We thoroughly and critically review studies reporting the real (refractive index) and imaginary (absorption index) parts of the complex refractive index of silica glass over the spectral range from 30 nm to 1000 \( \mu \)m. The general features of the optical constants over the electromagnetic spectrum are relatively consistent throughout the literature. In particular, silica glass is effectively opaque for wavelengths shorter than 200 nm and larger than 3.5–4.0 \( \mu \)m. Strong absorption bands are observed (i) below 160 nm due to the interaction with electrons, absorption by impurities, and the presence of OH groups and point defects; (ii) at \( \sim 2.73–2.85, 3.5, \text{ and } 4.3 \mu \text{m} \) also caused by OH groups; and (iii) at \( \sim 9–9.5, 12.5, \text{ and } 21–23 \mu \text{m} \) due to Si—O—Si resonance modes of vibration. However, the actual values of the refractive and absorption indices can vary significantly due to the glass manufacturing process, crystallinity, wavelength, and temperature and to the presence of impurities, point defects, inclusions, and bubbles, as well as to the experimental uncertainties and approximations in the retrieval methods. Moreover, new formulas providing comprehensive approximations of the optical properties of silica glass are proposed between 7 and 50 \( \mu \)m. These formulas are consistent with experimental data and substantially extend the spectral range of 0.21–7 \( \mu \)m covered by existing formulas and can be used in various engineering applications. © 2007 Optical Society of America

1. Introduction

Silicon dioxide (SiO\(_2\) or silica) has many forms including three main crystalline varieties: quartz, tridymite, and cristobalite [1,2]. Silica can also exist in noncrystalline form as silica glass or vitreous silica [1], also referred to as amorphous silica and glassy silica. There are four basic types of commercial silica glasses [3,4]: (1) Type I is obtained by the electric melting of natural quartz crystal in vacuum or in an inert gas at low pressure, (2) Type II is produced from quartz crystal powder by flame fusion, (3) Type III is synthetic and produced by the hydrolyzation of SiCl\(_4\) when sprayed into an OH flame, and (4) Type IV is also synthetic and produced from SiCl\(_4\) in a water vapor-free plasma flame.

Each type of silica glass has its own impurity level and optical properties. For example, Type I silica glasses tend to contain metallic impurities [3,4]. On the other hand, Types III and IV are much purer than Type I and feature greater ultraviolet (UV) transmission [3,4]. However, Type III silica glasses have, in general, higher water content, and infrared (IR) transmission is limited by strong water absorption peaks at wavelengths around 2.2 and 2.7 \( \mu \)m [3–5]. Type IV is similar to Type III but contains less water and thus has better IR transmission. Suppliers provide various product grades for different optical applications [3,4,6]. Trade names for Type I silica glass are Infrasil, IR-Vitreosil, Pursil, and GE 105, for example. On the other hand, KU, Herosil, Homosil, Vitrasil, and OG Vitreosil are Type II. Moreover, KU-1, KV, Suprasil I, Tetrasil, Spectrosil, and Corning 7940 are known as synthetic fused silica and classified as Type III. Finally, KI, Suprasil W, Spec-
The term “fused quartz” is usually used for Type I and “fused silica” is used for synthetic fused silica, i.e., Types III or IV. In practice, however, those terms are often used interchangeably.

Because of its favorable physical, chemical, and optical characteristics, silica glass has been used in numerous applications: (1) as laboratory glassware, (2) in optics, as lenses or beam splitters, (3) for lighting and IR heating, (4) in telecommunications, as fiber optics, (5) in micro- and optoelectronics, as a dielectric insulator, a waveguide, photonic crystal fibers, or projection masks for photolithography, and (6) in thermal protection systems, as fibrous thermal insulation. In all these applications, optical properties are essential to predict and optimize the optical and thermal radiation performances of this material. Silica fiber optics, for example, are used in the near-IR at ~1.31 and 1.55 μm due to their low optical attenuation and optical dispersion [7,8]. In lens designing, one often needs to fit and interpolate refractive index data that are reported or measured at discrete wavelengths over certain spectral regions [9,10]. On the other hand, astronomers and atmospheric scientists are interested in the optical properties of interstellar and atmospheric silica particles in the mid- and far-IR region of the spectrum [11–13].

The complex refractive index, \( n_\lambda \), of silica glass at wavelength \( \lambda \) is defined as

\[
m_\lambda = n_\lambda + ik_\lambda,
\]

where \( n_\lambda \) is the refractive index and \( k_\lambda \) is the absorption index. The wavelength \( \lambda \) is related to other quantities such as frequency \( \nu \) and wavenumber \( \eta \) according to

\[
\lambda = \frac{c_\lambda}{\nu} = \frac{1}{\eta},
\]

where \( c_\lambda \) is the speed of light at wavelength \( \lambda \) in vacuum. Therefore, the refractive and absorption indices can also be expressed as functions of frequency and denoted by \( n_\nu \) and \( k_\nu \) or as functions of wavenumber and denoted by \( n_\eta \) and \( k_\eta \).

