Low Temperature Plasma
Technology Laboratory

Diffusion of an annular plasma for positron acceleration

Francis F. Chen

LTP-707 June, 2007

Electrical Engineering Department
Los Angeles, California 90095-1594

UNIVERSITY OF CALIFORNIA • LOS ANGELES

Diffusion of an annular plasma for positron acceleration

Francis F. Chen, University of California, Los Angeles 90095-1594

Electron acceleration to multi-GeV energies has been demonstrated using plasma wakefields in a tunnel-ionized plasma. However, coherent wakefields for positron acceleration may require hollow plasmas pre-ionized by a laser beam. The lifetime of such a plasma is determined by an unusual diffusion problem in which the diffusion rate varies by an order of magnitude inside the hole. The problem is solved by numerical differentiation without using a PIC code.

I. Introduction

Energy doubling of the 42-GeV Stanford Linear Accelerator electron beam has been demonstrated by Blumenfeld et al.[1] using the wakefield acceleration mechanism.[2] The plasma is created by tunnel ionization[3,4,5] of lithium vapor at the leading edge of the electron beam pulse ~10μm in diameter and ~15μm long. In the blow-out regime[6], the beam density is larger than the plasma density, and the electric field of the beam expels the cold electrons, which all end up at approximately the same radius. These electrons are drawn back in by the ions they left behind, and their oscillation forms a plasma wave on which electrons in the tail of the beam pulse surf to gain higher energy. The same mechanism does not work as well for positron acceleration, since the beam now *attracts* cold electrons from a spread in radius, and a coherent wakefield is not formed. A solution is to pre-form a hollow plasma by ionizing lithium vapor with an annular laser beam. The positron beam then attracts cold electrons starting from a well defined radius, thus improving the chances of creating a coherent wake.

The time delay between the laser pulse and the positron pulse is determined by the diffusion of the annular plasma. In classical diffusion of a cylindrical plasma, the initial fine structure decays rapidly until the lowest Bessel function mode remains, and this then decays at an exponential rate.[7] If one assumes a cylindrical shell with absorbing boundaries at r_1 and r_2, a Bessel function solution can easily be found, and it would decay with a time constant of about 2 msec. The current problem differs in two major respects. First, the diffusion coefficient varies by an order of magnitude with density, and hence with radius. Second, there is no absorbing boundary in the hole, and plasma fills the hole rapidly because of the strong gradients there. Higher harmonics can be generated rather than damped.

II. Experimental parameters

Single-photon ionization of oven-heated lithium vapor has been used as a plasma target for successful wakefield acceleration of 28-GeV positron beams.[8,9] The 193-nm argon fluoride laser was focused into a 1.4-m long lithium oven with neutral density $n_n = 2 \times 10^{15}$ cm^{-3}. In this paper, we assume that the laser beam is of order 500 μm in radius, and a beam block creates a hole of order 200 μm in radius. The laser intensity is adjusted to give a maximum density n_0 of order 2×10^{14} cm^{-3}. These parameters were chosen to match experiments that have been attempted, but both the density and the size can be varied from those in this initial survey.

Near the density peak, diffusion is dominated by ion-ion collisions. At low density, it is dominated by ion-neutral charge-exchange collisions. Ions starting near the peak diffuse slowly because of frequent collisions with one another. As they spread out, their density decreases so that the rate of expansion is slowed only by the less frequent ion-neutral collisions. The situation is analogous to the bursting of a helium balloon. Initially, the helium gas expands at a rate determined only by He-He collisions. When the helium is mixed with air, it diffuses via He-air collisions. In a plasma, once the ion diffusion rate is determined, electrons will create an ambipolar electric field to speed up the diffusion.

