}^{(1)}$ is 90° out of phase with $\mathbf{E}^{(1)}$, while $\mathbf{B}^{(1)}$ is in phase with $\mathbf{E}^{(1)}$. Consequently, $\mathbf{v}^{(1)} \times \mathbf{B}^{(1)}$ identically vanishes when averaged over an oscillation. Similarly, $\langle \mathbf{v}^{(1)} \cdot \nabla \mathbf{v}^{(1)} \rangle$ also vanishes, and there is no quasilinear drift. If the electromagnetic wave is an extraordinary wave, however, $\mathbf{E}^{(1)}$ has a longitudinal component. This component is 90° out of phase with the transverse component and gives rise to a time-averaged force $-e\langle \mathbf{v}^{(1)} \times \mathbf{B}^{(1)} \rangle$. This force is partly cancelled by the $\mathbf{v}^{(1)} \cdot \nabla \mathbf{v}^{(1)}$ term, but there remains a dc force **F** in the y direction in the geometry of Fig. 2, leading to an $\mathbf{F} \times \mathbf{B}_0$ drift in the $-x$ direction (out of the plasma).[24] In this low-β theory, the momentum is taken up by the dc magnetic field.

If the plasma is not uniform, one might think that this quasilinear drift along ∇n_0 would cause a charge separation and thus create an electric field to move the ions. Unfortunately, this does not happen. When the effect of ∇n_0 is properly introduced in calculating $\mathbf{v}^{(1)}$ and $\mathbf{B}^{(1)}$, and the second-order terms in the equation of continuity are retained, this quasilinear drift identically cancels out.[25] There is, however, another force proportional to $\nabla(E^2/8\pi)$; that is, the local radiation pressure.

This effect, it turns out, had previously been treated in connection with the upstream acceleration of ions in laser-pellet experiments and with laser self-focussing. The case $B_0 \neq 0$ was discussed by Gurevich and Pitaevskii,[26] and the case $B_0 = 0$ was discussed by Hora *et al.*[27,28] and Lindl and Kaw.[29] In these papers the general formulism for the "ponderomotive force" in a non-uniform dielectric was used. This treatment neglects convective effects like $\mathbf{v}^{(1)} \cdot \nabla \mathbf{v}^{(1)}$ and the dependence of the dielectric constant on field strength, but these effects are small, and the results of Hora are confirmed by more accurate calculations.[30] According to this picture, the radiation pressure of a laser beam is greatly enhanced near a resonance or cutoff because of the large gradient of $|E|^2$ there (cf. Figs 1–4). A local force is imparted to both species, and the force can be modulated to excite or suppress low-frequency waves. In the region of standing waves (Fig. 1c), the force is alternately inward and outward in consecutive quarter-wavelengths, leading to a stratification of the plasma. It is clear that accurate computations of the E-field gradients in a plasma are needed for predicting the ponderomotive effect; Lindl and Kaw[29] have done this for some cases. In addition to gradients along the beam, finite-diameter beams also have a gradient in the transverse direction. The radial ponderomotive force is used to explain self-focussing; it can also be used for modulational coupling.

There is a less sophisticated effect which can cause nonlinear coupling, and that is simple plasma heating. If the resistivity is finite, focussing a laser beam at a small spot will heat the plasma and cause a local ∇p force. Feedback stabilization by microwave heating at $\omega = \omega_h$ and $\omega = \omega_c$, respectively, has been reported by Hendel *et al.*[31] and by Keen and Fletcher.[32] It is likely that heating would be the dominant nonlinear mechanism in small-scale, low-temperature experiments. On the other hand, in Wong's experiment[3] the rise in electron temperature was measured directly and found to be negligible. In high-temperature plasmas heating would occur by Landau or cyclotron damping. Heating can be enhanced by modulation at ω_c or $\frac{1}{2}\omega_c$.[33]

Kaw and Dawson[34] have recently suggested another mechanism for coupling with ions. At electromagnetic wave intensities below the threshold for parametric instabilities, the frequency of ion waves can be shifted toward the region of Landau damping. Further nonlinear mechanisms may be possible in finite-Larmor-radius theories if the distortion of the electron orbits is taken into account. As far as we know, this question has not been explored.