The experimental data for the refractive and absorption indices vary in precision depending on the measurement techniques used and on the approximations made in retrieving the intrinsic optical properties. One should also keep in mind that these optical properties may be sensitive to the presence of impurities, crystallinity, point defects, inclusions, bubbles, wavelength, temperature, and the glass manufacturing process. In addition, when considering the literature it is often difficult to choose the set of experimental data or formula to use and to assess its validity. The objectives of this paper are (1) to critically review and compare the experimental data reported in the literature for the complex refractive index of silica glass and (2) to develop formulas that provide a comprehensive approximation of the measured data near room temperature. Given the wide range of engineering and scientific applications, the spectral range from 30 nm to 1000 μm is considered.

2. Experimental Methods

A. Refractive Index, \( n_\lambda \)

Various experimental techniques and procedures have been used to retrieve the real part of the complex refractive index \( n_\lambda \). The most accurate is the minimum deviation angle method [14], which relies on measuring the minimum deviation angle \( \theta_{\text{min}} \) of an isosceles triangular prism made of the silica glass placed in air. This method is based on Snell’s law [14], and the refractive index \( n_\lambda \) can be estimated by

\[
n_\lambda = \frac{\sin \left( \frac{\theta_{\text{min}} + \phi}{2} \right)}{\sin \left( \frac{\phi}{2} \right)} n_{\text{air}},
\]

where \( \phi \) is an apex angle of the prism sample and \( n_{\text{air}} \) is the refractive index of air \( (n_{\text{air}} = 1) \). This method is often used to accurately measure the refractive index of highly transparent glass for which the absorption index \( k_\lambda \), i.e., the imaginary part of the complex refractive index, is negligibly small.

Alternatively, the interferometric method is also used to measure \( n_\lambda \). It is based on observing the interference fringes created when light is incident normally upon a silica glass plate [15,16]. Other techniques include the Abbe’s or the Pulfrich’s refractometers whose accuracy on the index of refraction is within ±2 × 10⁻³ and ±5 × 10⁻⁵, respectively [17].

Moreover, when absorption cannot be ignored, both \( n_\lambda \) and \( k_\lambda \) can be retrieved from the directional or hemispherical reflectance and/or emittance of a slab of known thickness. Electromagnetic wave theory can be used to retrieve both \( n_\lambda \) and \( k_\lambda \), assuming optically smooth surfaces and accounting for internal reflection [18]. Finally, the Kramers–Kröönig relations [18] can also be used to predict either the refractive index from the absorption index, or vice versa, at frequency \( \nu [18,19]:

\[
n_\nu = 1 + \frac{2}{\pi} P \int_0^\nu \frac{n_{\nu'}}{\nu'^2 - \nu^2} \, d\nu',
\]

\[
k_\nu = -\frac{2\nu}{\pi} P \int_0^\nu \frac{n_{\nu'}}{\nu'^2 - \nu^2} \, d\nu',
\]

where \( P \) denotes the Cauchy principle value of the integral [18]. Alternatively, the refractive and absorption indices can be simultaneously obtained from reflectance. First, the phase angle of the complex reflection coefficient \( \Theta \) at frequency \( \nu_0 \) can be expressed by the Kramers–Kröönig relations [20]:

\[
\frac{n_{\nu_0}}{\sin \Theta} = \frac{k_{\nu_0}}{\cos \Theta} = \frac{2}{\pi} P \int_0^{\nu_0} \frac{n_{\nu'}}{\nu'^2 - \nu^2} \, d\nu'.
\]
Equations (11)–(13) can be solved as a quadratic in the exponential factor in terms of \( k_\lambda \). After some algebraic manipulation, one obtains the following expression for \( k_\lambda \) as a function of the refractive index, \( n_\lambda \), the sample thickness, \( L \), and the spectral normal transmittance, \( T_{0,\lambda} \):

\[
k_\lambda = -\left( \frac{\lambda}{4\pi L} \right) \ln \left[ \frac{(1 - \rho_\lambda)^2 + 4\rho_\lambda^2 T_{0,\lambda} - (1 - \rho_\lambda)}{2\rho_\lambda^2 T_{0,\lambda}} \right].
\]  

Alternatively, the absorption index, \( k_\lambda \), can also be determined from measurements of the spectral normal emittance, \( \epsilon_{\lambda,0} \), by using the following expression [22]:

\[
k_\lambda = \left( \frac{\lambda}{4\pi L} \right) \ln \left[ \frac{1 - \rho_\lambda - \rho_\lambda \epsilon_{\lambda,0}}{1 - \rho_\lambda - \epsilon_{\lambda,0}} \right].
\]  

Note that the above expressions for the absorption index \( k_\lambda \) given by Eqs. (14) and (15) are valid if both \((n_\lambda - 1)\) and \((n_\lambda + 1)\) are much larger than \( k_\lambda \). In either case, an expression for \( n_\lambda \) is necessary to estimate \( \rho_\lambda \). As discussed later in this paper, the Sellmeier equation proposed by Malitson [23] can be used for that purpose between 0.21 and 6.7 \( \mu \)m.

