27

Corr.: p.10

136

COLLISIONAL AND CONVECTIVE THRESHOLDS FOR RAMAN BACKSCATTER

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I. INTRODUCTION AND ABSTRACT

Various expressions for the threshold of stimulated Raman backscatter appear in the literature[1,2]. For an infinite, homogeneous plasma, one has

$$\frac{v_o^2}{c^2} = \frac{4\gamma_1\gamma_2\omega_1\omega_2}{\omega_o^2\omega_p^2} \qquad (1)$$

which depends on the finiteness of both damping rates γ_1 and γ_2 (the symbols will be defined later). For an inhomogeneous plasma with density scalelength L_n, one has

$$\frac{v_o^2}{c^2} = \frac{2}{k_o L_n}, \qquad (2)$$

which does not depend on collisions at all. For convective instability in homogeneous plasmas of finite length, one has[1]

$$2\gamma_o > \left(\frac{\gamma_1}{V_1} + \frac{\gamma_2}{V_2}\right)(V_1V_2)^{1/2}, \qquad (3)$$

where γ_o is the homogeneous growth rate and the V's are group velocities. Eq. (3) depends on collisions but not on the plasma length L. More puzzling is the fact that Eq. (3) is also the threshold for absolute instability in an infinite plasma.

We derive all three of these thresholds from a single straightforward treatment which clarifies the relationship among them and removes the mystery of their widely disparate functional forms.

II. WAVE EQUATIONS

We neglect pump depletion and take the incident (0) and reflected (2) light waves and the electrostatic plasma wave (1) to have the following forms:

$$\underline{\tilde{E}}_o = \hat{y}\, E_o \cos(k_o x - \omega_o t) \equiv \hat{y} E_o \cos \Phi_o \qquad (4)$$

$$\underline{\tilde{E}}_2 = \hat{y} E_2(x,t) \cos(k_2 x - \omega_2 t) \equiv \hat{y} E_2(x,t) \cos \Phi_2 \qquad (5)$$

$$\tilde{n}_1 = n_1(x,t) \cos(k_1 x - \omega_1 t + \delta) \equiv n_1(x,t) \cos \Phi_1 \qquad (6)$$

Here the ω's and k's are purely real, E_o is the constant pump amplitude, and $E_2(x,t)$ and $n_1(x,t)$ are the slowly varying peak amplitudes of the decay waves. To avoid complexity, the polarizations and k-vectors have been aligned. The phase δ for E_o and E_2 has been set equal to 0 by choice of origin for x and t. Note that k_2 is negative for backscatter and $k_1 \approx 2k_o$ in an underdense plasma.

The reflected wave

Maxwell's equations give, for a transverse wave,

$$-c^2 \underline{\nabla} \times \underline{\nabla} \times \underline{\tilde{E}} = c^2 \nabla^2 \underline{\tilde{E}} = \frac{\partial^2 \underline{\tilde{E}}}{\partial t^2} + 4\pi \frac{\partial \underline{j}}{\partial t} \tag{7}$$

Since $\nabla^2 \underline{\tilde{E}} = \hat{y} \partial^2 \tilde{E}_y / \partial x^2$, we have for the wave E_2

$$\frac{\partial \tilde{E}_2}{\partial x} = E_2' \cos \Phi_2 - k_2 E_2 \sin \Phi_2 \tag{8}$$

$$\frac{\partial^2 \tilde{E}_2}{\partial x^2} = E_2'' \cos \Phi_2 - 2k_2 E_2' \sin \Phi_2 - k_2^2 E_2 \cos \Phi_2, \tag{9}$$

where the prime indicates an x-derivative on the slow spatial scale. We therefore neglect the E_2'' term and write

$$-\nabla^2 \tilde{E}_2 = k_2^2 E_2 \cos \Phi_2 + 2k_2 E_2' \sin \Phi_2 \tag{10}$$

Similarly, neglecting the second derivative on the slow time scale yields

$$-\frac{\partial^2 \tilde{E}_2}{\partial t^2} = \omega_2^2 E_2 \cos \Phi_2 - 2\omega_2 \dot{E}_2 \sin \Phi_2. \tag{11}$$

The electron current $\underline{j}$ is due to the quiver velocity v_2 of wave 2 plus a nonlinear coupling term j_{NL}:

$$j = -en_o v_2 + j_{NL}. \tag{12}$$

Since KT has no effect on transverse waves, v_2 is given by the simple equation of motion

$$m \frac{\partial \tilde{v}_2}{\partial t} = -e\tilde{E}_2 - m\nu_e \tilde{v}_2, \tag{13}$$

where ν_e is the electron collision frequency with ions or neutrals. In the small collision term we may replace v_2 by the solution of

$$\frac{\partial \tilde{v}_2}{\partial t} = -\frac{e}{m} E_2 \cos \Phi_2,$$

or

$$\tilde{v}_2 = \frac{eE_2}{m\omega_2} \sin \Phi_2. \tag{14}$$

Thus

$$4\pi \frac{\partial j}{\partial t} = 4\pi \frac{\partial j_{NL}}{\partial t} + 4\pi e n_o \frac{e}{m} E_2 \left(\cos \Phi_2 + \frac{\nu_e}{\omega_2} \sin \Phi_2\right)$$

$$= 4\pi \frac{\partial j_{NL}}{\partial t} + \omega_p^2 E_2 \left(\cos \Phi_2 + \frac{\nu_e}{\omega} \sin \Phi_2\right). \tag{15}$$

Inserting Eqs. (10), (11), and (15) into Eq. (7), we obtain

$$(\omega_2^2 - \omega_p^2 - c^2k_2^2)E_2 \cos \Phi_2 - 2(\omega_2\dot{E}_2 + c^2k_2E_2' + \frac{\omega_p^2}{\omega_2}\frac{\nu_e}{2} E_2) \sin \Phi_2 = 4\pi \frac{\partial j_{NL}}{\partial t}.$$

Thus, a uniform plane wave ($E_2' = 0$) that is undriven has the frequency

$$\omega_2^2 = \omega_p^2 + c^2k_2^2 \tag{16}$$

and the damping rate γ_2 given by

$$-\dot{E}_2 = \gamma_2 E_2 = \frac{\omega_p^2}{\omega_2^2}\frac{\nu_e}{2} E_2. \tag{17}$$

Writing ν_e in terms of γ_2, we finally have for the wave E_2:

$$(\omega_2^2 - \omega_p^2 - c^2k_2^2)E_2 \cos \Phi_2 - 2(\omega_2\dot{E}_2 + c^2k_2E'_2 + \gamma_2\omega_2E_2) \sin \Phi_2 = 4\pi \frac{\partial j_{NL}}{\partial t}. \tag{18}$$

Here $\partial j_{NL}/\partial t$ stands for the derivative on the ω_2 time scale of the component of j_{NL} at frequency ω_2.

The plasma wave

The governing equations are

$$\underline{\nabla}\cdot\underline{E}_1 = -4\pi e n_1 \tag{19}$$

$$\frac{\partial n_1}{\partial t} + n_o\underline{\nabla}\cdot\underline{v}_1 = 0 \tag{20}$$

$$m\,\frac{\partial v_1}{\partial t} = -eE_1 - 3KT_e\,\frac{\nabla n_1}{n_o} - m\nu_e v_1 + \frac{F_{NL}}{n_o}\,, \tag{21}$$

where F_{NL} is the ponderomotive force of waves E_o and E_2.

Defining $$v_e^2 \equiv 3KT_e/m, \tag{22}$$

we may write the divergence of Eq. (21) as

$$\frac{\partial}{\partial t}\,\underline{\nabla}\cdot\underline{v}_1 = -\frac{e}{m}\,\underline{\nabla}\cdot\underline{E}_1 - \frac{v_e^2}{n_o}\,\nabla^2 n_1 - \nu_e\underline{\nabla}\cdot\underline{v}_1 + \frac{\underline{\nabla}\cdot\underline{F}_{NL}}{mn_o}\,. \tag{23}$$

The ν_e term can be written in terms of $\partial n_1/\partial t$ by Eq. (20). Taking the time derivative of Eq. (20) and substituting Eqs. (23) and (19), we obtain

$$\frac{\partial^2 n_1}{\partial t^2} - \frac{en_o}{m}\,(-4\pi e n_1) - v_e^2\nabla^2 n_1 + \nu_e\frac{\partial n_1}{\partial t} + \frac{\nabla\cdot F_{NL}}{m} = 0,$$

$$\frac{\partial^2 n_1}{\partial t^2} + \omega_p^2 n_1 - v_e^2\,\nabla^2 n_1 + \nu_e\,\frac{\partial n_1}{\partial t} + \frac{\nabla\cdot F_{NL}}{m} = 0, \tag{24}$$

where n_1 is really $\tilde{n}_1$. Its derivatives are, from Eq. (6),

$$\frac{\partial\tilde{n}_1}{\partial t} = \dot{n}_1\cos\Phi_1 + \omega_1 n_1\sin\Phi_1, \tag{25}$$

$$\frac{\partial^2\tilde{n}_1}{\partial t^2} = 2\omega_1\dot{n}_1\sin\Phi_1 - \omega_1^2 n_1\cos\Phi_1, \tag{26}$$

where we have neglected $\ddot{n}_1 \cos \Phi_1$. Similarly, the x derivative is approximately

$$-\nabla^2 \tilde{n}_1 = k_1^2 n_1 \cos \Phi_1 + 2k_1 n_1' \sin \Phi_1. \tag{27}$$