We should point out the relation of the double-resonance effect to the rapidly growing field of parametric instabilities. Above a power threshold determined by damping effects, an electromagnetic wave will decay into an electron plasma wave and an acoustic wave, or will cause the so-called oscillating two-stream instability. Such parametric instabilities have been observed in the ionosphere.[35] The double-resonance effect differs from this in that the beam is modulated at an ion frequency, so that the threshold for nonlinear effects is considerably lower. This change in threshold has been demonstrated experimentally by Stenzel and Wong.[36] Another difference occurs in practice because the work on parametric instabilities up to now has been concerned mostly with the case $B_0 = 0$, whereas in the double-resonance work one is primarily interested in finite-B_0 instabilities, such as the drift or kink modes.

C. We now wish to consider how two CO_2-laser beams can be down-converted in frequency to radiation at $\omega \cong \omega_p$ or $\omega \cong \omega_h$ for densities of $10^{15} - 10^{16}$ cm^{-3}. The fundamental mechanism in this process is again the Lorentz force $-e\mathbf{v}^{(1)} \times \mathbf{B}^{(1)}$, where $\mathbf{v}^{(1)}$ is the electron velocity resulting from the E-field of one wave, and $\mathbf{B}^{(1)}$ is the magnetic field of the other wave. In the simplest case of two ordinary waves interacting with $B_0 = 0$, if $\mathbf{v}_1{}^{(1)}$ of wave 1 varies as $\sin \omega_1 t$, then $\mathbf{B}_2{}^{(1)}$ of wave 2 varies as $\cos \omega_2 t$. The product

$$\sin \omega_1 t \cos \omega_2 t = \tfrac{1}{2}[\sin(\omega_1 + \omega_2)t + \sin(\omega_1 - \omega_2)t] \tag{3}$$

gives sum and difference frequencies. In the previous section, we were concerned with the case $\omega_1 = \omega_2$ and therefore found no dc effect with O-waves alone. When $\omega_1 - \omega_2 = \omega_p$, however, plasma oscillations can be excited by the difference frequency, even without help from an X-wave, with its out-of-phase longitudinal component. In principle, it should also be possible to let $\omega_1 - \omega_2$ be an ion frequency, so that ion waves are excited by two frequencies within the same laser line; but this effect would be very weak.

The above-mentioned case of two ordinary waves with $B_0 = 0$ or with $\mathbf{k}_1, \mathbf{k}_2$ parallel to $\mathbf{B}_0$ was first treated in detail by Kroll, Ron, and Rostoker.[37] The vector relations are illustrated in Fig. 5(a). In Fig. 5, $\mathbf{k}_3$ and $\mathbf{E}_3$ refer to the difference-frequency wave. Since $\mathbf{v}_1{}^{(1)}$ is parallel to $\mathbf{E}^{(1)}$, it is clear that $\mathbf{v}_1{}^{(1)} \times \mathbf{B}_2{}^{(1)}$ is in the direction of $\mathbf{B}_0$ and therefore can excite an electrostatic wave along $\mathbf{B}_0$. There are also other possibilities. Weyl[38] has analyzed the case of Fig. 5(b), in which two ordinary waves propagate nearly perpendicularly to $\mathbf{B}_0$. If at least one of the waves has a component of $\mathbf{k}$ parallel to $\mathbf{B}_0$, there is a component of $\mathbf{E}_1{}^{(1)} \times \mathbf{B}_2{}^{(1)}$ along $\mathbf{B}_0$, and plasma oscillations at $\omega = \omega_p = \omega_1 - \omega_2$ can be excited along $\mathbf{B}_0$. Etievant *et al.*[39] treated the cases with $\mathbf{k}_1$ and $\mathbf{k}_2$ both perpendicular to $\mathbf{B}_0$. In Fig. 5(c), two O-waves are incident, producing an X-wave propagating in its lower passband. It is clear that $\mathbf{E}_1{}^{(1)} \times \mathbf{B}_2{}^{(1)}$ is perpendicular to $\mathbf{B}_0$ in this case, so that the product wave must have $\mathbf{E}_3 \perp \mathbf{B}_0$. If $\omega_1 - \omega_2$ is near ω_h, the X-wave becomes an electrostatic upper-hybrid oscillation, and the interaction is greatly enhanced because the small group velocity of this oscillation allows the energy to pile up. In Fig. 5(d), an O-wave and an X-wave interact to form an O-wave. In this case, it is seen that $E_1{}^{(1)}$ and $B_2{}^{(1)}$ are both in the $\mathbf{B}_0$ direction; the $\mathbf{v}_1{}^{(1)} \times \mathbf{B}_2{}^{(1)}$ term therefore vanishes, and the entire nonlinear effect comes from the $\mathbf{v}^{(1)} \cdot \nabla \mathbf{v}^{(1)}$ term. Two X-waves can also interact to form a third X-wave; this case (Fig. 5e) has been treated by Harker and Crawford.[40]