In addition, in the UV and IR regions of the spectrum when silica glass is strongly absorbing, most reported values of \( n_\lambda \) and \( k_\lambda \) were retrieved from near-normal reflectance measurements in combination with the Kramers–Krönig relations [24].

3. Experimental Data and Discussion

Table 1 summarizes representative references reporting experimental values of the complex refractive index of silica glass at room temperature for the spectral range from 30 nm to 1000 \( \mu \)m. For each study, the measurement method, the spectral range, as well as the sample thicknesses, compositions, and temperatures investigated are also reported when available. In addition, the absorption index \( k_\lambda \) was derived from transmittance measurements using Eq. (14) if it was not directly reported. Then, computation from transmittance and emittance data sometimes leads to negative values, particularly in the spectral region from 0.2 to 4.0 \( \mu \)m where silica glass is very weakly absorbing. This was the case for transmittance data from [25,26]. Hence, in this region, the experimental data should be used with care since the uncertainty for \( k_\lambda \) is very large and \( k_\lambda \) effectively vanishes.

Figure 1 shows the real and imaginary parts of the refractive index of silica glass, \( n_\lambda \) and \( k_\lambda \), as a function of wavelength \( \lambda \) over the spectral range from 30 nm to 1000 \( \mu \)m as reported in the references listed in Table 1. Because of the density of data points in some parts of the spectrum and for the sake of clarity, Figs. 2–5 show details of both the real \( n_\lambda \) and imaginary \( k_\lambda \) parts of the complex index of refraction of silica glass for wavelengths between 30 nm and 1 \( \mu \)m, 1 and
<table>
<thead>
<tr>
<th>Reference</th>
<th>Wavelength Range (μm)</th>
<th>Measurement Method</th>
<th>Reported Data</th>
<th>Temperature</th>
<th>Sample Thickness</th>
<th>Type</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>[12]</td>
<td>2–500</td>
<td>Reflection, Kramers–Krönig method</td>
<td>( R, n, k )</td>
<td>300, 200, 100, and 10 K</td>
<td>N/A</td>
<td>III</td>
<td>Suprasil, UV grade synthetic fused SiO₂</td>
</tr>
<tr>
<td>[13]</td>
<td>7–300</td>
<td>Transmission</td>
<td>( n, k )</td>
<td>RT</td>
<td>N/A</td>
<td>N/A</td>
<td></td>
</tr>
<tr>
<td>[15]</td>
<td>110–550</td>
<td>Interferometric</td>
<td>( T, n, k )</td>
<td>RT</td>
<td>2.1409 mm</td>
<td>I</td>
<td>Infrasil (low H₂O)</td>
</tr>
<tr>
<td>[16]</td>
<td>3–6.7</td>
<td>Interferometric</td>
<td>( n, T )</td>
<td>RT</td>
<td>0.23 mm</td>
<td>III</td>
<td>Suprasil 2 – 1000 ppm OH content</td>
</tr>
<tr>
<td>[23]</td>
<td>0.21–3.71</td>
<td>Minimum deviation method</td>
<td>( n )</td>
<td>20 °C</td>
<td>N/A</td>
<td>III</td>
<td>Corning code 7940, Dynasil high-purity synthetic SiO₂ glass and GE type 151</td>
</tr>
<tr>
<td>[25]</td>
<td>0.80–2.60</td>
<td>( k ) from Eq. (14)</td>
<td>( T )</td>
<td>298 K</td>
<td>1.6 mm</td>
<td>N/A</td>
<td></td>
</tr>
<tr>
<td>[26]</td>
<td>0.19–0.42</td>
<td>( k ) from Eq. (14)</td>
<td>( T )</td>
<td>298 K</td>
<td>2 mm</td>
<td>N/A</td>
<td></td>
</tr>
<tr>
<td>[27]</td>
<td>0.05–0.7</td>
<td>Reflection and Kramers–Krönig</td>
<td>( n, e^* )</td>
<td>RT</td>
<td>2 mm</td>
<td>N/A</td>
<td></td>
</tr>
<tr>
<td>[28]</td>
<td>7.19–9.06</td>
<td>Interferometric</td>
<td>( n, T, R )</td>
<td>RT</td>
<td>0.1956 mm</td>
<td>IV</td>
<td>Suprasil W2</td>
</tr>
<tr>
<td>[29]</td>
<td>0.2–3</td>
<td>Reflection and transmission</td>
<td>( n, k, R, T )</td>
<td>24 °C</td>
<td>N/A</td>
<td>N/A</td>
<td></td>
</tr>
<tr>
<td>[30]</td>
<td>16.7–25</td>
<td>Reflection</td>
<td>( R, n, k )</td>
<td>RT</td>
<td>N/A</td>
<td>N/A</td>
<td></td>
</tr>
<tr>
<td>[44]</td>
<td>0.35–3.51</td>
<td>Minimum deviation method</td>
<td>( n )</td>
<td>24 °C</td>
<td>N/A</td>
<td>N/A</td>
<td>Samples from General Electric, Heraeus, Nieder Fused Quartz, and Corning Glass Works</td>
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<tr>
<td>[45]</td>
<td>1.44–4.77</td>
<td>Interferometric</td>
<td>( n, T )</td>
<td>23.5 °C–481 °C</td>
<td>0.1994 mm</td>
<td>IV</td>
<td>Suprasil W2</td>
</tr>
<tr>
<td>[49]</td>
<td>7.14–50</td>
<td>Reflection</td>
<td>( n, k )</td>
<td>RT</td>
<td>N/A</td>
<td>II</td>
<td>KU and KI</td>
</tr>
<tr>
<td>[50]</td>
<td>0.0006–500</td>
<td>Kramers–Krönig relation</td>
