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The Floating Potential of Cylindrical Langmuir Probes

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ABSTRACT

The floating potential of a cylindrical probe is computed numerically, and the results are fitted to analytic functions. They differ significantly from the plane approximation.

The formula normally used to calculate the space potential from the measured floating potential is derived for plane probes and is erroneous when applied to cylindrical probes in the low-density plasmas ($n < 10^{12}$ cm^{-3}) used in industrial plasma processing. The floating potential V_f is that at which the collected ion and electron fluxes are equal. If A_p is the probe area, n_0 the density in the body of the plasma, and $V = 0$ the potential there, the electron flux ΓA_p to the probe is

$$I_e = A_p n_0 v_{th} \exp(V_f / KT_e), \qquad v_{th} \equiv (KT_e / 2\pi m)^{1/2}. \tag{1}$$

[Note: $I \equiv$ total particle current for plane probes and current per unit length for cylindrical probes; the electrical current $\pm eI$ is not used here.] For a plane probe, the ion current is given by the Bohm criterion at the sheath edge, defined as the point where the ions have an inward drift velocity c_s, having fallen to the potential $V = V_{sh} = -½\, KT_e$, where the density n is $n_s = n_0 \exp(-½)$. Thus,

$$I_i = A_p n_s c_s = \alpha_0 A_p n_0 c_s, \qquad c_s \equiv (KT_e / M)^{1/2}, \tag{2}$$

M being the ion mass and α_0 has the value exp(−½) = 0.61. A spread in ion energies can bring α_0 closer to the convenient value of 0.5. Setting $I_i = I_e$ yields the usual formula for the floating potential:

$$-\frac{eV_f}{KT_e} = \ln\left[\frac{1}{\alpha_0}\left(\frac{M}{2\pi m}\right)^{1/2}\right] \approx 5.18 \text{ in argon.} \tag{3}$$

The ion collection area for a *cylindrical* probe, however, depends on the radius R_{sh} of the sheath, which is not known *a priori*. In this case, there is no need for the artifice of a sharp sheath edge, since solutions of Poisson's equation can be extended to infinity. Two collisionless theories are available for calculating $V(r)$: the Bernstein-Rabinowitz[1] (BR) theory, which takes into account the angular momentum of the ions which orbit the probe; and the Allen-Boyd-Reynolds[2] (ABR) theory, which neglects orbiting, so that ions move only radially and axially. Chen[3] has recently shown that the BR theory overestimates the ions' angular momentum in partially ionized plasmas because of collisions in the presheath.

Hence, for plasma processing purposes, we shall employ the ABR equation as modified by Chen[4] for cylindrical probes:

$$\frac{\partial}{\partial\xi}\left(\xi\frac{\partial\eta}{\partial\xi}\right)=J\eta^{-\frac{1}{2}}-\xi e^{-\eta}\,, \tag{4}$$

where

$$\eta\equiv -eV/KT_e\,,\qquad \xi\equiv r/\lambda_D\,,\qquad \lambda_D\equiv\left(\varepsilon_0 KT_e/n_0e^2\right)^{\frac{1}{2}}. \tag{5}$$

The normalized ion current J is defined by

$$J\equiv\frac{1}{2\pi\sqrt{2}}\frac{I_i}{n_0}\frac{1}{\lambda_D c_s}\,. \tag{6}$$

For each value of J, Eq. (4) can be solved to yield $\eta(\xi)$ for all ξ. The constraint that the probe be floating can be expressed as follows. The radius ξ_p of a probe at floating potential can be found from the condition $I_i = I_e$ at the probe surface, where

$$I_e=2\pi R_p n_0 v_{th}\exp(-\eta_f)\,,\qquad I_i=2\pi R_p n_p v_i\,. \tag{7}$$

Here n_p is the ion density at the surface of the floating probe, and v_i is the ion velocity there, given from energy conservation by

$$v_i=(2\eta_f)^{\frac{1}{2}}c_s\,. \tag{8}$$

Setting $I_i = I_e$ yields

$$n_p=n_0\left(\frac{M}{2\pi m}\right)^{1/2}\frac{\exp(-\eta_f)}{(2\eta_f)^{\frac{1}{2}}}\,. \tag{9}$$