In the electromagnetic treatments[39,40] of the nonlinearly generated wave

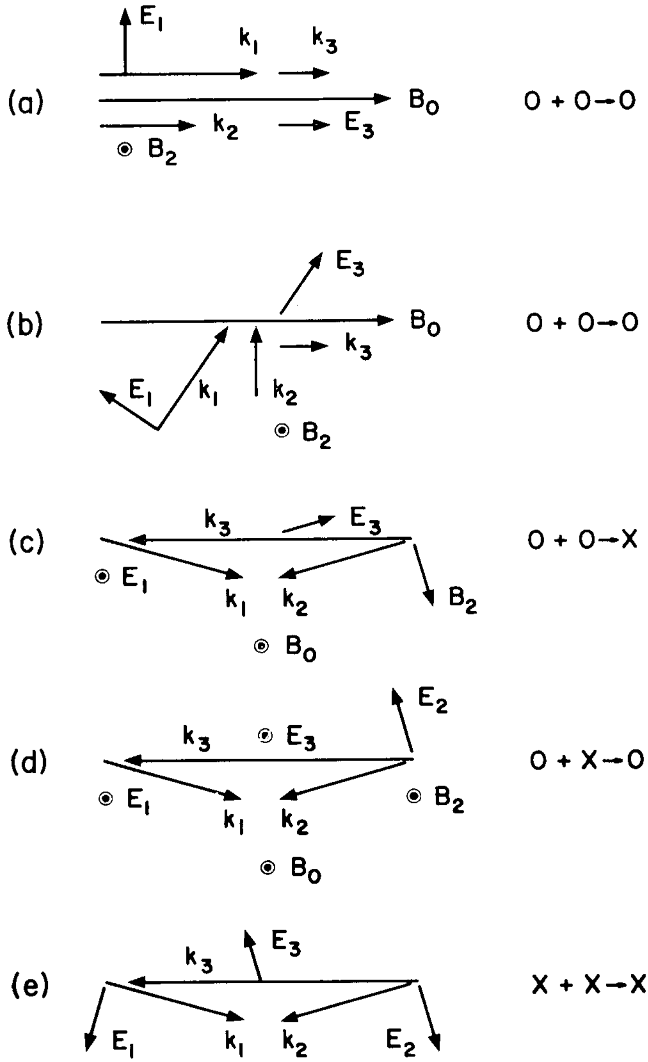
(a)
E1
k1
k3
B0
k2
E3
B2
O + O→O
(b)
E3
B0
k3
E1
k1
k2
B2
O + O→O
(c)
k3
E3
E1
k1
k2
B2
B0
O + O→X
(d)
E2
k3
E3
E1
k1
k2
B2
B0
O + X→O
(e)
E3
k3
E1
k1
k2
E2
X + X→X

FIGURE 5.