Substituting Eqs. (25-27) into Eq. (24), we obtain

$$(-\omega_1^2 + \omega_p^2 + k_1^2 v_e^2) n_1 \cos \Phi_1 + 2(\omega_1 \dot{n}_1 + k_1 v_e^2 n_1' + \omega_1 \frac{\nu_e}{2} n_1) \sin \Phi_1$$

$$= - \frac{\nabla \cdot F_{NL}}{m} - \nu_e \dot{n}_1 \cos \Phi_1. \tag{28}$$

We now neglect the last term in $\nu_e \dot{n}_1$, which is smaller than the $\omega_1 \dot{n}_1$ term if $\nu_e << \omega_1$. A uniform, undriven plane wave has the damping rate

$$-\dot{n}_1 = \gamma_1 n_1 = \frac{\nu_e}{2} n_1. \tag{29}$$

Replacing $\nu_e/2$ by γ_1, we finally have

$$(\omega_1^2 - \omega_p^2 - k_1^2 v_e^2) n_1 \cos \Phi_1 - 2(\omega_1 \dot{n}_1 + k_1 v_e^2 n_1' + \gamma_1 \omega_1 n_1) \sin \Phi_1 = \nabla \cdot F_{NL}/m. \tag{30}$$

III. NONLINEAR COUPLING

The nonlinear current j_{NL} that drives the wave E_2 is the result of the density modulation n_1 quivering in the wave E_o:

$$j_{NL} = -e \tilde{n}_1 \tilde{v}_o,$$

where v_o satisfies

$$m \frac{\partial \tilde{v}_o}{\partial t} = -eE_o \cos \Phi_o. \tag{31}$$

Thus we have

$$\tilde{v}_o = \frac{eE_o}{m\omega_o} \sin \Phi_o, \tag{32}$$

$$\tilde{n}_1 = n_1 \cos \Phi_1, \tag{33}$$

$$j_{NL} = -\frac{e^2}{m\omega_o} n_1 E_o \sin \Phi_o \cos \Phi_1. \tag{34}$$

$$= -\frac{e^2}{m\omega_o} n_1 E_o \frac{1}{2} [\sin(\Phi_o + \Phi_1) + \sin (\Phi_o - \Phi_1)]. \tag{35}$$

The frequency and k-number matching conditions are $\omega_o = \omega_1 + \omega_2$, $k_o = k_1 + k_2$, or

$$\Phi_o = \Phi_1 + \Phi_2 - \delta. \tag{36}$$

Thus the sin $(\Phi_o + \Phi_1)$ term is nonresonant with E_2 and can be discarded. We are left with

$$j_{NL} = -\frac{e^2 n_1 E_o}{2m\omega_o} \sin (\Phi_2 - \delta). \tag{37}$$

$$4\pi \frac{\partial j_{NL}}{\partial t} = \frac{\omega_p^2}{2} \frac{\omega_2}{\omega_o} \frac{n_1}{n_o} E_o \cos (\Phi_2 - \delta). \tag{38}$$

The ponderomotive force F_{NL} is due to the Lorentz interactions of waves E_o and E_2:

$$\underline{F}_{NL} = - en_o(\underline{v}_o \times \underline{B}_2 + \underline{v}_2 \times \underline{B}_o). \tag{39}$$

Eqs. (14) and (32) give v_2 and v_o, while $\underline{B}_j$ is the solution of

$$\frac{\partial \underline{B}_j}{\partial t} = - \underline{\nabla} \times \underline{E}_j \cos \Phi_j$$

$$= - \hat{z} \frac{\partial}{\partial x} [E_j \cos (k_j x - \omega_j t)]. \tag{40}$$

Thus $$B_j = (k_j/\omega_j) E_j \cos \Phi_j, \qquad j = 0, 2 \tag{41}$$

Using Eqs. (14), (32), and (41) in Eq. (39), we find the x component of F_{NL} to be

$$F_{NL} = - en_o \frac{eE_o}{m\omega_o} \sin \Phi_o \frac{k_2}{\omega_2} E_2 \cos \Phi_2 + (0 \leftrightarrow 2) \tag{42}$$

$$= -\frac{\omega_p^2}{\omega_o \omega_2} \frac{E_o E_2}{4\pi} (k_2 \sin \Phi_o \cos \Phi_2 + k_o \sin \Phi_2 \cos \Phi_o). \tag{43}$$

Here

$$\sin\Phi_o\cos\Phi_2 = \frac{1}{2}[\sin(\Phi_o + \Phi_2) + \sin(\Phi_o - \Phi_2)]$$

$$\sin\Phi_2\cos\Phi_o = \frac{1}{2}[\sin(\Phi_2 + \Phi_o) - \sin(\Phi_o - \Phi_2)]$$

Dropping the nonresonant terms and setting $\Phi_o - \Phi_2 = \Phi_1 - \delta$ and $k_2 - k_o = k_1$, we obtain

$$F_{NL} = \frac{\omega_p^2}{\omega_o\omega_2}\frac{E_oE_2}{8\pi}k_1\sin(\Phi_1 - \delta) \tag{44}$$

$$\frac{1}{m}\underline{\nabla}\cdot\underline{F}_{NL} = \frac{\omega_p^2k_1^2}{\omega_o\omega_2}\frac{E_oE_2}{8\pi m}\cos(\Phi_1 - \delta). \tag{45}$$

IV. COUPLED WAVE EQUATIONS

Inserting the nonlinear terms (38) and (45) into Eqs. (18) and (30), we obtain the result

$$(\omega_2^2 - \omega_p^2 - c^2k_2^2)E_2\cos\Phi_2 - 2(\omega_2\dot{E}_2 + c^2k_2E_2' + \gamma_2\omega_2E_2)\sin\Phi_2$$
$$= \frac{\omega_p^2}{2}\frac{\omega_2}{\omega_o}\frac{n_1}{n_o}E_o\cos(\Phi_2 - \delta). \tag{46}$$

$$(\omega_1^2 - \omega_p^2 - v_e^2k_1^2)n_1\cos\Phi_1 - 2(\omega_1\dot{n}_1 + v_e^2k_1n_1' + \gamma_1\omega_1n_1)\sin\Phi_1$$
$$= \frac{\omega_p^2}{\omega_o\omega_2}\frac{k_1^2}{m}\frac{E_oE_2}{8\pi}\cos(\Phi_1 - \delta). \tag{47}$$

V. INFINITE, HOMOGENEOUS PLASMA

Here $E_2' = n_1' = 0$, and at threshold $\dot{E}_2 = \dot{n}_1 = 0$ also. The lowest threshold will occur for exact frequency matching, so that the first terms in Eqs. (46) and (47) vanish. We then have

$$-2\gamma_2\omega_2 E_2 \sin\Phi_2 = \frac{\omega_p^2}{2}\frac{\omega_2}{\omega_o}\frac{n_1}{n_o} E_o\cos(\Phi_2 - \delta) \tag{48}$$

$$-2\gamma_1\omega_1 n_1 \sin\Phi_1 = \frac{\omega_p^2}{\omega_o\omega_2}\frac{k_1^2}{m}\frac{E_o E_2}{8\pi}\cos(\Phi_1 - \delta). \tag{49}$$

Choosing $\delta = -90^o$ and multiplying these together, we have

$$4\gamma_1\gamma_2\omega_1\omega_2 = \frac{\omega_p^4 k_1^2}{n_o m\omega_o^2}\frac{E_o^2}{16\pi}. \tag{50}$$

The threshold can be written in terms of the <u>peak</u> oscillating velocity v_o by using Eq. (32):

$$4\gamma_1\gamma_2\omega_1\omega_2 = \frac{\omega_p^2 k_1^2}{n_o m\omega_o^2}\cdot\frac{m^2\omega_o^2}{e^2}\frac{v_o^2}{16\pi}\cdot\frac{4\pi n_o e^2}{m} = \frac{v_o^2}{4\pi}k_1^2\omega_p^2,$$

$$\boxed{v_o^2 = \frac{16\gamma_1\gamma_2\omega_1\omega_2}{k_1^2\omega_p^2}}\ . \tag{51}$$

This is the usual homogeneous threshold, Eq. (1). Substituting $k_1 \simeq 2k_o$ and the γ's given by Eqs. (17) and (29), we can write this as

$$v_o^2 = \frac{\omega_1}{\omega_2}\frac{\nu_e^2}{k_o^2} \quad \text{or} \quad \frac{v_o^2}{c^2} \simeq \frac{\omega_p}{\omega_o}\frac{\nu_e^2}{\omega_o^2}. \tag{52}$$