Substituting Eqs. (8) and (9) into I_i and I_i into Eq. (6) gives

$$J=\frac{1}{\sqrt{2}}\xi_p\left(\frac{M}{2\pi m}\right)^{1/2}e^{-\eta_f}\,, \tag{10}$$

so that

$$\eta_f=\ln\left[\frac{\xi_p}{J}\left(\frac{M}{4\pi m}\right)^{1/2}\right]. \tag{11}$$

Solution of Eq. (4) yields the potential distribution

$$\eta=\eta(J,\xi)\,. \tag{12}$$

Integration of Eq. (4) is non-trivial, and care must be taken to join smoothly to the quasineutral solution at large radii. For each J, Eqs. (11) and (12) give two curves whose intersection yields a pair of values (η_f, ξ_p), as illustrated in Fig. 1.

Varying J generates the function $\eta_f(\xi_p)$, shown in Fig. 2 for argon, which approaches the plane limit of 5.18. If we now define

$$\alpha\equiv\sqrt{2}J/\xi_p\,, \tag{13}$$

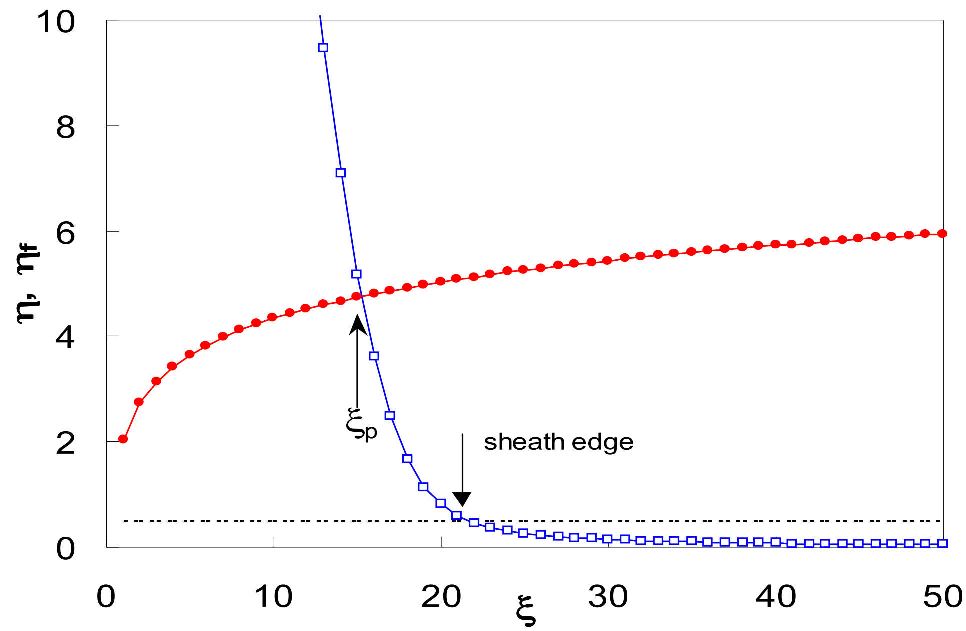

Fig. 1. The potential profile $\eta(\xi)$ ( ) and the floating potential condition $\eta_f(\xi)$ (•) for the case $J = 10$, $\xi_p = 15$ in argon. The Bohm criterion is met at the "sheath edge" where $\eta = ½$ (– – –).