ω_3, the electrons are taken as a cold fluid. This work has also been extended to warm plasmas.[41] The interaction is always peaked near resonance, where the amplitude of ω_3 is limited by collisions. Since ω_3 becomes an electrostatic wave at resonance, it is possible to assume ω_3 to be electrostatic *a priori*. This is the approach followed by Kroll *et al.*,[37] Weyl,[38] and Barrett *et al.*[42] If one uses the Vlasov equation to describe the product wave, the limit imposed by Landau damping on its amplitude can be calculated. This has been done for $B_0 = 0$ by Kroll *et al.*[37] and for $B_0 \neq 0$ by Boyd and Turner.[43]

It should be noted that all of these theories are for homogeneous plasmas, and the wave amplification effect discussed in Section A would, in an inhomogeneous plasma, lower the threshold. Moreover, some of the restrictions imposed by applications to diagnostics need not be met in the present application to plasma stabilization. For instance, Etievant *et al.*[39] were interested in detecting ω_3 for density measurements; the accessibility of ω_3 to the outside of the plasma led to the restriction $\omega_p < \sqrt{2}\omega_c$. This condition need not be fulfilled if ω_3 is to couple to lower frequencies inside the plasma. In the scheme of Fig. 5(b), Weyl[38] found that the angle between $\mathbf{k}_1$ and $\mathbf{k}_2$ had to be smaller than the optimum value in order to avoid Landau damping of the plasma wave. In our application, it is not necessary to detect the wave $\mathbf{k}_3$; as long as energy is absorbed, there will be a gradient of the intensity of the incoming waves, leading to a ponderomotive force. The magnitude of the nonlinear effect, as predicted in Refs. 37–39, is such as to be easily observable with the megawatt lasers which are commonplace today. With the help of the wave amplification effect, multimegawatt lasers could possibly create a difference-frequency wave large enough to cause a low-frequency effect. This remains to be tested experimentally.

Recent work by Kaufman and Watson[44] indicates that the energy of two laser beams with $\omega_1 - \omega_2 \approx \omega_p$ can be coupled efficiently to low frequencies by a cascade process. By successive emission of plasmons of frequency ω_p, electromagnetic waves would decay to $\omega_2 - \omega_p$, $\omega_2 - 2\omega_p$, $\omega_2 - 3\omega_p$, etc. The emitted plasma oscillations, which are phase coherent, then decay parametrically into ion waves. The efficiency of energy absorption in a 1 cm length can be of the order of 30%, the only energy loss being the emission of sum-frequency waves. The threshold for this process is about 10^{13} W/cm^2.

Finally, we wish to review the experimental evidence for these nonlinear effects. Using 9 and 18 GHz microwaves launched into a plasma of 10^{12} cm^{-3} density in a magnetic field, Tetenbaum *et al.*[45] detected the generation of sum and difference frequencies with $\frac{1}{2}$–4 W of incident power. Stern and Tzoar[46] observed the decay of an electromagnetic wave into a plasma oscillation and an acoustic wave in a $B_0 = 0$ experiment. The threshold was about 3W at 4.4 GHz. In 1967, Hiroe and Ikegami[47] did a similar experiment with $B_0 \neq 0$. Above a threshold of about 2W, an incident X-wave with $\omega \approx \omega_h$ was found to excite an oscillation at the lower hybrid frequency. The double-resonance experiment of Wong *et al.*[3] was done in an alkali–metal plasma (Q-machine) with 2–4 GHz microwaves and densities below 10^{11} cm^{-3}. An incident X-wave power of only 0.3 mW/cm^2 was sufficient to excite electrostatic ion cyclotron waves of $n_1/n_0 = 5\%$ amplitude, and careful measurements with probes showed that this coupling occurred at the radius of resonance $\omega = \omega_h$. Since the distance **d** (Eq. 2) was smaller than the free-space wavelength in this experiment, the resonance was readily accessible by tunneling. In further experiments,[48] Wong detected a coupling also at cutoff, $\omega = \omega_R$; the density was adjusted so that hybrid resonance did not occur in the plasma. He also measured coupling with O-waves at their cutoff, $\omega = \omega_p$. The power required for cutoff-coupling was about an order of magnitude higher than for resonance-coupling, presumably because the field gradients are weaker at cutoff (Figs. 1 and 2). Cutoff-coupling was also observed in emission by Tetenbaum and Bailey.[49] An experiment by Phelps, Rynn, and van Hoven[50] was similar to that by Wong *et al.*[3] except that the 3 GHz microwaves were cavity-coupled rather than horn antenna-coupled to the plasma, and the cavity modes were carefully taken into account. Both plasma waves and ion acoustic waves could be generated by the beating of two microwave frequencies differing by the appropriate amount. In the case of ion waves, the power detected by probes was very small—about 100 db below the incident power—but it must be remembered that ion waves are heavily damped in a Q-machine with $T_e/T_i \approx 1$.