The growth rate γ is found by setting $\dot{E}_2 = \gamma E_2$, $\dot{n}_1 = \gamma n_1$ in Eqs. (46) and (47) and again neglecting the space derivative and the frequency mismatch. Far enough above threshold that $\gamma >> \gamma_1, \gamma_2$, the damping terms can also be neglected. The value of γ is then called the homogeneous growth rate γ_o:

$$-2\omega_2\gamma_o E_2 \sin\Phi_2 = \frac{\omega_p^2}{2}\frac{\omega_2}{\omega_o}\frac{n_1}{n_o} E_o\cos(\Phi_2 - \delta) \tag{53}$$

$$-2\omega_1\gamma_o n_1 \sin\Phi_1 = \frac{\omega_p^2}{\omega_o\omega_2}\frac{k_1^2}{m}\frac{E_o E_2}{8\pi}\cos(\Phi_1 - \delta) \tag{54}$$

These are the same equations as (48) and (49), except that γ_o^2 replaces $\gamma_1\gamma_2$. Thus the solution from Eq. (51) is

$$\boxed{\gamma_o^2 = \frac{v_o^2}{16}\,\frac{k_1^2\,\omega_p^2}{\omega_1\omega_2}}\quad , \tag{55}$$

or, for $k_1 \simeq 2k_o$, $\omega_2 \cong \omega_o = ck_o$, $\omega_1 \simeq \omega_p$,

$$\gamma_o^2 = \frac{1}{4}\,\frac{v_o^2}{c^2}\,\omega_o\omega_p, \tag{56}$$

$$\gamma_o = \frac{1}{2}\,\frac{v_o}{c}\,(\omega_o\omega_p)^{\frac{1}{2}}\ . \tag{57}$$

This is the usual formula for the homogeneous growth rate of SRS. Note that the threshold (51) can be expressed as

$$\gamma_o^2 = \gamma_1\gamma_2. \tag{58}$$

VI. FINITE, HOMOGENEOUS PLASMAS

Since the plasma is homogeneous within an interaction length L, we can assume perfect frequency matching for $0 < x < L$ and neglect the first terms in Eqs. (46) and (47). We consider a steady state with $\dot{E}_2 = \dot{n}_1 = 0$, obtaining

$$-2(c^2k_2E_2' + \gamma_2\omega_2E_2)\sin\Phi_2 = \frac{\omega_p^2}{2}\,\frac{\omega_2}{\omega_o}\,\frac{n_1}{n_o}\,E_o\cos(\Phi_2 - \delta) \tag{59}$$

$$-2(v_e^2k_1n_1' + \gamma_1\omega_1n_1)\sin\Phi_1 = \frac{\omega_p^2}{\omega_o\omega_2}\,\frac{k_1^2}{m}\,\frac{E_oE_2}{8\pi}\cos(\Phi_1 - \delta) \tag{60}$$

We can write these in terms of the group velocities V_1 and V_2 of waves 1 and 2. Differentiating the plasma wave dispersion relation

$$\omega_1^2 = \omega_p^2 + k_1^2v_e^2\ , \tag{61}$$

we find

$$V_1 = \frac{d\omega_1}{dk_1} = \frac{k_1v_e^2}{\omega_1}\ . \tag{62}$$

Similarly, from Eq. (16) we find

$$V_2 = \left|\frac{k_2 c^2}{\omega_2^2}\right|. \tag{63}$$

Here k_2 is negative, but V_2 is defined as a positive quantity. Dividing Eqs. (59) and (60) by $-2\omega_2$ and $-2\omega_1$, respectively, we have

$$(-V_2 E_2' + \gamma_2 E_2)\sin\Phi_2 = -\frac{\omega_p^2}{4\omega_o}\frac{n_1}{n_o}E_o\cos(\Phi_2 - \delta) \tag{64}$$

$$(V_1 n_1' + \gamma_1 n_1)\sin\Phi_1 = -\frac{\omega_p^2}{2\omega_o\omega_2}\frac{k_1^2}{m\omega_1}\frac{E_o E_2}{8\pi}\cos(\Phi_1 - \delta). \tag{65}$$

Again the phase δ of $\tilde{n}_1$ is obviously -90°, and the other factors are just what we called γ_o^2 in Eq. (56). Thus

$$\left(-V_2 E_2' + \gamma_2 E_2\right)\left(V_1 n_1' + \gamma_1 n_1\right) = \gamma_o^2 n_1 E_2 ,$$

$$\left(-V_2\frac{E_2'}{E_2} + \gamma_2\right)\left(V_1\frac{n_1'}{n_1} + \gamma_1\right) = \gamma_o^2 . \tag{66}$$

Let E_2 and n_1 have the same spatial e-folding rate κ, with E_2 growing in the -x direction and n_1 in the +x direction:

$$E_2' = -\kappa E_2 , \qquad n_1' = \kappa n_1 . \tag{67}$$

Now we have

$$\left(\kappa + \frac{\gamma_2}{V_2}\right)\left(\kappa + \frac{\gamma_1}{V_1}\right) = \frac{\gamma_o^2}{V_1 V_2} , \tag{68}$$

$$\kappa^2 + \kappa\left(\frac{\gamma_1}{V_1} + \frac{\gamma_2}{V_2}\right) + \frac{\gamma_1\gamma_2}{V_1 V_2} = \frac{\gamma_o^2}{V_1 V_2} , \tag{69}$$

$$2\kappa = -\left(\frac{\gamma_1}{V_1} + \frac{\gamma_2}{V_2}\right) \pm \left[\left(\frac{\gamma_1}{V_1} + \frac{\gamma_2}{V_2}\right)^2 + 4\,\frac{\gamma_o^2 - \gamma_1\gamma_2}{V_1 V_2}\right]^{\frac{1}{2}}. \tag{70}$$

There is a positive root κ, indicating growth, as soon as γ_o exceeds the homogeneous threshold $\gamma_o^2 = \gamma_1\gamma_2$. However, for appreciable growth in a length L, we require $\kappa L > 1$. If the first term in the discriminant dominates, this severe condition cannot be satisfied by increasing γ_o. Therefore, when convection is the limiting factor, the γ_o^2 term in the discriminant must dominate. Assuming $\gamma_o^2 >> \gamma_1\gamma_2$, we then have

$$2\gamma_o > \left(\frac{\gamma_1}{V_1} + \frac{\gamma_2}{V_2}\right)\sqrt{V_1V_2} \quad , \tag{71}$$

which is Eq. (3). What this means is that in any given Δx within the interaction length the amount of wave energy gained by instability is larger than that lost by damping. The total length L does not matter. Of course, if the plasma is infinite, this condition means that the wave can grow everywhere at all times, which means absolute instability.

Suppose now that γ_o satisfies Eq. (71). Then κ is approximately

$$\kappa \simeq \gamma_o(V_1V_2)^{-\frac{1}{2}} \tag{72}$$

Appreciable growth still requires $\kappa L >> 1$, or, from Eqs. (62) and (63),

$$\gamma_o L >> (V_1V_2)^{\frac{1}{2}} = \left(\frac{ck_1v_e^2}{\omega_1}\right)^{\frac{1}{2}} \tag{73}$$

Using Eq. (57) for γ_o and noting that

$$\frac{v_e^2}{\omega_p^2} = 3\lambda_D^2 \quad , \tag{74}$$

we can write the convective threshold for underdense plasmas as

$$\boxed{\frac{v_o^2}{c^2} >> 24\,\frac{\lambda_D^2}{L^2}} \; . \tag{75}$$

VII. INHOMOGENEOUS PLASMAS

When ω_p or v_e varies with x, perfect phase matching cannot be maintained throughout the interaction region. Since ω_1 is fixed by $\omega_1 = \omega_o - \omega_2$, k_1 must vary in space to satisfy the local dispersion relation

$$\omega_1^2 = \omega_p^2(x) + v_e^2(x)k_1^2(x). \tag{76}$$

Let
$$k_1(x) = k_{10} + \Delta(x), \tag{77}$$

where $k_{10} = k_1(0)$ satisfies $k_o = k_{10} + k_2$, the origin $x = 0$ being defined as the point where k-matching is perfect. To simplify the discussion, we neglect the gradient in T_e and the relatively small changes in k_o and k_2 in an underdense plasma. Taking $\delta = -\pi/2$, as we found previously, we can write $\tilde{n}_1$ as $\tilde{n}_1 = n_1 \cos \Phi_1$, where

$$\Phi_1(x) = \int_0^x k_1(x')dx' - \omega_1 t - \frac{\pi}{2}$$

$$= k_{10}x - \omega_1 t - \frac{\pi}{2} + \phi(x), \tag{78}$$

with
$$\phi(x) \equiv \int_0^x \Delta(x')dx' , \tag{79}$$

$$\Phi_1'(x) = k_{10} + \Delta(x) = k_1(x). \tag{80}$$

The spatial derivatives of $\tilde{n}_1$ are now

$$\frac{\partial \tilde{n}_1}{\partial x} = n_1' \cos \Phi_1 - k_1(x) n_1 \sin \Phi_1 \tag{81}$$

$$-\frac{\partial^2 \tilde{n}_1}{\partial x^2} \simeq k_1^2 n_1 \cos \Phi_1 + 2k_1 n_1' \sin \Phi_1 + k_1' n_1 \sin \Phi_1 \tag{82}$$