The ion-ion collision frequency is[10]

$$\nu_{\perp}^{ii} = 1.4\times 10^{-7} A^{-1/2} T_i^{-3/2} n\lambda_{ii} \ \text{sec}^{-1}, \tag{1}$$

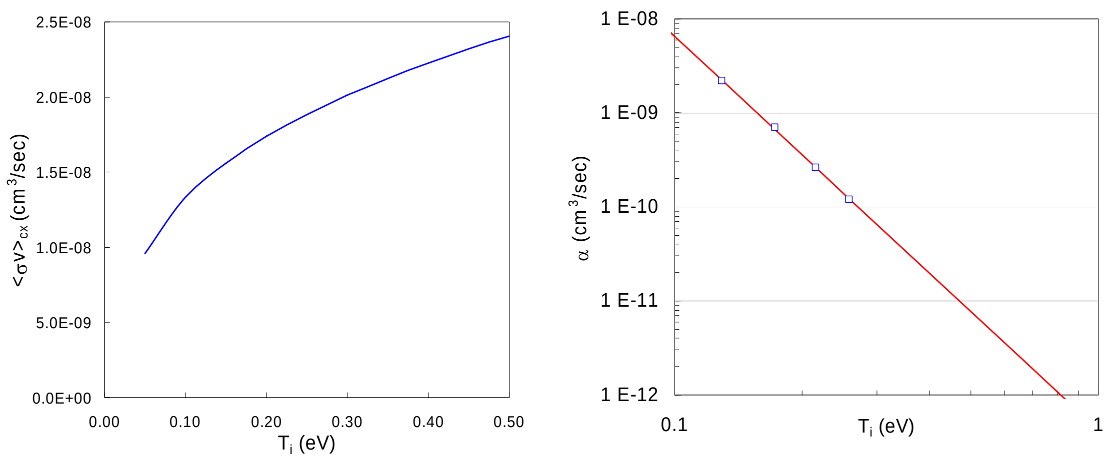
where A is the atomic number, T_i the ion temperature in eV, n the plasma density in cm^{-3}, and λ_{ii} the appropriate Coulomb logarithm, given by

$$\lambda_{ii} = 22.7 - \ln(n^{1/2}/T_i^{3/2}) . \tag{2}$$

This frequency falls linearly with n. The Li^+-Li resonant charge exchange cross section σ_{cx} was given most conveniently by Lorentz et al.[11], who fitted experimental points down to 14 eV with the formula

$$\sigma_{cx}(\text{Li}) = (a - b\log E_{eV})^2 \quad \text{cm}^2, \tag{3}$$

where a = 19.3 × 10^{-8} and b = 3.5 × 10^{-8}. This formula was extrapolated to the lower energies of this problem. The collision probability $<\sigma v>_{cx}$ was computed by integrating over a Maxwellian ion distribution at T_i and is shown in Fig. 1a.



(a) (b)

Fig. 1. (a) Ion-neutral collision probability and (b) recombination coefficient vs. lithium ion temperature.

The ions are heated by electrons and cooled by the neutrals. Since the photon energy of the 193-nm laser is 6.45 eV and the lithium ionization energy is 5.39 eV, electrons are created with approximately 1 eV of average energy, corresponding to $(3/2)T_e$. Thus, $T_e \approx$ 0.67 eV. The neutral gas has the oven temperature of 750° C (≈1000° K), so that the ion start with the temperature $T_0 \approx$ 0.088 eV. The equilibration time between ions and electrons is[10]

$$t_{eq} = 3.25\times10^8 AT_{eV}^{3/2} / n\lambda_{ei} . \tag{4}$$

For T_e =0.67 eV and $n \approx 2 \times 10^{14}$ cm^{-3}, this gives t_{eq} = 2.4 μsec, which is much longer than the 50-nsec timescales we shall find. Hence, T_i remains at ≈ 0.09 eV, at which $<\sigma v>_{cx} \approx 1.3 \times 10^{-8}$ cm^3/sec. Thus the ion-neutral collision frequency is

$$\nu_{io} = 1.3\times10^{-8} n_n \quad \sec^{-1} \tag{5}$$

The measured radiative recombination coefficient α for hydrogen-like atoms has been given by Curry[12] (Fig. 1b) for T_e up to 3000°K and densities up to 10^{15} cm^{-3}. Extrapolating to 0.67 eV (7700°K) yields $\alpha \approx 2.3 \times 10^{-12}$ cm^3/sec at $n = 2 \times 10^{14}$ cm^{-3}. This turns out to be at most a 1% effect, but it is trivial to include it.