<td>( n, k )</td>
<td>RT</td>
<td>N/A</td>
<td>N/A</td>
<td>Compilation of data</td>
</tr>
<tr>
<td>[51]</td>
<td>1.00–7.5</td>
<td>Transmission</td>
<td>( T, k )</td>
<td>RT</td>
<td>5 μm to 3.06 m</td>
<td>N/A</td>
<td></td>
</tr>
<tr>
<td>[54]</td>
<td>0.10–0.16</td>
<td>Reflection and Kramers–Krönig</td>
<td>( n, k )</td>
<td>RT</td>
<td>N/A</td>
<td>N/A</td>
<td>Samples produced by Electro-Quartz</td>
</tr>
<tr>
<td>[55]</td>
<td>1.35–4.85</td>
<td>Interferometric</td>
<td>( n, T )</td>
<td>23.5 °C</td>
<td>0.2345 mm</td>
<td>IV</td>
<td>Suprasil W2</td>
</tr>
<tr>
<td>[56]</td>
<td>8.13–9.63</td>
<td>Reflection</td>
<td>( R, n, k )</td>
<td>RT</td>
<td>N/A</td>
<td>N/A</td>
<td></td>
</tr>
<tr>
<td>[57]</td>
<td>7.84–12.9</td>
<td>Reflection</td>
<td>( n, k )</td>
<td>RT</td>
<td>N/A</td>
<td>N/A</td>
<td></td>
</tr>
<tr>
<td>[58]</td>
<td>0.23–3.37</td>
<td>Minimum deviation method</td>
<td>( n )</td>
<td>26 °C, 471 °C, and 828 °C</td>
<td>N/A</td>
<td>III</td>
<td>Corning 7940</td>
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<tr>
<td>[59]</td>
<td>7.14–11.11</td>
<td>Reflection, transmission, and Kramers–Krönig relation</td>
<td>( n, k )</td>
<td>RT</td>
<td>N/A</td>
<td>II</td>
<td>KU</td>
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<td>[60]</td>
<td>7.14–14.29</td>
<td>Reflection</td>
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<td>RT</td>
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<td>Infrasil</td>
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<td>[61]</td>
<td>0.5–4.5</td>
<td>Minimum deviation</td>
<td>( n )</td>
<td>RT</td>
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<td>N/A</td>
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<tr>
<td>[62]</td>
<td>0.37, 0.44, 0.55, 1.01, and 1.53</td>
<td>Minimum deviation</td>
<td>( n )</td>
<td>294, 240, 180, and 120 K</td>
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<td>N/A</td>
<td></td>
</tr>
<tr>
<td>[63]</td>
<td>0.06–40</td>
<td>( n, k )</td>
<td>RT</td>
<td>N/A</td>
<td>N/A</td>
<td></td>
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<tr>
<td>[64]</td>
<td>0.2–3.5</td>
<td>Transmission</td>
<td>( k, T )</td>
<td>RT—1500 °C</td>
<td>0.953 mm</td>
<td>I, III</td>
<td>GE IR type 105, GE UV type 151, and Corning Vycor IR Glass Number 7905</td>
</tr>
<tr>
<td>[65]</td>
<td>2.00–6.00</td>
<td>( k ) from Eq. (14)</td>
<td>( T )</td>
<td>25 °C and 400 °C</td>
<td>2.8 mm</td>
<td>I</td>
<td>GE Type 105</td>
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<tr>
<td>[66]</td>
<td>0.31–3.97</td>
<td>( k ) from Eq. (14)</td>
<td>( T )</td>
<td>RT</td>
<td>5.45 mm</td>
<td>N/A</td>
<td></td>
</tr>
<tr>
<td>Reference</td>
<td>Wavelength (nm)</td>
<td>Optic</td>
<td>Type</td>
<td>Details</td>
<td></td>
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<td>[67]</td>
<td>0.16–0.30</td>
<td></td>
<td></td>
<td>from Eq. (14)</td>
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<tr>
<td>[68]</td>
<td>1.00–4.62</td>
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<td></td>
<td>from Eq. (14)</td>
<td></td>
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<td></td>
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<td>[69]</td>
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<td>[70]</td>
<td>7.41–50</td>
<td></td>
<td></td>
<td>Reflection and dispersion analysis</td>
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<td>[71]</td>
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<td></td>
<td>Transmission and reflection</td>
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<td></td>
<td>Transmission</td>
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<td>[73]</td>
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<td>[74]</td>
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<td>Transmission</td>
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<tr>
<td>[75]</td>
<td>0.029–1.77</td>
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<td>Transmission, reflection, and Kramers–Krönig</td>
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<td>[77]</td>
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<td>Internal reflection and Kramers–Krönig</td>
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<tr>
<td>[78]</td>
<td>2–35</td>
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<td></td>
<td>Reflection and Kramers–Krönig</td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