Eq. (18) for E_2 remains unchanged as long as we neglect the variation in k_2, but Eq. (30) now becomes

$$[\omega_1^2 - \omega_p^2(x) - v_e^2 k_1^2(x)]n_1 \cos \Phi_1 - 2[\omega_1 \dot{n}_1 + v_e^2 k_1(x) n_1' + \gamma_1 \omega_1 n_1$$

$$+ \frac{1}{2} v_e^2 k_1'(x) n_1] \sin \Phi_1 = \nabla \cdot F_{NL}/m. \tag{83}$$

The ponderomotive term, from Eq. (44), is

$$F_{NL} = \frac{\omega_p}{\omega_o \omega_2} \frac{E_o E_2}{8\pi} (k_o - k_2) \sin(\Phi_o - \Phi_2). \tag{84}$$

From Eq. (78) and the definition of k_{10}, we have

$$k_o - k_2 = k_{10}$$

and

$$\Phi_o - \Phi_2 = k_{10} x - \omega_1 t = \Phi_1 + \frac{\pi}{2} - \phi(x), \tag{85}$$

so that

$$F_{NL} = \frac{\omega_p^2}{\omega_o \omega_2} \frac{E_o E_2}{8\pi} k_{10} \sin(\Phi_1 + \frac{\pi}{2} - \phi). \tag{86}$$

Using Eqs. (79), (80), and (77), we have

$$\frac{1}{m} \frac{\partial}{\partial x} F_{NL} = \frac{\omega_p^2}{\omega_o \omega_2} \frac{E_o E_2}{8\pi m} k_{10} \cos(\Phi_1 + \frac{\pi}{2} - \phi)[k_1(x) - \Delta(x)]$$

$$= - \frac{\omega_p^2}{\omega_o \omega_2} \frac{E_o E_2}{8\pi m} k_{10}^2 \sin(\Phi_1 - \phi). \tag{87}$$

Similarly, the other coupling term, from Eqs. (35) and (85), is

$$j_{NL} = - \frac{e^2}{2m\omega_o} n_1 E_o \sin(\Phi_o - \Phi_1)$$

$$= - \frac{e^2}{2m\omega_o} n_1 E_o \sin(\Phi_2 + \frac{\pi}{2} - \phi) = - \frac{e^2}{2m\omega_o} n_1 E_o \cos(\Phi_2 - \phi) \tag{88}$$

Thus

$$4\pi \frac{\partial}{\partial t} j_{NL} = - \frac{\omega_p^2}{2} \frac{\omega_2}{\omega_o} \frac{n_1}{n_o} E_o \sin(\Phi_2 - \phi). \tag{89}$$

The coupled equations (46) and (47) now become

$$[\omega_1^2 - \omega_p^2(x) - v_e^2 k_1^2(x)] n_1 \cos \Phi_1 - 2(\omega_1 \dot{n}_1 + v_e^2 k_1 n_1' + \gamma_1 \omega_1 n_1 + \frac{1}{2} v_e^2 k_1' n_1) \sin \Phi_1$$

$$= - \frac{\omega_p^2}{\omega_o \omega_2} \frac{E_o E}{8\pi m} k_{10}^2 \sin[\Phi_1 - \phi(x)] \tag{90}$$

$$(\omega_2^2 - \omega_p^2 - c^2k_2^2)E_2\cos\Phi_2 - 2(\omega_2\dot{E}_2 + c^2k_2E_2' + \gamma_2\omega_2E_2)\sin\Phi_2$$

$$= -\frac{\omega_p^2}{2}\frac{\omega_2}{\omega_o}\frac{n_1}{n_o}E_o\sin[\Phi_2 - \phi(x)]. \qquad (91)$$

If $k_1(x)$ satisfies the local dispersion relation (76), the first terms can be dropped, and again introducing the group velocities (62) and (63), we obtain

$$(\dot{n}_1 + V_1n_1' + \gamma_1n_1 + \tfrac{1}{2}V_1'n_1)\sin\Phi_1 = \frac{\omega_p^2}{\omega_o\omega_2}\frac{E_oE_2}{16\pi}\frac{k_{10}^2}{m\omega_1}\sin(\Phi_1 - \phi) \qquad (92)$$

$$(\dot{E}_2 - V_2E_2' + \gamma_2E_2)\sin\Phi_2 = \frac{\omega_p^2}{4}\frac{E_o}{\omega_o}\frac{n_1}{n_o}\sin(\Phi_2 - \phi). \qquad (93)$$

We learned in Eq. (50) that the product of the factors on the r.h.s. is just $\gamma_o^2n_1E_2$. These two equations can be symmetrized by splitting the difference. We therefore introduce the new variables

$$A_1 \equiv n_1/C , \qquad A_2 \equiv CE_2 , \qquad (94)$$

where

$$C \equiv \frac{k_{10}}{2}\left(\frac{n_o}{\pi m\omega_1\omega_2}\right)^{\frac{1}{2}} = \frac{k_{10}}{4\pi e}\left(\frac{\omega_p^2}{\omega_1\omega_2}\right)^{\frac{1}{2}} . \qquad (95)$$

In exponential notation Eqs. (92) and (93) can now be written

$$\frac{\partial A_1}{\partial t} + V_1\frac{\partial A_1}{\partial x} + \gamma_1A_1 + \tfrac{1}{2}V_1'A_1 = \gamma_oA_2e^{-i\phi(x)} \qquad (96)$$

$$\frac{\partial A_2}{\partial t} - V_2\frac{\partial A_2}{\partial x} + \gamma_2A_2 = \gamma_oA_1e^{-i\phi(x)} . \qquad (97)$$

With the exception of the odd term in Eq. (96), these are the usual equations[1] for an inhomogeneous plasma. The ratio of the V_1' term to the V_1 term is

$$\frac{V_1'A_1}{2V_1A_1'} \simeq \frac{k_1'}{2k_1}\frac{A_1}{A_1'} \simeq \frac{L}{2}\frac{k_1'}{k_1} ,$$

where L is the interaction length. From Eq. (76), we have

$$\frac{k_1'}{k_1} = -\frac{\omega_p^{2'}}{v_e^2}\frac{1}{2k_1^2} = -\frac{1}{2}\frac{\omega_p^2}{k_1^2}\frac{1}{L_n v_e^2} ,$$

where L_n is the density scalelength. Thus

$$\left|\frac{V_1'A_1}{2V_1A_1'}\right| \simeq \frac{1}{4}\frac{L}{L_n}\frac{1}{3k_1^2\lambda_D^2} \simeq \frac{1}{48}\frac{L}{L_n}\frac{1}{k_o^2\lambda_D^2} . \qquad (98)$$

In a typical experiment $k_1^2\lambda_D^2$ might be 10^{-2} and $L_n/L = 10$ or more. Thus the ratio (98) is of order unity, and the neglect of the V_1' term is not justified.

The customary procedure is not only to neglect the V_1' term but also to assume V_1, γ_1, and γ_o to be constant in Eqs. (96) and (97), when in fact all these vary with density. Then one uses a Laplace transform $\partial A_1/\partial t = \gamma A_1$, $\partial A_2/\partial t = \gamma A_2$ to obtain

$$V_1\frac{\partial A_1}{\partial x} + (\gamma + \gamma_1)A_1 = \gamma_o A_2 e^{-i\phi(x)} \qquad (99)$$

$$-V_2\frac{\partial A_2}{\partial x} + (\gamma + \gamma_2)A_2 = \gamma_o A_1 e^{-i\phi(x)} , \qquad (100)$$

the function $\phi(x)$ depending on the density profile. One variable can be eliminated to give a single second-order differential equation. By a standard change of variable, the first derivative term can then be eliminated to bring the equation into WKB form. For particular density profiles, the equation can be brought into the form of a standard confluent hypergeometric equation and can be solved. For profiles like linear or parabolic density distributions, the equation can be solved in the WKB approximation to give the eigenvalues of γ. The instability criterion is then $\gamma > 0$.