Nonlinear generation of electromagnetic waves, as predicted by Etievant *et al.*,[39] was tested by Cano, Fidone, and Zanfagna[51] with 9 and 21 GHz microwaves in an arc with $n = 10^{10}$–10^{12} cm^{-3} and $B \leqq 16$ kG. With about 5 W of incident power, sum and difference frequencies at about 10^{-12} W were detected. The poor efficiency was due to the fact that the coupling was done off-resonance. The only experiment reported so far using lasers is that of Stansfield, Nodwell, and Meyer,[52] which was designed to test the theory of Kroll *et al.*[37] ($B_0 = 0$). Light at about 7400 Å from a ruby laser was frequency-shifted by dye cells so that two beams differing in frequency by ω_p were incident in an arc plasma with $n \simeq 10^{16}$ cm^{-3}. The excitation of plasma oscillations was barely detectable by laser scattering with a third beam.

This last experiment pioneers the experimental study of the nonlinear optics of plasmas. This subject has both fundamental interest and possible application to controlled fusion. Further experimentation on the effects described above would be greatly facilitated by the development of a powerful far-infrared laser in the 300–800 μm range.

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References

1. J. M. Dawson, Phys. Fluids **7,** 981 (1964).
2. J. M. Dawson, A. Hertzberg, R. E. Kidder, G. C. Vlases, H. G. Ahlstrom, and L. C. Steinhauer, *Plasma Physics and Controlled Nuclear Fusion Research 1971,* (International Atomic Energy Agency, Vienna, 1971), Vol. I, p. 673.
3. A. Y. Wong, D. R. Baker, and N. Booth, Phys. Rev. Lett. **24,** 804 (1970); *Feedback and Dynamic Control of Plasmas,* T. K. Chu and H. W. Hendel, Eds. (Amer. Inst. of Physics, New York, 1970), pp. 84–93.
4. L. E. Sharp and A. T. Wetherell, Culham Laboratory Report CLM-P280 (1971).
5. L. M. Brekhovskikh, *Waves in Layered Media* (Academic Press, New York, 1970), p. 189 ff.
6. K. G. Budden, *Radio Waves in the Ionosphere* (Cambridge Univ. Press, 1961), p. 319 ff.
7. V. L. Ginzburg, *Propagation of Electromagnetic Waves in Plasmas* (Pergamon Press, New York, 1964), p. 192 ff.
8. Ibid., p. 366.
9. R. Papoular, J. Phys. Radium **22,** 749 (1961), for instance.
10. K. G. Budden, loc. cit., pp. 476–481.
11. V. L. Ginzburg, loc. cit., pp. 287–292.
12. R. Sugihara, J. Phys. Soc. Japan **21,** 786 (1966).
13. F. W. Sluijter, Proc. Int'l Conf. on Ioniz. Phenomena in Gases, Belgrade, 1966, **II,** 264 (1967).
14. G. M. Weyl, Phys. Rev. Lett. **25,** 1417 (1970).
15. A. Baños, Jr., UCLA PPG-124 and 125 (1972).
16. F. F. Chen and R. B. White, UCLA PPG-126 (1972).
17. T. H. Stix, Phys. Rev. Lett. **15,** 878 (1965).