- **RT** is room temperature.
- *b* Reference [24].

**Overall:** The reported values for the real and imaginary parts of the complex index of refraction are unreliable [24]. For example, data reported by Ellis et al. [27] agree relatively well with one another. However, discrepancies larger than those for some authors, especially for wavelengths below 9 nm, are consistent among all experimental data except for those reported by Tan [28], Khosravi and Nassif [29] (0.2 μm < λ < 2.2 μm). The data for wavelengths below 9 nm as reported by various authors are considered less reliable. For wavelengths between 9 and 50 μm, the data agree considerably less well. The data above 50 μm have been reported for various authors.

- For wavelengths below 9 nm, the data reported by Tan [28], Khosravi and Nassif [29] (0.2 μm < λ < 2.2 μm) agree relatively well with one another. However, the reported values are generally considered less reliable. For wavelengths above 50 μm, the data agree considerably less well. The data for wavelengths between 9 and 50 μm have been reported for various authors.
Fig. 1. Real $n_\ell$ and imaginary $k_\ell$ parts of the complex refractive index of silica glass reported in the literature and summarized in Table 1. The solid curve (present study) was obtained with Eqs. (21)–(24) by using coefficients listed in Table 2.
Fig. 2. Real $n$, and imaginary $k$, parts of the complex refractive index of silica glass between 30 nm and 1 μm as reported in the literature and summarized in Table 1.
Fig. 3. Real $n$, and imaginary $k$, parts of the complex refractive index of silica glass between 1 and 15 $\mu$m as reported in the literature and summarized in Table 1. The solid curve (present study) was obtained with Eqs. (21)–(24) by using coefficients listed in Table 2.
Fig. 4. Real $n_\lambda$ and imaginary $k_\lambda$ parts of the complex refractive index of silica glass between 15 and 100 $\mu$m as reported in the literature and summarized in Table 1. The solid curve (present study) was obtained with Eqs. (21)–(24) by using coefficients listed in Table 2.
Fig. 5. Real $n_i$ and imaginary $k_i$ parts of the complex refractive index of silica glass between 100 and 1000 μm as reported in the literature and summarized in Table 1.
of silica glass for photomask material in 157 nm photolithography using F$_2$ excimer lasers [32,37,38]. An acceptable transmittance at $\sim$ 157 nm has been achieved by minimizing the OH content of silica [37] or by doping silica glass with network modifiers such as fluorine, which relaxes the glass structure and eliminates strained Si--O--Si bonds [33]. Experimental measurements and theoretical calculations of the electronic structure of SiO$_2$ has been reviewed by Griscom [39] and spectroscopic data for wavelengths between 90 and 350 nm have been discussed by Sigel [34]. An interaction between the UV radiation and electrons and point defects is also responsible for the steep increase of $n_\lambda$ for wavelengths less than 300 nm.

In the IR part of the spectrum, silica glass is effectively opaque for a wavelength larger than 3.5--4.0 $\mu$m. Beyond this wavelength, three major absorption bands can be observed (Fig. 1) due to the resonance of Si--O--Si vibrations. The absorption peak between 9.0 and 9.5 $\mu$m can be attributed to the asymmetric stretching vibration of Si--O--Si bridges [17,24]. The absorption band at $\sim$ 12.5 $\mu$m is due to the symmetric stretching vibration of the Si--O--Si bridge involving the displacement of the O atom perpendicular to the Si--Si direction in the Si--O--Si plane [24]. The third band between 21 and 23 $\mu$m is the consequence of the O--Si--O bending vibration but has also been attributed to the “rocking” mode of Si--O--Si bonds caused by the displacement of an oxygen atom out of the Si--O--Si plane [24]. The resonance of Si--O--Si vibrations are also responsible for the sharp decreases in $n_\lambda$ around the resonance wavelengths [17]. The reader is referred to [24, pp. 63--77] for a detailed discussion on vibrational spectroscopy of silica glass at these wavelengths. Moreover, smaller absorption bands at $\sim$ wavelengths 2.73--2.85, 3.5, and 4.3 $\mu$m correspond to the presence of OH groups in the structure of the glass [17,40--42]. The magnitude of the absorption depends on the melting technology and in particular on the partial pressure of water vapor above the melt during the melting process [17]. The concentration of OH groups in silica glass can be computed from the absorption band at $\sim$ wavelength 2.73--2.85 $\mu$m [17]. To the best of our knowledge, no model or approximate equation has been proposed for the absorption index of silica glass. This is the subject of Section 4.