Though mathematically elegant, this procedure is not guaranteed to give an accurate threshold because Eqs. (99) and (100) are themselves very approximate. We therefore employ a simpler procedure. We consider the plasma to be homogeneous between the turning points of the WKB problem; that is, the points where the accumulated phase error $\delta(x)$ is of order $\pi/2$. We then apply the distance between turning points to the finite-length criterion, Eq. (75). In a homogeneous plasma, Eqs. (99) and (100) are accurate. Multiplying them together, we see that

$$\left(\gamma + \gamma_1 + V_1 \frac{A_1'}{A_1}\right)\left(\gamma + \gamma_2 - V_2 \frac{A_2'}{A_2}\right) = \gamma_o^2 e^{-2i\phi(x)} . \tag{101}$$

The driving term on the r.h.s. goes to 0 when $2\phi(x) \simeq \pi/2$, or $\phi(x) \simeq \pi/4$. Hence, we take this to be the interaction length $\phi(L) \simeq \pi/4$. We may expand $\Delta(x)$ in Eq. (77) in a Taylor series around the phase-matched point, $x = 0$:

$$\Delta(x) = k_1(x) - k_o + k_2 = x\Delta'(0) + \frac{1}{2} x^2 \Delta''(0) + \ldots \tag{102}$$

Let $n = n_o$, $\omega_p = \omega_{po}$ at $x = 0$. Then the plasma wave dispersion relation (76) can be written (if T_e = const.)

$$\omega_1^2 = \omega_{po}^2 \frac{n(x)}{n_o} + v_e^2 k_1^2(x). \tag{103}$$

Successive derivatives with fixed ω_1 give

$$\begin{aligned} \frac{\omega_{po}^2}{n_o} n'(x) &= -2v_e^2 k_1 k_1'(x) \\ \frac{\omega_{po}^2}{n_o} n''(x) &= -2v_e^2 (k_1 k_1'' + k_1'^2) \end{aligned} \tag{104}$$

Thus

$$\begin{aligned} \Delta'(x) &= k_1'(x) = -\frac{\omega_{po}^2 n'(x)}{2n_1 v_e^2 k_1} \\ \Delta''(x) &= k_1''(x) = -\frac{1}{k_1}\left(\frac{\omega_{po}^2 n''(x)}{2n_o v_e^2} + k_1'^2\right) \end{aligned} \tag{105}$$

Linear profile

Let $$n(x) = n_o\left(1 \pm \frac{x}{L_n}\right) . \tag{106}$$

Then $$n'(x) = \pm \frac{n_o}{L_n} , \qquad n''(x) = 0 , \tag{107}$$

and $$\Delta' = - \frac{\omega_{po}}{2n_1 v_e^2 k_1} \left(\pm \frac{n_o}{L_n} \right) = \mp \frac{1}{2} \frac{\omega_{po}}{v_e^2} \frac{1}{k_1 L_n} = \mp (6k_1 L_n \lambda_D^2)^{-1} , \tag{108}$$

$$\Delta'' = - \frac{k_1'^2}{k_1} = - \frac{\Delta'^2}{k_1} , \tag{109}$$

where $k_1 = k_1(0)$. The turning-point condition is

$$\delta(L) = \int_o^L \Delta(x)dx = \pm \frac{\pi}{4} \simeq \pm 1. \tag{110}$$

Substituting from Eq. (102), we have

$$\int_o^L [x\Delta'(0) + \frac{1}{2} x^2 \Delta''(0)] \, dx = \pm 1. \tag{111}$$

Thus

$$\Delta'(0) \frac{1}{2} L^2 + \Delta''(0) \frac{1}{6} L^3 = \pm 1,$$

$$\frac{L^2}{2} \Delta' \left(1 - \frac{L}{3} \frac{\Delta'}{k_1} \right) = \pm 1. \tag{112}$$

If the second term were negligible, we would have $|\Delta'| = 2/L^2$. Substituting this into the second term, we find its magnitude to be $|L\Delta'/3k_1| = 2/3k_1 L \simeq (3k_o L)^{-1} << 1$. Thus we may neglect the Δ'' term and choose the sign in Eq. (110) so that L^2 is positive regardless of the sign in Eq. (106). Eq. (108) then gives

$$L^2 = 12k_1 L_n \lambda_D^2 . \tag{113}$$

Inserting this into the finite length threshold (75), we have

$$\frac{v_o^2}{c^2} >> \frac{2}{k_1 L_n} \simeq \frac{1}{k_o L_n} \tag{114}$$

This is within a factor of 2 of the usual condition for convective instability in an inhomogeneous plasma, Eq. (2). Note that no plasma parameters appear except for density scalelength.

Parabolic profile

Now let

$$n = n_o\left(1 \pm \frac{x^2}{L_n^2}\right), \tag{115}$$

so that

$$n' = \pm 2n_o x/L_n^2 , \qquad n'' = \pm 2n_o/L_n^2 , \tag{116}$$

$$\Delta'(0) = 0 , \qquad \Delta''(0) = \mp \frac{\omega_{po}^2}{2n_o k_1 v_e^2} \frac{2n_o}{L_n^2} = \mp (3k_1 L_n^2 \lambda_D^2)^{-1} \tag{117}$$

Eq. (111) now gives, for either a density hill or density trough,

$$L^3 = 18 k_1 L_n^2 \lambda_D^2. \tag{118}$$

Eq. (75) can be written

$$\frac{v_o^3}{c^3} >> 24^{3/2} \lambda_D^3/L^3 ,$$

so that Eq. (118) leads to

$$\frac{v_o^3}{c^3} >> \left(\frac{128}{3}\right)^{1/2} \frac{\lambda_D}{k_1 L_n^2} \cong \left(\frac{32}{3}\right)^{1/2} \frac{\lambda_D}{L_n} \frac{1}{k_o L_n} . \tag{119}$$

This is equal to the linear-profile threshold with the same L_n when

$$\frac{v_o}{c} = \left(\frac{32}{3}\right)^{1/2} \frac{\lambda_D}{L_n} .$$

For reasonable plasma parameters, this corresponds to a 10-μm intensity of

$\simeq 10^5$ W/cm^2, so that at higher intensities than this the threshold for a parabolic profile is much more easily satisfied than for a linear profile. The "exact" solution for a parabolic profile[1] works out to be

$$\frac{v_o^3}{c^3} \gg 6^{\frac{1}{2}} \frac{\lambda_D}{L_n} \frac{1}{k_o L_n}, \tag{120}$$

in remarkable agreement with the approximate result (119). These results can be trusted only to within a factor 3 or so because they are sensitive to whether the integral in Eq. (111) is taken from -L/2 to +L/2 or from 0 to L, and to the constant chosen on the right-hand side. The thresholds in any case refer to a number of e-foldings above noise which is not specified.

VIII. A NEW THRESHOLD

A numerical example will show that none of the conventional thresholds given above is appropriate to experiments in cool, underdense plasmas. The homogeneous growth rate γ_o, as given in Eq. (55), can be written in terms of pump intensity I_o for $\omega_2 \simeq \omega_o$, $\omega_1 \simeq \omega_p$, $k_1 \simeq 2k_o$, $I_o = cE_o^2/8\pi$ as

$$\gamma_o^2 = \frac{2\pi e^2}{m^2 c^3} \frac{\omega_p}{\omega_o} I_o , \tag{121}$$

or

$$\gamma_o^2 = 6.47 \times 10^{20} (n/n_c)^{\frac{1}{2}} I_9 \tag{122}$$

where I_9 is I_o in GW/cm^2. The damping rates are

$$\gamma_1 = \tfrac{1}{2}\nu_{ei} + \gamma_{LD} , \qquad \gamma_2 = (n/n_c)(\nu_{ei}/2) , \tag{123}$$

where

$$\nu_{ei} = 2.9 \times 10^{-6} Zn_e \ln \Lambda / T_{eV}^{3/2} , \tag{124}$$

$$\ln \Lambda = 24 - \ln(n^{\frac{1}{2}}/T_{eV}) , \tag{125}$$

$$\gamma_{LD} = (\pi/\varepsilon^3)^{1/2} \omega_p \zeta^3 e^{-\zeta^2} , \tag{126}$$

$$\zeta = \omega_p/k_1 v_{th} , \qquad \varepsilon = 2.718... \tag{127}$$

The group velocities are

$$V_2 \simeq c , \qquad V_1 \simeq 3k_o v_{th}^2/\omega_p = 6k_o \lambda_D^2 \omega_p . \tag{128}$$

Let us take, for example, $T_{eV} = 10$, $n = 10^{17}$ cm^{-3}, and $\lambda_o = 9.55$ μm. The homogeneous threshold $\gamma_o^2 = \gamma_1\gamma_2$ then gives

$$I_o = 1.3 \times 10^8 \text{ W/cm}^2,$$

which is exceeded in almost all experiments. The collisional-convective threshold, Eq. (71), gives

$$I_o = 3.1 \times 10^{12} \text{ W/cm}^2 ,$$

which is not exceeded in most small laboratory experiments. Thus the use of Eq. (75) is not justified in calculating the inhomogeneous thresholds.