III. Computations

Diffusion of a cylindrically symmetric plasma is given by

$$\frac{\partial n}{\partial t} = D_a(n)\left(\frac{\partial^2 n}{\partial r^2} + \frac{1}{r}\frac{\partial n}{\partial r}\right) - \alpha n^2 , \tag{6}$$

where D_a is the ambipolar diffusion coefficient[7]

$$D_a = D_i(1 + T_e / T_i) \tag{7}$$

and

$$D_i = KT_i / M\nu_{tot} . \tag{8}$$

Here D_a depends on n but not explicitly on r. The total collision frequency ν_{tot} is the sum of the ion-ion part [Eq.(1)] and the ion-neutral part [Eq.(5)]. For A = 3, T_e = 0.67 eV, and T_i = 0.09 eV, we have

$$\nu_{tot} = 1.3\times10^{-8} n_n + 8.3\times10^{-6} n \ \sec^{-1}. \tag{9}$$

At 10% ionization, the ion-ion term is larger by a factor 64. This causes D_a to increase from 140 to 1300 cm^2/sec as n drops from 2×10^{14} cm^{-3} at the peak to 2×10^{13} cm^{-3} in the hole and outside edge. Diffusion is an order of magnitude faster in the hole and foot of the density distribution.

The density initial density profile can be modeled with two three-parameter functions, one for $r \le r_0$, and one for $r \ge r_0$, where r_0 is the radius of the beam block:

$$\frac{n}{n_0} = 1 + (f-1)\left[1 - \left(\frac{r}{r_0}\right)^s\right]^t , \qquad r \le r_0 \tag{10}$$

$$\frac{n}{n_0} = \left[1 - \left(\frac{r - r_0}{w}\right)^u\right]^v , \qquad r \ge r_0 . \tag{11}$$

Here f is the fractional density at the center of the hole, and w is the half-width of the laser beam. These functions are chosen to join smoothly at $r = r_0$. The k^{th} derivative of Eq. (10) vanishes for $k \le t$, and the k^{th} derivative of Eq.(11) vanishes for $k \le u$. Figure 2 illustrates density profiles obtainable by adjusting the six free parameters.

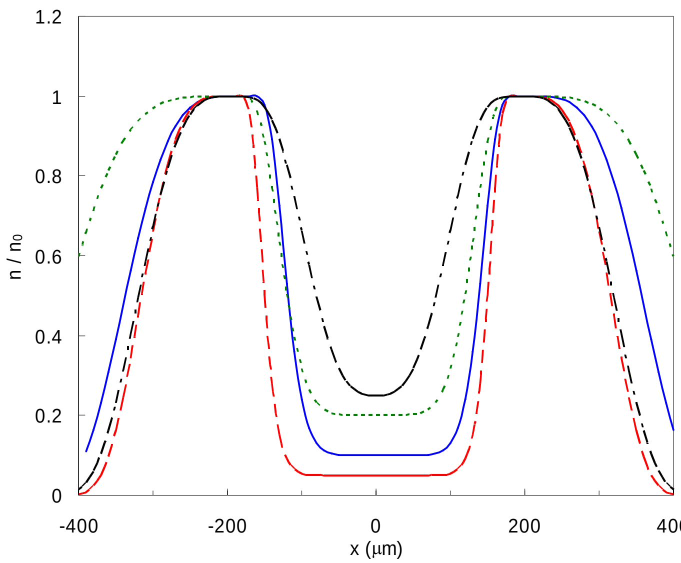


Fig. 2. Hollow density profiles obtainable by varying the parameters f, s, t, u, v, and w for r_0 = 200 μm