4. Optical Constant Theory

The complex index of refraction, $m_\lambda = n_\lambda + ik_\lambda$, and the complex relative dielectric permittivity, $\epsilon(\lambda) = \epsilon'(\lambda) + i\epsilon''(\lambda)$ are related by the expression $\epsilon(\lambda) = m_\lambda^2$, i.e., [21]

$$\epsilon'(\lambda) = n_\lambda^2 - k_\lambda^2, \quad \epsilon''(\lambda) = 2n_\lambda k_\lambda.$$  

Numerous physical models such as the Lorentz model, the Drude model, and the Debye relaxation model have been proposed to predict the optical properties of solids [18]. The Lorentz model assumes that electrons and ions in the material are harmonic oscillators subject to the force applied by time-dependent electromagnetic fields. Then the complex relative dielectric permittivity can be expressed in terms of frequency $\nu$ as follows [21]:

$$\epsilon(\nu) = 1 + \frac{\nu_{pj}^2}{\nu_j^2 - \nu^2 - i\gamma_j \nu} = 1 + \frac{\nu_{pj}^2(\nu_j^2 - \nu^2) + i\gamma_j \nu_{pj}^2 \nu}{(\nu_j^2 - \nu^2 + \gamma_j^2 \nu^2)},$$  

(17)

where $\nu_{pj}$ and $\nu_j$ are the plasma and resonance frequencies, respectively. The parameter $\gamma_j$ is the damping factor of the oscillators. Only when $\nu$ is very close to one of the resonance frequencies $\nu_j$, the imaginary terms in Eq. (17) are important [43]. Thus, $\gamma_j$ is negligibly small compared with $(\nu_j^2 - \nu^2)$ for silica glass for a wavelength below 7 $\mu$m and $\epsilon''$ is virtually equal to 0.0. Hence, after substituting Eq. (2) into Eq. (17), $\epsilon$ can be expressed in terms of $\lambda$ as follows:

$$\epsilon(\lambda) = \epsilon'(\lambda) = 1 + \sum_j \frac{A_j^2 \lambda^2}{(\lambda^2 - \lambda_j^2)},$$  

(18)

where $A_j = v_{pj} \lambda_j/\epsilon_j$ with $\lambda_j$ being the resonance wavelength. Moreover, as $\epsilon'(\lambda)$ vanishes, the medium is weakly absorbing and $k_\lambda$ is negligibly small compared with $n_\lambda$. Then $\epsilon'(\lambda)$ is equal to $n_\lambda^2$ and given by the Sellmeier dispersion formula:

$$\epsilon'(\lambda) = n_\lambda^2 = 1 + \sum_j \frac{A_j^2 \lambda^2}{(\lambda^2 - \lambda_j^2)}.$$  

(19)

Different formulas for the refractive index of silica glass as a function of wavelength and based on the Sellmeier dispersion formula have been proposed in the literature [23,44,45] for different spectral regions. Rodney and Spindler [44] suggested a formula for $n_\lambda$ over a spectral range from 0.347 to 3.508 $\mu$m at 20 °C. Rodney and Spindler [44] suggested a formula for $n_\lambda$ over a spectral range from 0.347 to 3.508 $\mu$m at 20 °C. Malitson [23] fitted experimental data with the following three-term Sellmeier equation:

$$n_\lambda^2 = 1 + \frac{0.6961663 \lambda^2}{\lambda^2 - (0.0684043)^2} + \frac{0.4079426 \lambda^2}{\lambda^2 - (0.1162414)^2} + \frac{0.8974794 \lambda^2}{\lambda^2 - (9.896161)^2}.$$  

(20)

Tan [16] confirmed the validity of Eq. (20) for wavelengths up to 6.7 $\mu$m. Furthermore, for a spectral range over 8 $\mu$m, an approximate piecewise linear fit was given by Dombrovsky [46]. However, no physics-
Based on formulas have been developed for the spectral range beyond 8 \( \mu \text{m} \).