We therefore go back to Eq. (70) for the spatial growth rate κ:

$$2\kappa = -B + [B^2 + \frac{4}{V_1V_2}(\gamma_o^2 - \gamma_1\gamma_2)]^{1/2} , \tag{129}$$

where
$$B \equiv \frac{\gamma_1}{V_1} + \frac{\gamma_2}{V_2} . \tag{130}$$

It is safe to neglect $\gamma_1\gamma_2$. If the collisional-convective threshold (71) is greatly exceeded, B can be neglected, and $\kappa \propto I_o^{1/2}$. But in the important intermediate regime, B^2 is large, and $\kappa \propto I_o$. Expanding Eq. (129), we have

$$2\kappa \simeq B[-1 + (1 + 2\gamma_o^2/V_1V_2B^2)],$$

$$\kappa = \gamma_o^2(\gamma_1V_2 + \gamma_2V_1)^{-1}. \tag{131}$$

Thus the relevant threshold is

$$\kappa L = \gamma_o^2 L \ (\gamma_1V_2 + \gamma_2V_1)^{-1} >> 1. \tag{132}$$

Since $\gamma_2 \ll \gamma_1$ and $V_1 \ll V_2$, this condition is close to

$$\frac{\gamma_o^2 L}{c\gamma_1} \gg 1, \qquad (133)$$

which is the threshold normally used for Brillouin scattering but never quoted for Raman scattering. The only difference between SBS and SRS is that SBS is insensitive to density gradients, so that L is determined primarily by the interaction length, while in SRS L is determined by the density scalelength L_n. From Eqs. (113) and (118) we have, for SRS,

$$L^2 = 24\ k_o L_n \lambda_D^2 \qquad \text{(linear profile)} \qquad (134)$$

$$L^3 = 36\ k_o L_n^2 \lambda_D^2 \qquad \text{(parabolic profile).} \qquad (135)$$

The last three equations give the correct SRS threshold in most situations.

IX. NUMERICAL RESULTS

The exact growth length κ^{-1}, computed from Eq. (70), is plotted vs. I_o in Fig. 1 for $\lambda_o = 9.55$ μm. Four combinations of n and T_e are shown. Case A is a low threshold case where the combination of collisional and Landau damping is minimized. At low I_o, the curve becomes vertical at the homogeneous threshold I_{hom}, which is marked at the top of the graph. The central part of the curve has $\kappa \propto I_o$ (ideal slopes are indicated by dashed lines). At intensities above the absolute threshold I_{abs} [Eq. (71)], marked by the sign $\int$, the slope gradually changes to $I_o^{1/2}$. The effective interaction length L is marked by the point P for a parabolic profile and the point L for a linear profile. In both cases L_n is taken as 10 cm. For case A the condition $\kappa L \gg 1$ and $I_o \gg I_{abs}$ are identical for the parabolic profile, but the agreement is accidental. The other curves show that the criteria are different. Case B has a higher threshold but still shows the three regions of different slope. Case C is collision dominated, and I_{hom} is high; the absolute threshold is extremely high. Case D is Landau damping dominated and has a very high I_{abs} and a very low I_{hom} (identical, in fact, to that of case A). The large disparity between I_{hom} and I_{abs} is because in this case γ_2 is small and γ_1 is large, and I_{hom} depends on the product while I_{abs}

depends of the sum. The result is a very large middle range where $\kappa \propto I_o$. From these curves one sees that the instability criterion of Eq. (71), and formulas such as Eqs. (75), (114), and (119) which are based on it, are not valid most of the time. One should use Eq. (70) and the condition $\kappa L >> 1$.

Fig. 2 shows the result of doing this; i.e. solving for κ from Eq. (70), calculating L from Eq. (135) for a parabolic profile with $L_n = 10$ cm, and setting $\kappa L = 10$. The threshold intensity is plotted vs n for various T_e. The turnover in the high-T_e curves is not real; it is the result of using the Landau damping formula (126), which is not valid for $\zeta \lesssim 1$.

Fig. 3 shows contours of constant I_o on the n-T_e plane, obtained by cross-plotting from Fig. 2. The minimum I_o is shown by the dashed lines in Figs. 2 and 3. On these lines the value of $k_1\lambda_D$ is approximately constant at 0.2. If the plasma is made by a theta pinch, the nT_e product is fixed by the $\beta = 1$ condition and the maximum B-field the capacitor bank can produce. For instance, if $nT_e = 2 \times 10^{18}$ cm^{-3} eV, the plasma lies along the thin straight line in Fig. 3. From this one sees that the threshold intensity is about 5×10^{11} W/cm^2, occurring when $n \simeq 7 \times 10^{16}$ cm^{-3} and $T_e \simeq 26$ eV. If L_n is not 10 cm, κ varies as 1/L or $L_n^{-2/3}$, so that the threshold intensity I_o, if it lies in the unity-slope region of Fig. 1, varies as $L_n^{-2/3}$. For instance, to reduce the threshold to 10^{11} W/cm^2 from 5×10^{11} W/cm^2 would require a scalelength increase to $L_n = 112$ cm.

For comparison, we have also computed the so-called "absolute" threshold given by Eq. (71). This is plotted in Fig. 4 vs. n for various T_e. Cross-plotting gives the constant-I_o contours of Fig. 5. This threshold is about a factor 5 lower than that of Fig. 3, but the two thresholds would be comparable if $L_n \gtrsim 100$ cm. Note that the curves of Fig. 5 are narrower than those of Fig. 2, but the sharpness of the minima does not greatly affect the shape of the constant-I_o contours of Figs. 3 and 5. Rather, it is the range of I_o which is affected; Figs. 3 and 5 differ by two orders of magnitude in the variation of I_o with n and T_e.