Eq. (6) is solved by numerical differentiation using Stirling's rule. However, the procedure is not completely straightforward. Where the density is low, at the bottom of the trough and on the outside, the diffusion is extremely fast, and time steps have to be small in order to avoid unphysical peaks and oscillations in the density there. On the other hand, repeated double differentiations generate noise which cannot be smoothed without losing the shape of the curve. Thus, only two or three small time steps can be computed at a time. This problem was solved by fitting the last smooth curve to the analytic formulas of Eqs. (10) and (11) with different values of the parameters. When the nonlinearity introduced by $D(n)$ became too large, the fitting was done with a commercial curve-fitting program[13] which uses, for instance, 20th order Tchebyshev polynomials. The procedure is repeated many times.

IV. Results

Figure 3 shows the evolution of a hollow profile characterized by r_0 = 200 μm, f = 0.1, s = 8, t = 20, u = 3.5, v = 8, w = 300, and $n_0 = 2 \times 10^{14}$ cm^{-3} in Eqs. (10) and (11). This profile has a large hole at a very low density, only 10% of the maximum. The center density fills in slowly because of the small gradients near the axis, changing noticeably only after about 10 nsec. Figure 4 shows the outer region in more detail. The profiles can overlap at late times because the curvature can change non-monotonically in nonlinear diffusion. In any case, the outside region is not of interest, and we focus on the density in the hole. Figures 5 and 6 compare the diffusion of two initial profiles differing in the slope of the density inside the hole. When the hole is not as sharply defined (Fig. 6) the larger gradients cause faster diffusion, and the center density changes noticeably after only about 2 nsec. We conclude that there is a 1- 10 nsec window in which the positron bunches can be fired to generate wakefields, depending on the sharpness of the edge of the density hole.

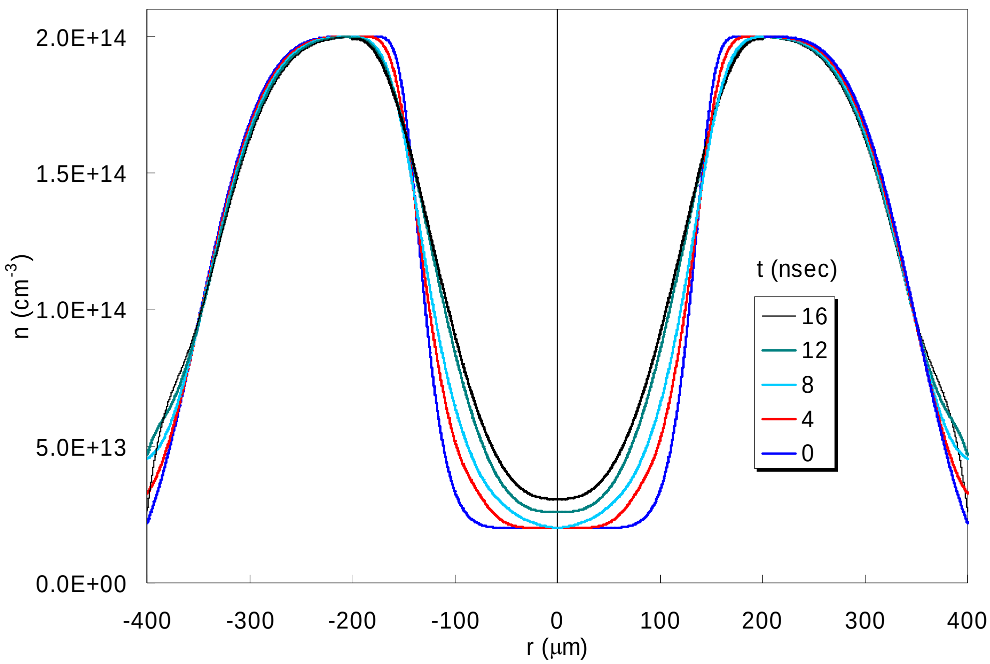


Fig. 3. Decay of an annular plasma with a deep, wide hole. The order of the curves is obvious (color online).