In parts of the spectrum where \( k_\lambda \) cannot be neglected or when the frequency \( \nu \) is very close to the resonance frequencies, the Sellmeier equation for \( n_\lambda \) is no longer valid and an alternative model must be used. Recently, Meneses et al. [47] proposed a new dielectric function model based on the causal version of the Voigt function. The model was validated by fitting the IR spectra of two different glasses and confirmed to be more appropriate than the Lorentz model [47]. Moreover, the authors proposed another simplified model based on Gaussian functions [48]. Then, the dielectric constant can be written as

\[
\epsilon(\eta) = \epsilon'(\eta) + i\epsilon''(\eta) = \epsilon_0 + \sum_j \left[ g^{kk \text{g}}_j(\eta) + ig'_j(\eta) \right],
\]

where the high frequency dielectric constant is denoted by \( \epsilon_0 \). In addition, the Gaussian functions \( g_j(\eta) \) and \( g^{kk \text{g}}_j(\eta) \) are defined as

\[
g_j(\eta) = \alpha_j \exp\left[-4 \ln 2 \left( \frac{\eta - \eta_0}{\sigma_j} \right)^2 \right] - \alpha_j \times \exp\left[-4 \ln 2 \left( \frac{\eta + \eta_0}{\sigma_j} \right)^2 \right],
\]

\[
g^{kk \text{g}}_j(\eta) = \frac{2\alpha_j}{\pi} \left[ D(2, \ln 2 \frac{\eta + \eta_0}{\sigma_j}) - D(2, \ln 2 \frac{\eta - \eta_0}{\sigma_j}) \right].
\]

Here, \( \alpha_j \) is the amplitude, \( \eta_0 \) is the peak position, \( \sigma_j \) is the full width at half-maximum, and \( D(x) \) is an operator defined as

\[
D(x) = \frac{1}{\Gamma(\frac{x}{2}) \Gamma(\frac{1-x}{2})} \int_0^1 \frac{t^{(x-1)/2} (1-t)^{\frac{1-x}{2}}}{e^t + 1} \, dt
\]

\[
D(x) = \frac{1}{\Gamma(\frac{x}{2}) \Gamma(\frac{1-x}{2})} \int_0^1 \frac{t^{(x-1)/2} (1-t)^{\frac{1-x}{2}}}{e^t + 1} \, dt
\]

Table 2. Parameters Used to Interpolate the Refractive Index \( n_\lambda \) and Absorption Index \( k_\lambda \) of Silica Glass by Using Eqs. (21)–(24)\(^\text{a,b}\)

<table>
<thead>
<tr>
<th>( j )</th>
<th>( \alpha_j )</th>
<th>( \eta_0_j )</th>
<th>( \sigma_j )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.7988</td>
<td>1089.7</td>
<td>31.454</td>
</tr>
<tr>
<td>2</td>
<td>0.46089</td>
<td>1187.7</td>
<td>100.46</td>
</tr>
<tr>
<td>3</td>
<td>1.2520</td>
<td>797.78</td>
<td>91.601</td>
</tr>
<tr>
<td>4</td>
<td>7.8147</td>
<td>1058.2</td>
<td>63.153</td>
</tr>
<tr>
<td>5</td>
<td>1.0313</td>
<td>446.13</td>
<td>275.111</td>
</tr>
<tr>
<td>6</td>
<td>5.3757</td>
<td>443.00</td>
<td>45.220</td>
</tr>
<tr>
<td>7</td>
<td>6.3305</td>
<td>465.80</td>
<td>22.680</td>
</tr>
<tr>
<td>8</td>
<td>1.2948</td>
<td>1026.7</td>
<td>232.14</td>
</tr>
</tbody>
</table>

\(^{a} \epsilon_0 = 2.1232. \)

\(^{b} \)These parameters were obtained by fitting the equations to data of Popova et al. [49].

Fig. 6. Residuals between experimental [49] and predicted values of \( n_\lambda \) and \( k_\lambda \). The predicted values are based on Eqs. (21)–(24) with coefficients listed in Table 2.
Fig. 7. Residuals between the experimental data on the refractive index and absorption index and values predicted in this work by using Eqs. (21)–(24) along with coefficients listed in Table 2.
In the present study, this model is used to interpolate the refractive index $n_\lambda$ and absorption index $k_\lambda$ for wavelengths $\lambda$ between 7 and 50 $\mu$m. It enables one to describe the experimental data with a reduced set of parameters [48] over a wide spectral range, including the spectral range where $k_\lambda$ may be large. Note that the above simplified model satisfies the Kramers–König relation.

The practical procedure for fitting complex refractive index data conducted in this paper is as follows: (i) the spectral reflectivity at normal incidence, $R(\eta)$, is computed from Eq. (7), (ii) parameters of $\epsilon(\eta)$ in Eqs. (21) and (22) are determined by curve fitting for $R(\eta)$ expressed as

$$ R(\eta) = \frac{\sqrt{\epsilon(\eta) - 1}}{\sqrt{\epsilon(\eta) + 1}}, $$

and (iii) $n_\lambda$ and $k_\lambda$ are computed using Eq. (16). The advantage of this procedure is that (a) fitting the reflectivity is easier than fitting $n_\lambda$ and $k_\lambda$ independently, (b) both $n_\lambda$ and $k_\lambda$ can be derived from a single curve fitting, and (c) the result automatically satisfies the Kramers–König relations.