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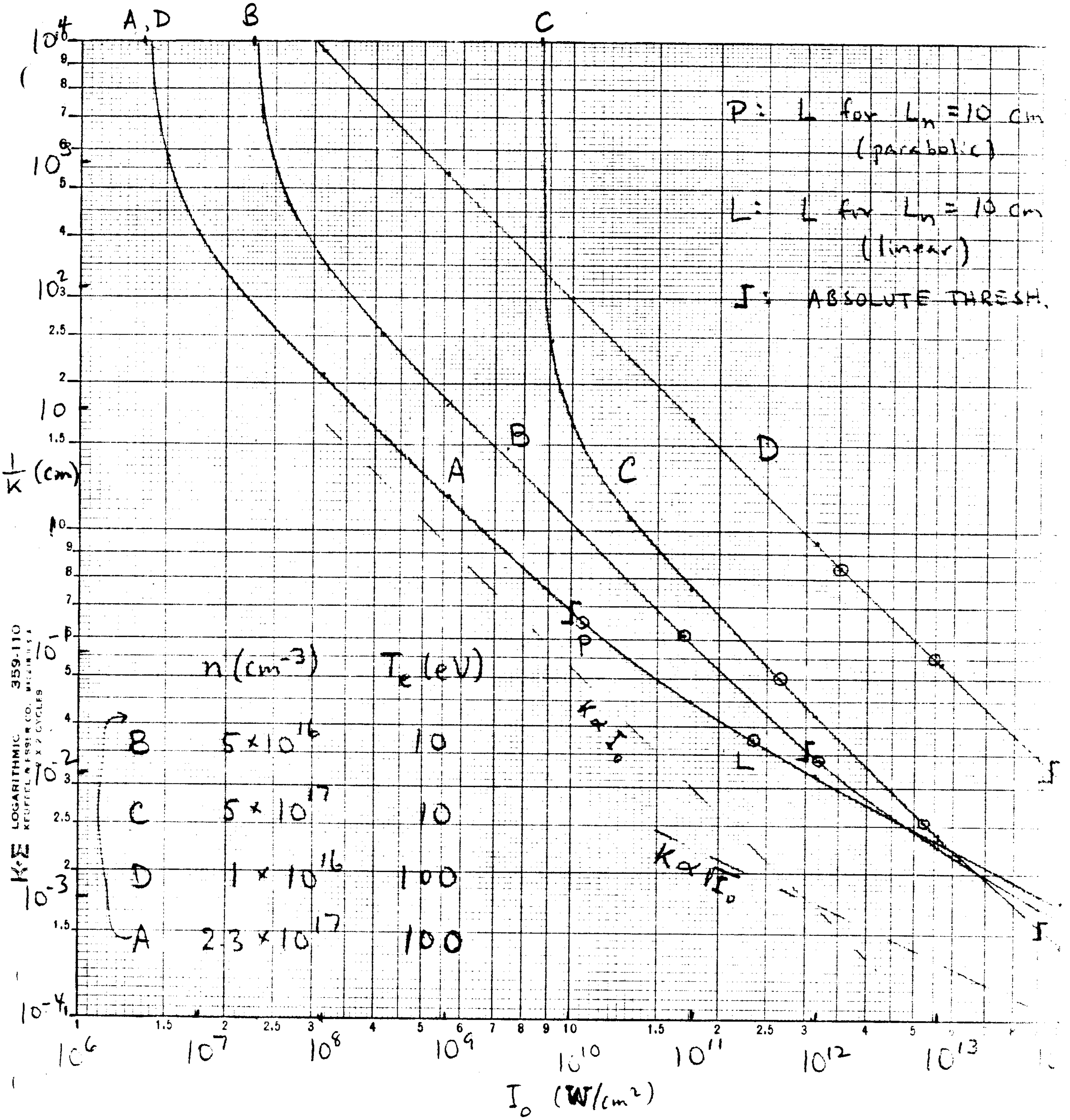
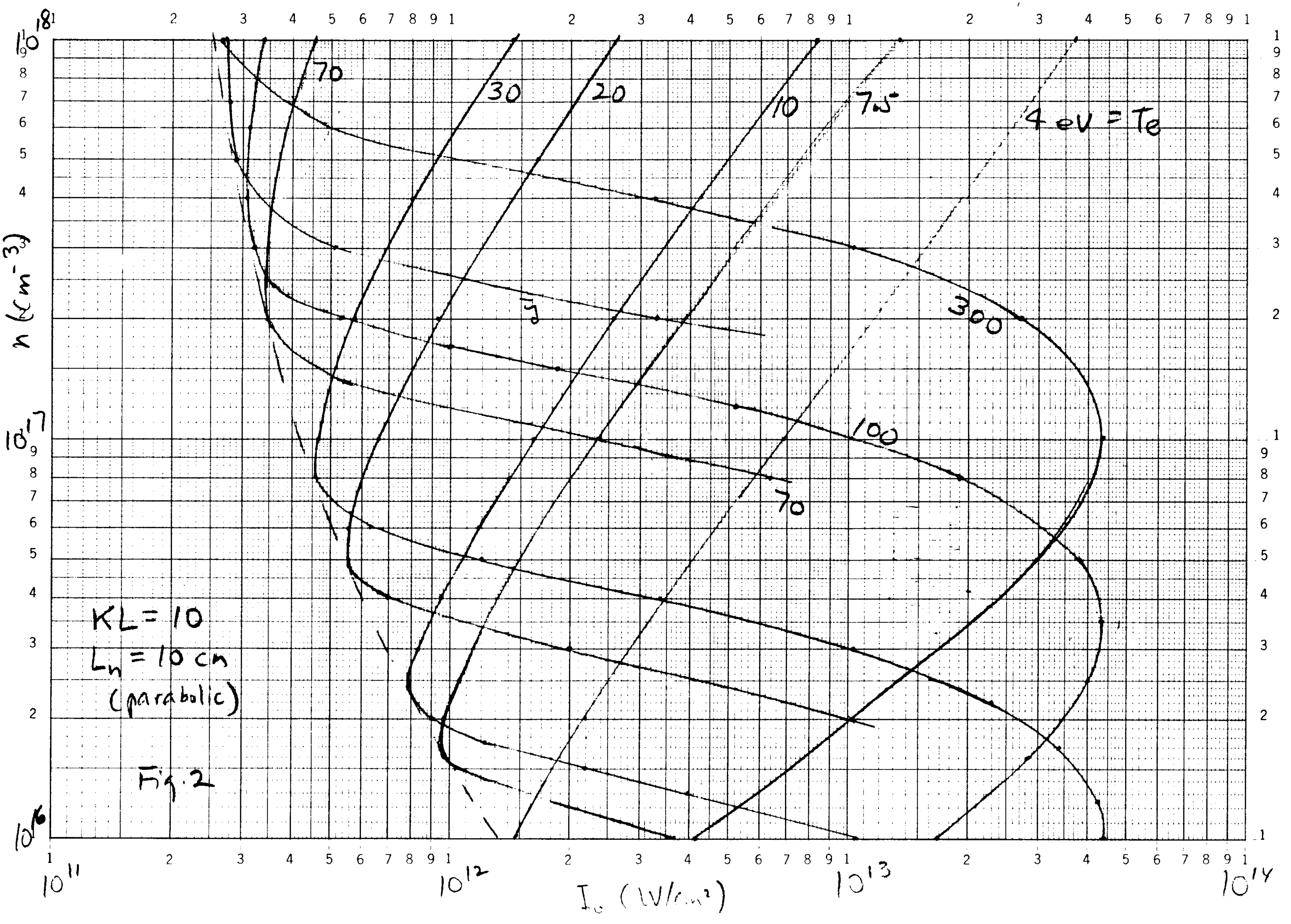
REFERENCES

1. C. S. Liu, M. N. Rosenbluth, and R. B. White, Phys. Fluids 17, 1211 (1974).
2. D. W. Forslund, J. M. Kindel, and E. L. Lindman, Phys. Fluids 18, 1002 (1975).



A, D
B
C
P: L for L_n = 10 cm (parabolic)
L: L for L_n = 10 cm (linear)
ABSOLUTE THRESH.
$\frac{1}{K}$ (cm)
A
B
C
D
P
L
$K \propto I_0$
$K \propto \sqrt{I_0}$
n (cm^{-3}) T_e (eV)
B 5 × 10^{16} 10
C 5 × 10^{17} 10
D 1 × 10^{16} 100
A 2.3 × 10^{17} 100
10^4
10^3
10^2
10
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^6
10^7
10^8
10^9
10^{10}
10^{11}
10^{12}
10^{13}
I_0 (W/cm^2)
K+E LOGARITHMIC 359-110

Fig. 1



70
30
20
10
7.5
4 eV = Te
15
300
100
70
10^{18}
10^{17}
10^{16}
n (cm⁻³)
10^{11}
10^{12}
10^{13}
10^{14}
I₀ (W/cm²)
KL = 10
Lₙ = 10 cm
(parabolic)

Fig. 2

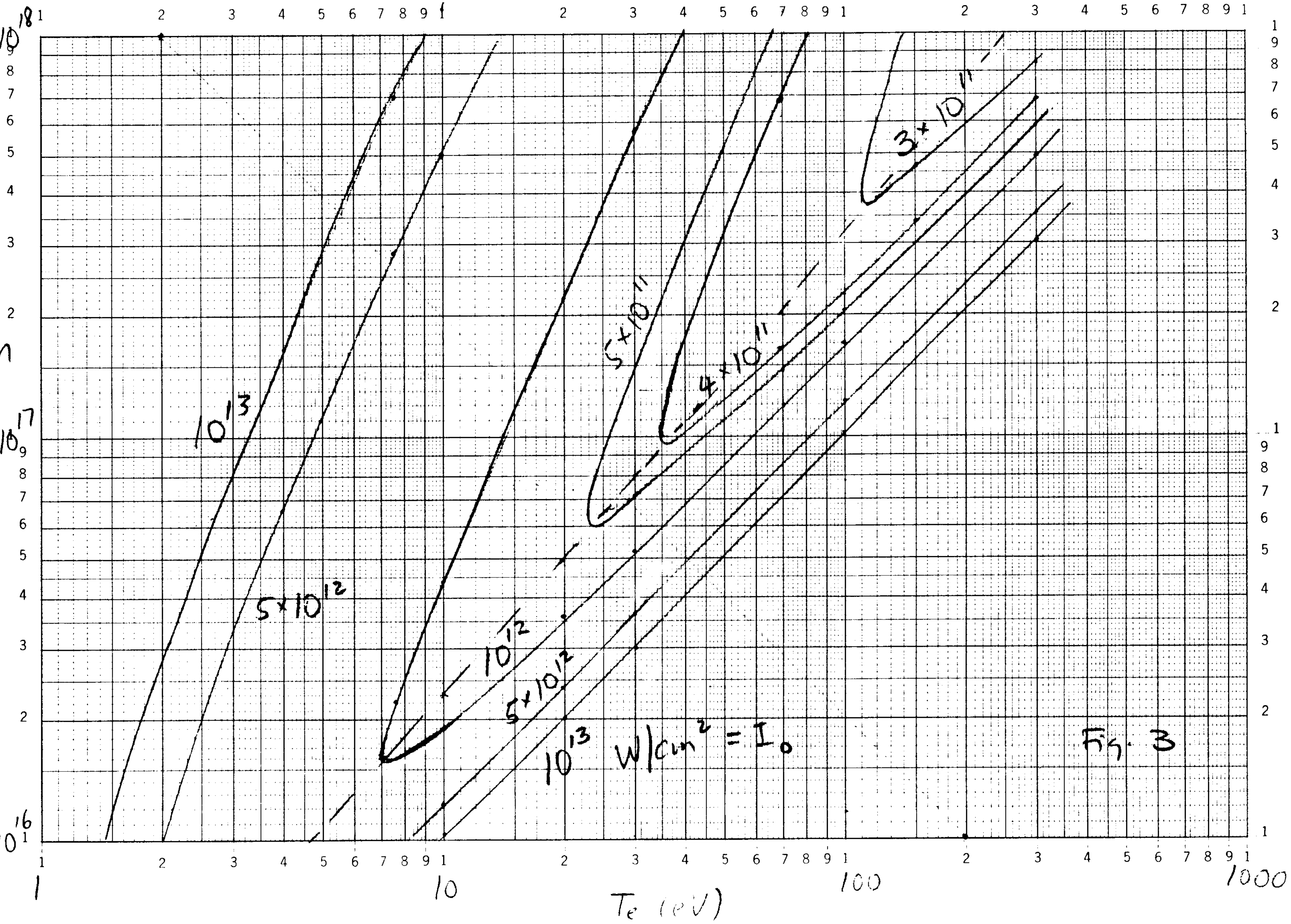

10^{18}
10^{17}
10^{16}
1
10
100
1000
T_e (eV)
10^{13}
5×10^{12}
10^{12}
5×10^{12}
10^{13}
5×10^{11}
4×10^{11}
3×10^{11}
W/cm² = I_0