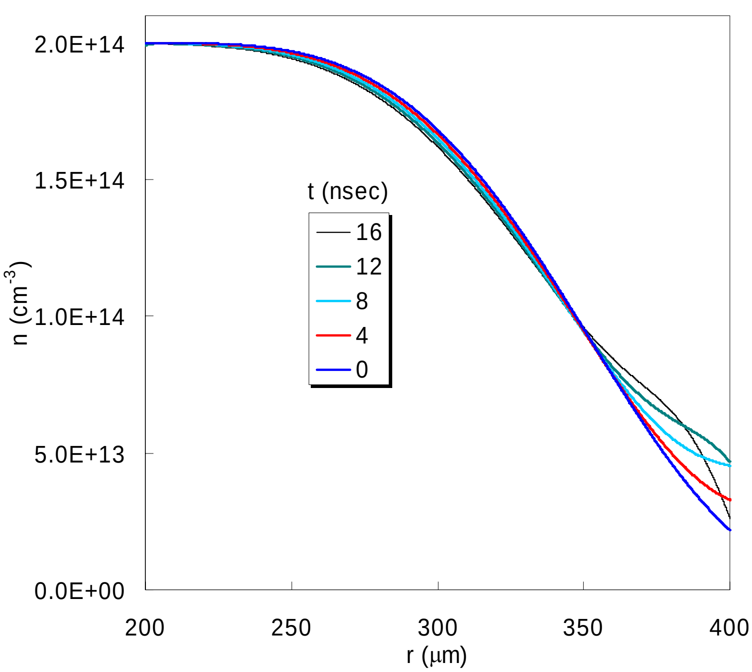


Fig. 4. Detail of the outside part of Fig. 3 (color online).

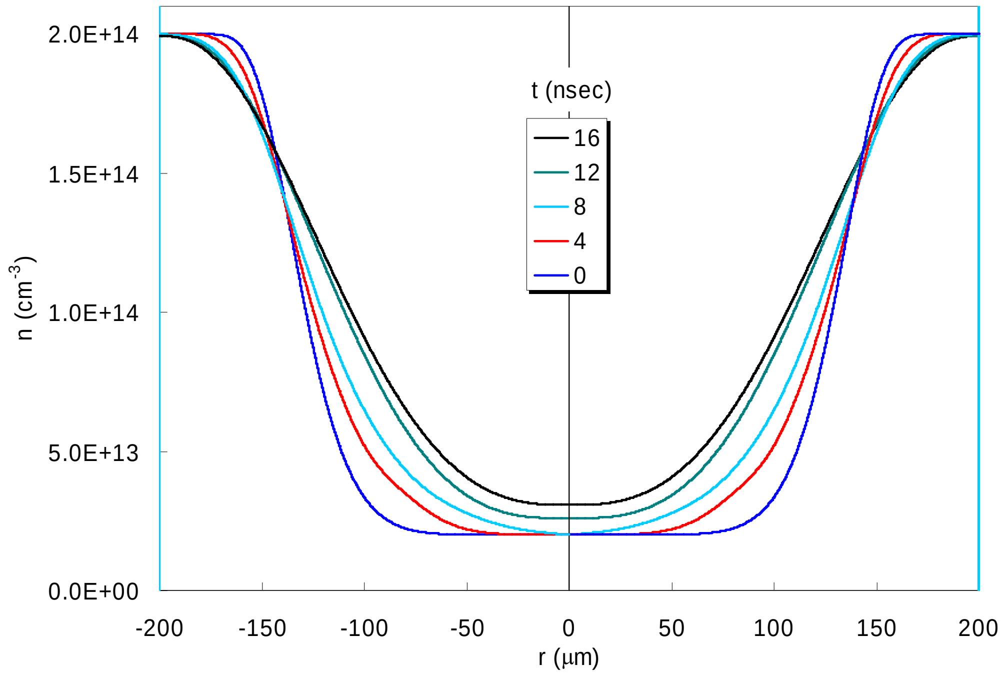


Fig. 5. Detail of the inner region of Fig. 3 (color online).