The experimental data of Popova et al. [49] were selected to develop formulas for both $n_\lambda$ and $k_\lambda$ because these data cover a wide spectral range from 7 to 50 $\mu$m, and both the refractive and absorption indices are reported at the same wavelength enabling the calculation of $R(\eta)$. In the spectral region from 0.2 to 7 $\mu$m, the absorption index of silica glass is very small and may be assumed to be zero for all practical purposes as suggested by Fig. 1. Moreover, reported data, including that of Popova et al. [49], indicates that the refractive index is satisfactorily predicted by the Sellmeier formula reported by [16] and Malitson [23] between 0.2 and 7 $\mu$m and given by Eq. (20). Therefore, the present study focuses on the spectral range between 7 and 50 $\mu$m.

To fit the model with experimental data, the FOCUS software was used [47,48]. By adding terms in Eq. (21) one by one, eight terms were found to best fit the experimental data. The fitting curves obtained in this study are shown in Figs. 3 and 4, and the associated parameters $\alpha_j$, $\eta_0_j$, and $\sigma_j$ used in Eqs. (21)–(23) are summarized in Table 2. The fitting curves for $n_\lambda$ and $k_\lambda$ obtained in the present study agree well with the data of Popova et al. [49] and with most of other data shown in Figs. 3 and 4. The differences (or residuals) between the data of Popova et al. [49] and the model predictions for both $n_\lambda$ and $k_\lambda$ using the parameters given in Table 2 are shown in Fig. 6. It indicates that the residuals are less than 0.06 except near 9 and 22.5 $\mu$m, where they reach up to 0.3 around 9 $\mu$m and 0.14 around 22.5 $\mu$m. This can be attributed to the fact that the refractive index changes greatly in those wavelength regions. However, these residuals are small compared with much larger difference observed in the experimental data reported for $n_\lambda$. For example, one can see differences larger than 0.5 among reported refractive index data in the spectral range between 9 and 10 $\mu$m. Thus, the approximation obtained here is acceptable and can be useful in engineering applications.

Moreover, the residuals between the experimental data for the refractive and absorption indices listed in Table 1, and the model predictions using the parameters given in Table 2 are shown in Fig. 7. Here again, the residuals are less than 0.3 except at $\sim$9.0 and 22.5 $\mu$m. This indicates that the model predictions using the parameters given in Table 2 agree with all data reported in the literature at the spectral range except at $\sim$9 and 22.5 $\mu$m. However, large discrepancies in reported experimental data sets can be observed around these two wavelengths. In general, the model should be used with care when applied outside the spectral range from 0.2 to 50 $\mu$m.

Finally, given its widespread use, particular attention was paid to the compilation of data reported by Philipp [50] for the refractive and absorption indices of silica glass over the spectral range from 0.006 to 500 $\mu$m. The compilation consists of experimental data reported by various authors [15,20,51–54] as well as unpublished data. Philipp [50] also retrieved optical properties from the absorption coefficient as well as computer generated values. Qualitatively, good agreement between the data compiled by Philipp [50] and the other data is observed in Figs. 2–5. Moreover, residuals with the present model range from $-0.42$ to 0.19 for $n_\lambda$ and $-0.25$ to 0.37 for $k_\lambda$ from 7 to 50 $\mu$m.

5. Conclusions

A thorough review of experimental data for the complex refractive index of silica glass at near room temperatures over a spectral range from 30 nm to 1000 $\mu$m implies that the values reported in the literature can vary significantly due to numerous sample features and experimental methods and conditions. Hence, it is essential to report the silica glass synthesis method, composition, impurity, and defects level, sample thickness, surface roughness, and temperatures as well as the retrieval method and the underlying assumptions when one reports optical properties of glass. However, the general features of the complex refractive index spectra are relatively consistent throughout the region of the spectrum considered. Silica glass is effectively opaque for wavelengths shorter than 200 nm and larger than 3.5–4.0 $\mu$m. Strong absorption bands are observed (i) below 160 nm due to interaction with electrons, absorption by impurities, and the presence of OH groups and point defects, (ii) at $-2.73$–2.85, 3.5, and 4.3 $\mu$m also caused by OH groups, (ii) at $-9$–9.5, 12.5, and 21–23 $\mu$m due to Si—O—Si resonance modes of vibration. New formulas for both the real and imaginary parts of the complex refractive index are pro-
posed over a wide spectral range between 7 and 50 μm, thus complementing the existing analytical formula for $n_0$ in the range of 0.21–7 μm [16,23]. The imaginary part of the complex refractive index can be neglected in much of this range (0.21–4 μm). The differences between various experimental data are comparable and greater than the differences between the results of these formulas and the experimental data used to develop them. Hence it is believed that the formulas proposed are useful for practical engineering applications such as simulations and optimizations of optical and thermal systems.

The data collected and presented in this study are available in digital form online [79] or directly from the corresponding author upon request.

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