Fig. 3

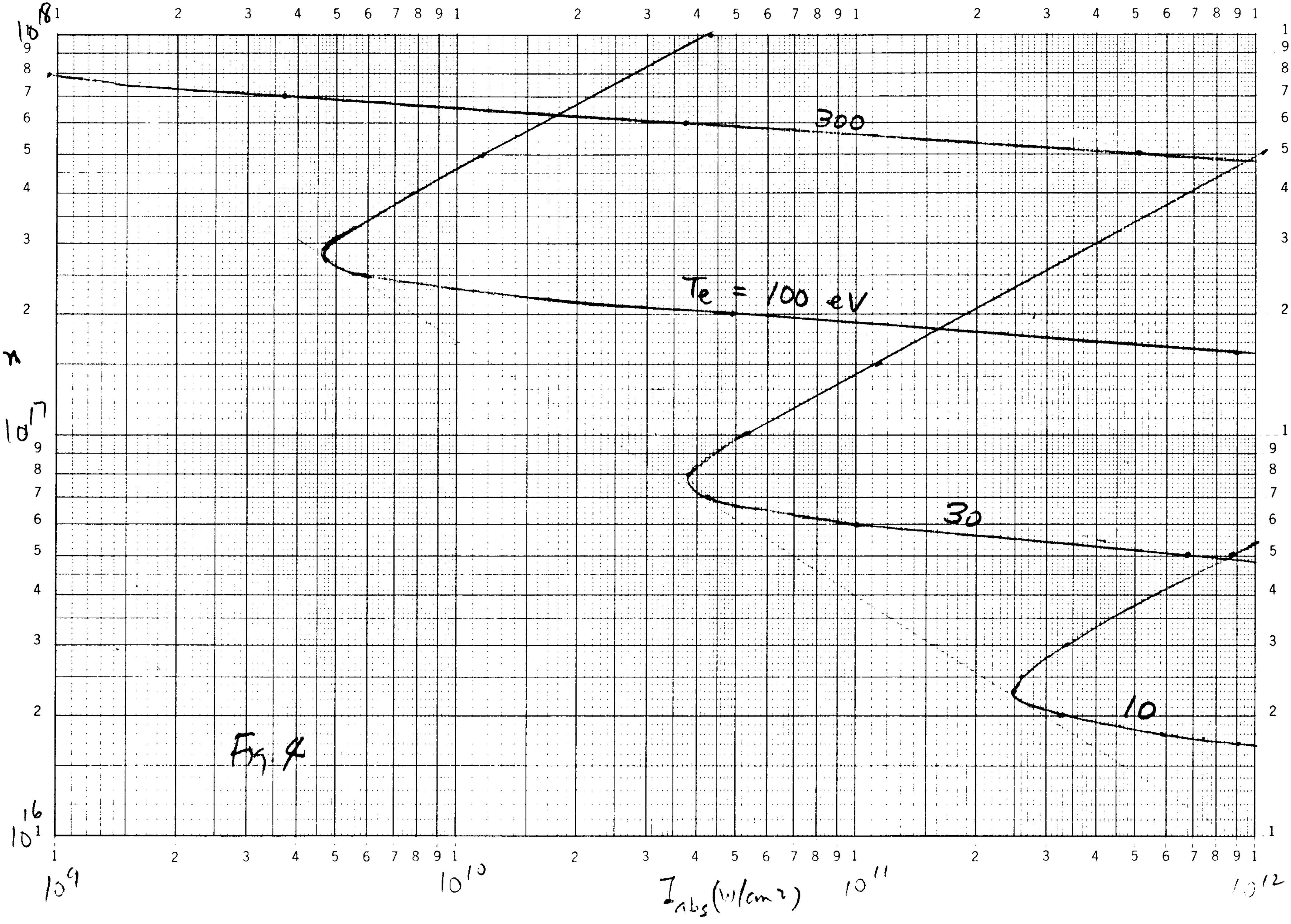

T_e = 100 eV
300
30
10
n
10^{18}
10^{17}
10^{16}
10^{9}
10^{10}
10^{11}
10^{12}
I_{abs} (W/cm²)

Fig. 4

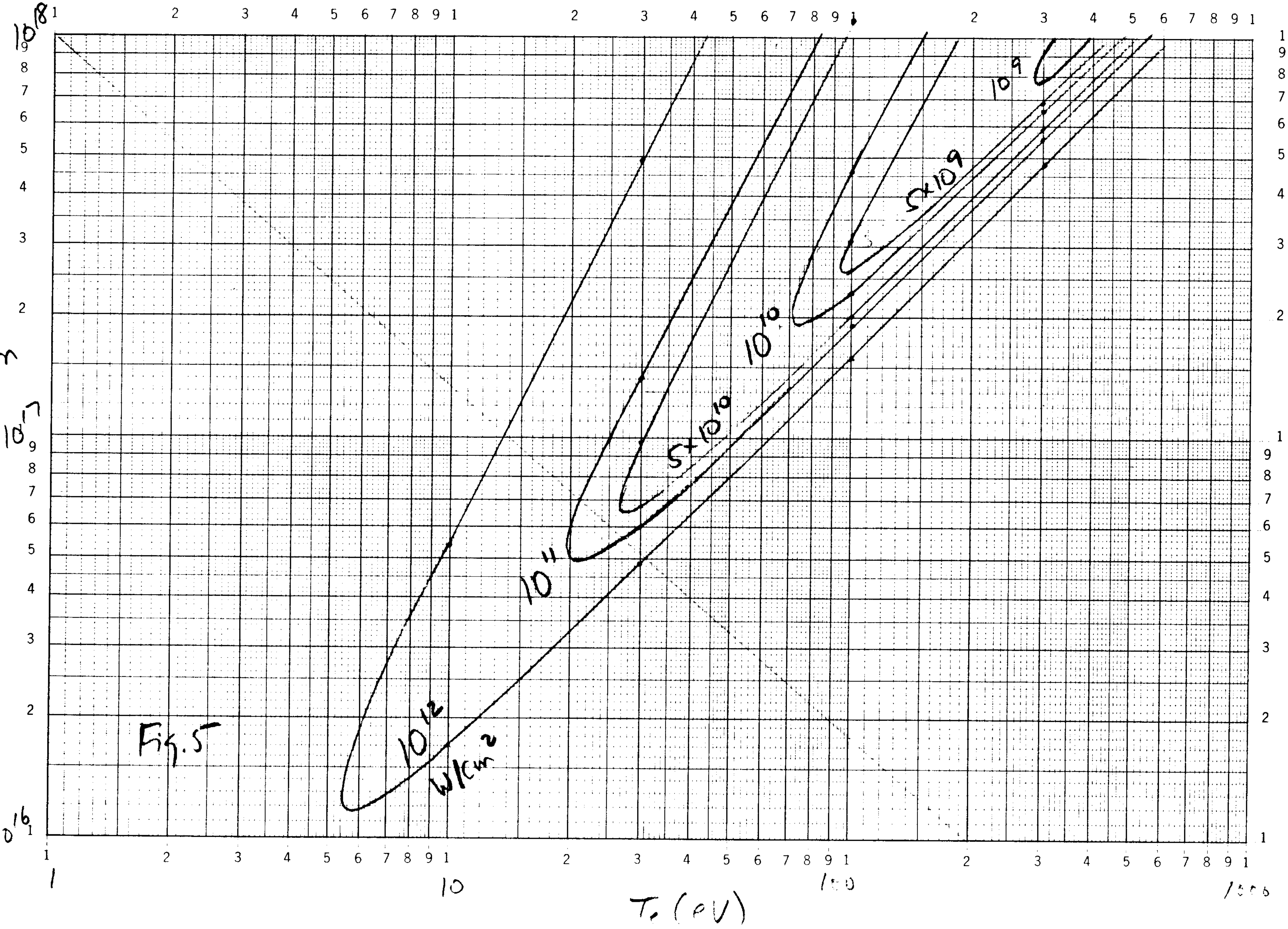

10^{18}
10^{17}
10^{16}
n
10^9
5×10^9
10^{10}
5×10^{10}
10^{11}
10^{12} W/cm²
1
10
100
1000
T_e (eV)

Fig. 5