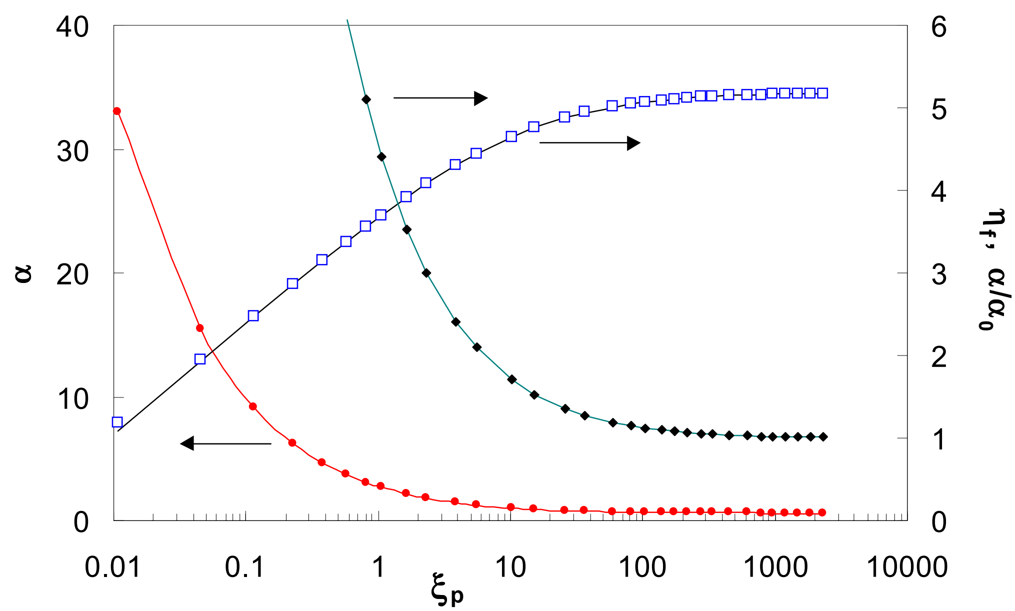

Fig. 2. Decrease of η_f ( ) with decreasing $\xi_p = R_p/\lambda_D$ due to the increase in sheath area as measured by α (•) and α/α_0 (♦). The line through the η_f points is an analytic fit.

Eq. (11) takes the same form as Eq. (3), with α in place of α_0. Thus, from Eq. (2), αA_p is the effective collection area of a floating probe, and the ratio α/α_0 expresses the expansion of this area as ξ_p is decreased. The functions α and α/α_0 are also shown in Fig. 2. There is no need to define a "sheath edge"; but if one is defined at the radius R_{sh} where $\eta = ½$, as in Fig. 1, conservation of current requires $I_i = 2\pi R_{sh} n_s c_s$. However, n_s is not $0.61 n_0$ as in the plane case, since quasineutrality has not been assumed at R_{sh}, and $n_i \neq n_e$ there. Using Eqs. (13) and (6), we can conveniently express the ion current to a floating probe in terms of the function $\alpha(\xi_p)$:

$$I_i = 2\pi R_p \alpha n_0 c_s\,, \tag{14}$$

with α acting as an effective Bohm coefficient.

The following analytic fits to the computed curves may be useful for probe analysis:

$$\frac{1}{(\eta_f)^6} = \frac{1}{(A\ln\xi_p + B)^6} + \frac{1}{(C\ln\xi_p + D)^6}\,, \tag{15}$$

where $A = 0.583$, $B = 3.732$, $C = -0.027$, and $D = 5.431$; and

$$\frac{\alpha}{\alpha_0} \approx \frac{R_{sh}}{R_p} = 1 + E\exp(-F\xi_p^G)\,, \tag{16}$$

where $E = 4000$, $F = 7.01$, and $G = 0.096$. In the plane probe limit $\xi_p \to \infty$, η_f approaches the value of 5.18 for argon, and α and α/α_0 approach 0.61 and 1, respectively. In the range ξ_p = 1−10 commonly encountered in rf discharges, η_f is of order 3.7 – 4.6 for argon, significantly less than the usual value of 5.2. The reason is that the sheath thickness at V_f causes a cylindrical probe of given area to collect more ion current than a plane probe, and thus the sheath drop has to be lowered to permit more electron flow.

FIGURE CAPTIONS

Fig. 1. The potential profile $\eta(\xi)$ ( ) and the floating potential condition $\eta_f(\xi)$ (•) for the case $J = 10$, $\xi_p = 15$ in argon. The Bohm criterion is met at the "sheath edge" where η = ½ (– – –).

Fig. 2. Decrease of η_f ( ) with decreasing $\xi_p = R_p/\lambda_D$ due to the increase in sheath area as measured by α (•) and α/α_0 (♦). The line through the η_f points is an analytic fit.

REFERENCES

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[1] I.B. Bernstein, and I.N. Rabinowitz, Phys. Fluids **2**, 112 (1959).
[2] J.E. Allen, R.L.F. Boyd, and P. Reynolds, Proc. Phys. Soc. (London) **B70**, 297 (1957).
[3] F.F. Chen, Phys. Plasmas (2001).
[4] F.F. Chen, Plasma Physics **7**, 47 (1965).