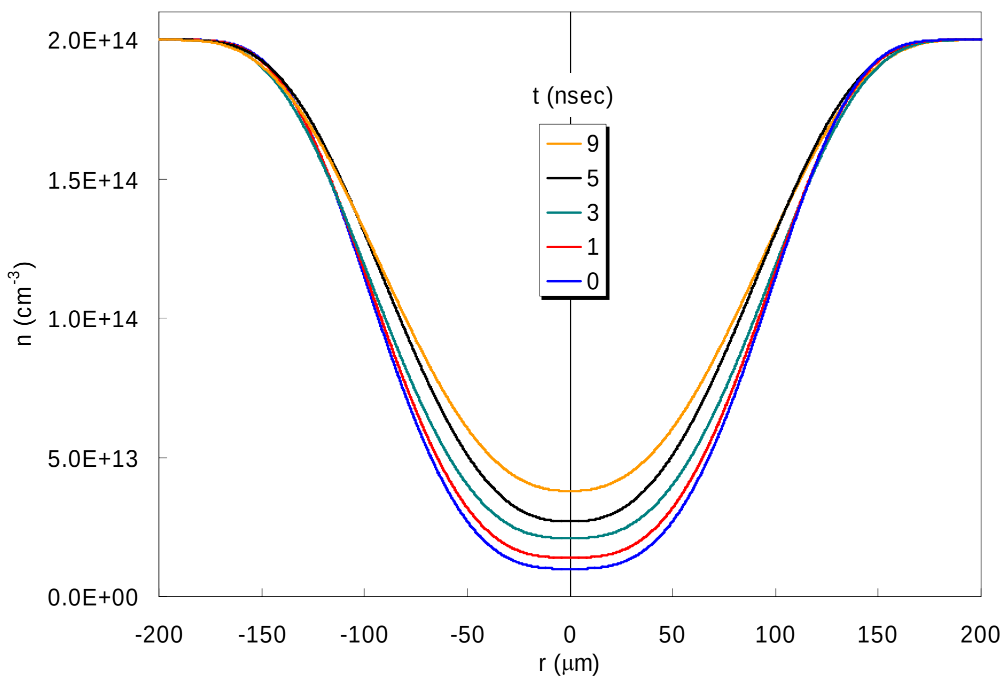


Fig. 6. Diffusion of a profile with r_0 = 200 μm, f = 0.05, s = 3, t = 6, and $n_0 = 2 \times 10^{14}$ cm^{-3}.

We are indebted to Chan Joshi for suggesting this problem, which turned out to be both interesting and non-trivial.

REFERENCES

[1] Blumenfeld et al., Nature **445**, 741 (2007).

[2] P. Chen, Phys. Rev. Lett. **54**, 693 (1985).

[3] W.P. Leemans et al., Phys. Rev. Lett. **68**, 321 (1992) ; Phys. Rev. A **46**, 1091 (1992).

[4] D.L. Bruhwiler, Phys. Plasmas **10**, 2022 (2003).

[5] C.L. O'Connell et al., Phys. Rev. Special Topics **9**, 101301 (2006).

[6] W. Lu, C. Huang, M. Zhou, W.B. Mori, and T. Katsouleas, Phys. Rev. Lett. 96, 165002 (2006).

[7] F.F. Chen, Intro. to Plasma Physics and Controlled Fusion, Vol. 1, 2nd ed. (Plenum Press, New York, 1984), Sec. 5.2.3.

[8] M.J. Hogan et al., Phys. Rev. Lett. **90**, 205002 (2003).

[9] B.E. Blue, Phys. Rev. Lett. **90**, 214801 (2003).

[10] D.L. Book, *NRL Plasma Formulary*, Naval Research Laboratory Publication 0084-4040 (1987).

[11] D.C. Lorenz, G. Black, and O. Heinz, Phys. Rev. **137**, A1049 (1965).

[12] B.P. Curry, Phys. Rev. A **1**, 166 (1970).

[13] Systat TableCurve 2D.