18. H. H. Kuehl, Phys. Rev. **154,** 124 (1967).
19. T. W. Tang, Phys. Fluids **13,** 121 (1970).
20. C. L. Hedrick, Jr., Thesis, UCLA (1970).
21. V. L. Ginzburg, loc. cit., p. 213 ff.
22. J. P. Freidberg, R. W. Mitchell, R. L. Morse, and L. I. Rudsinski, Phys. Rev. Lett. **28,** 795 (1972).
23. L. D. Pearlstein and D. Bhadra, Phys. Fluids **12,** 213 (1969).
24. F. F. Chen and C. Etievant, Phys. Fluids **13,** 687 (1970).
25. N. Booth, Thesis, UCLA (1970).
26. A. V. Gurevich and L. P. Pitaevskii, Sov. Phys. JETP **18,** 855 (1964).
27. H. Hora, D. Pfirsch, and A. Schlüter, Z. Naturforsch. **22**a, 278 (1967).
28. H. Hora, Phys. Fluids **12,** 182 (1969) and in *Laser Interaction and Related Plasma Phenomena,* H. J. Schwarz and H. Hora, Eds. (Plenum Press, New York, 1971), p. 383 ff.
29. J. D. Lindl and P. K. Kaw, Phys. Fluids **14,** 371 (1971).
30. S. Bodner (private communication).
31. H. W. Hendel, T. K. Chu, F. W. Perkins, and T. C. Simonen, Phys. Rev. Lett. **24,** 90 (1970).
32. B. E. Keen and W. H. W. Fletcher, Phys. Rev. Lett. **25,** 350 (1970).
33. A. V. Gurevich and A. B. Shvartzburg, Geomagnetism and Aeronomy **8,** 893 (1968).
34. P. K. Kaw and J. M. Dawson, Phys. Fluids **14,** 792 (1971).
35. A. Y. Wong and R. J. Taylor, Phys. Rev. Lett. **27,** 644 (1971).
36. R. Stenzel and A. Y. Wong, Phys. Rev. Lett. **28,** 274 (1972).
37. N. M. Kroll, A. Ron, and N. Rostoker, Phys. Rev. Lett. **13,** 83 (1964).
38. G. Weyl, Phys. Fluids **13,** 1802 (1970).
39. C. Etievant, S. Ossakow, E. Ozizmir, C. H. Su, and I. Fidone, Phys. Fluids **11,** 1778 (1968).
40. K. J. Harker and F. W. Crawford, J. Appl. Phys. **40,** 3247 (1969).
41. R. Cano, C. Etievant, I. Fidone, and G. Granata, Nucl. Fusion **9,** 223 (1969).
42. E. B. Barrett, R. F. Whitmer, and S. J. Tetenbaum, Phys. Rev. A**135,** 369 (1964).

43. T. J. M. Boyd and J. G. Turner, J. Phys. **A5**, 132 (1972).
44. B. I. Cohen, A. N. Kaufman and K. M. Watson, Phys. Rev. Lett. **29**, 581 (1972).
45. S. J. Tetenbaum, R. F. Whitmer, and E. B. Barrett, Phys. Rev. **A135**, 374 (1964).
46. R. A. Stern and N. Tzoar, Phys. Rev. Lett. **15**, 485 (1965).
47. S. Hiroe and H. Ikegami, Phys. Rev. Lett. **19**, 1414 (1967).
48. A. Y. Wong *et al.*, *Fourth International Conf. on Plasma Physics and Controlled Nuclear Fusion Research*, Madison, Wisconsin (International Atomic Energy Agency, Vienna, 1971), Vol. I, p. 335.
49. S. J. Tetenbaum and H. N. Bailey, Phys. Rev. Lett. **19**, 12 (1967).
50. D. Phelps, N, Rynn, and G. van Hoven, Phys. Rev. Lett. **26**, 688 (1971).
51. R. Cano, I. Fidone, and B. Zanfagna, Phys. Fluids **14**, 811 (1971).
52. B. L. Stansfield, R. Nodwell, and J. Meyer, Phys. Rev. Lett. **26**, 1219 (1971).